

Detecting the traders' strategies in minority–majority games and real stock-prices

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Abstract

Price dynamics is analyzed in terms of a model which includes the possibility of effective forces due to trend followers or trend adverse strategies. The method is tested on the data of a minority–majority model and indeed it is capable of reconstructing the prevailing traders' strategies in a given time interval. Then we also analyze real (NYSE) stock-prices dynamics and it is possible to derive an indication for the "sentiment" of the market for time intervals of at least one day. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

The simplest representation of price dynamics is usually considered as a simple random walk (RW). It is easy to realize, however, that many important deviations are also present. The most studied are the problems of the "fat tails" (in the distribution of price returns), the volatility clustering and various other elements related to the nonstationarity of the process [1,2]. The arbitrage condition implies that no simple correlations can be present. A large effort is therefore devoted to the identification of complex correlations of various types. These correlations arise from the collective behavior of traders, which lastly, define the price.

In this perspective a simple classification of trading strategies can be made in terms of trend followers or trend adverse. Usually these different strategies are taken as input in models which represent the behavior of traders.

Here we would like to consider the complementary point of view. Namely, given a time series, is it possible to identify, from the data, the strategies of the traders? In order to address this question we use a new approach which is based on a RW plus a force which depends on the distance of the price from some suitable moving averages [3,4]. This idea is that, with such an analysis, one can identify the "sentiment" of the market in a given time interval.

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In this paper, we first perform some statistical tests of the method to clear its signal to noise ratio. Then we apply the method to time series generated by a minority–majority model [5]. This is an important test because, in this case, one knows the prevailing strategy of the traders. The results are rather encouraging because the method can indeed identify these strategies. Finally we apply the method to real stock prices data of the NYSE and the preliminary results show that it is possible to derive statistically significant information on the prevailing trading strategy for a single day or larger time periods.

2. The effective potential model

In recent papers [3,4], the idea that the stock-price dynamics can be influenced by a moving average of the price itself in the previous time steps has been introduced. Hence, at every time step t , one can introduce a moving average of the previous M time steps

$$P_M(t) = \frac{1}{M} \sum_{\tau=0}^{M-1} P(t - \tau). \quad (1)$$

In Fig. 1 is plotted the time evolution of a RW with Gaussian random noise together with the moving average of the price. One can investigate if there could be a relation between the next price increment $P(t+1) - P(t)$ and the difference $P(t) - P_M(t)$. The simplest assumption is to adopt a linear dependence

$$P(t+1) - P(t) \propto P(t) - P_M(t). \quad (2)$$

In this case, the price dynamics can be described in terms of a RW with the existence of a linear force. This force can be either repulsive or attractive depending on the sign of the constant of proportionality between $P(t+1) - P(t)$ and $P(t) - P_M(t)$. Therefore, the dynamical equation of the price is a RW with the presence of a force that is the gradient of a quadratic potential Φ [4],

$$P(t+1) = P(t) - b(t) \frac{d}{dP(t)} \Phi(P(t) - P_M(t)) + \omega(t), \quad (3)$$

where $\omega(t)$ corresponds to a random noise with unitary variance and zero mean. $P_M(t)$ is the moving average described in Eq. (1).

The potential Φ together with the pre-factor $b(t)$ describe the interaction between the price and the moving average. In simple assumption of a linear force [4], Φ results to be quadratic:

$$\phi(P(t) - P_M(t)) = (P(t) - P_M(t))^2. \quad (4)$$

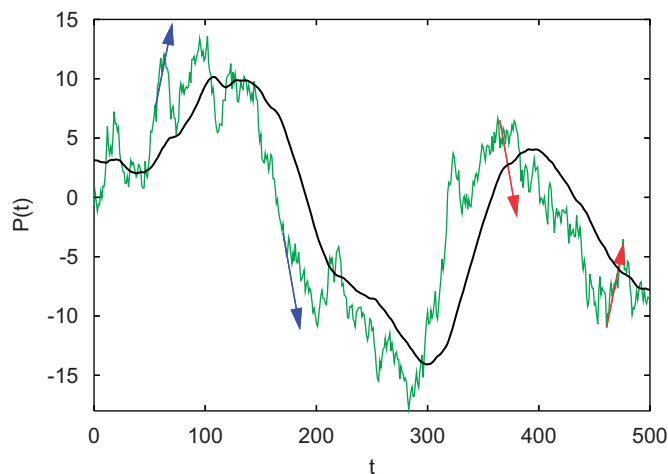


Fig. 1. Example of a model of price dynamics (in this case a simple RW) together with its moving average defined as the average over the previous 50 points. The idea is that the distance of the price from its moving average can lead to repulsive (blue arrows) or attractive (red arrows) effective forces.

We can simulate a process whose dynamical stochastic equation is given by Eq. (3). The time evolution of the “price” of such a process, is shown in Fig. 2, where we can observe three cases in which the potential is attractive, repulsive and constant (simple RW). From this simulation, one can reconstruct the force of the process plotting $P(t + 1) - P(t)$ as a function of $P(t) - P_M(t)$. Then, integrating from the center, one can obtain the potential. In Fig. 3 are shown the potentials obtained from a simulation of the process described in Eq. (3) in the case of attractive force for various values of M . We can observe that the potentials have an amplitude (that is the slope of the linear force) which depends on M . In Ref. [4] is shown that such a dependence can be eliminated rescaling the potential by a factor $(M - 1)$.

In Fig. 4 are shown the potentials plotted in Fig. 3, rescaled by the factor $(M - 1)$. Indeed we can observe a good data collapse.

This idea of assuming a linear force in Eq. (3) has been tested on real data. In Ref. [4] a series of data from the Yen–Dollar exchange rates have been analyzed. The potential analysis for the case of the Yen–Dollar exchange rates indeed leads to the observation of rather quadratic potentials. Anyhow, other kind of force can

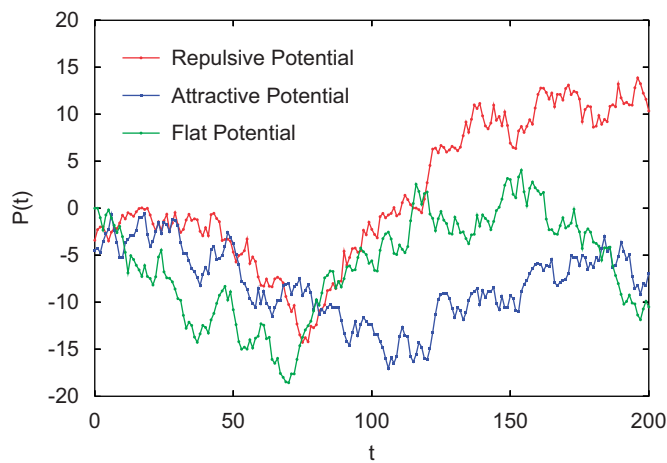


Fig. 2. Time evolution of the price described by Eq. (3). Three different behaviors are plotted. The red line represents the time evolution of a RW in a repulsive quadratic potential while the blue line is in an attractive quadratic potential. The green line is the case of flat potential (simple RW). The parameters are fixed to $M = 20$ and $b = \pm 1$. We can observe an over diffusion (under diffusion) in the case of repulsive (attractive) potential.

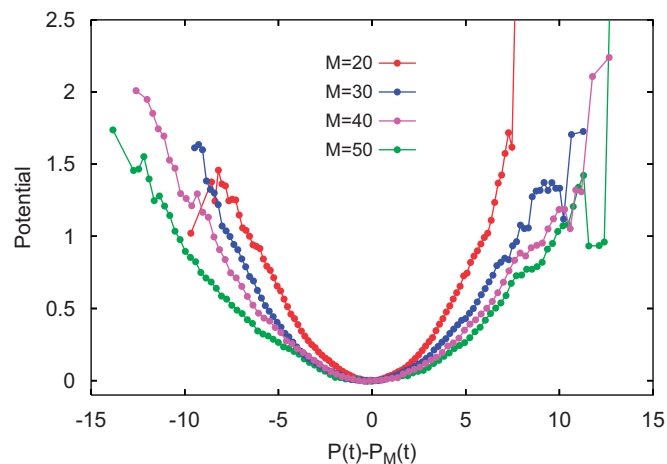


Fig. 3. The plot shows the shapes of the quadratic attractive potentials defined by Eqs. (3) and (4). We can see that the amplitude of the potentials depends on the choice of the parameter M .

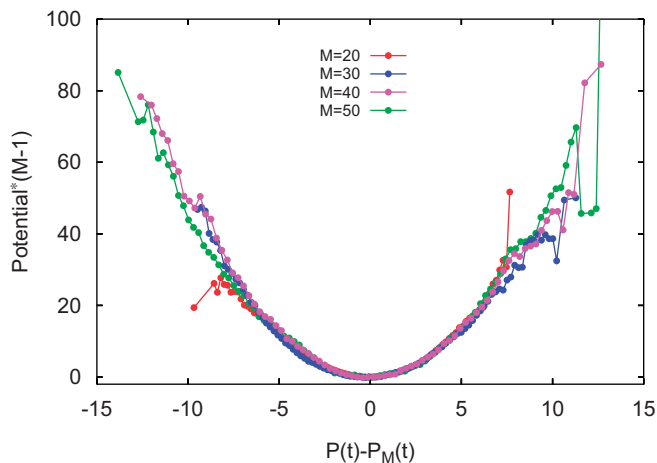


Fig. 4. The different potential plotted in Fig. 3 are re-plotted scaling the curves with the factor $(M - 1)$. We can see that in this way we obtain a good data collapse.

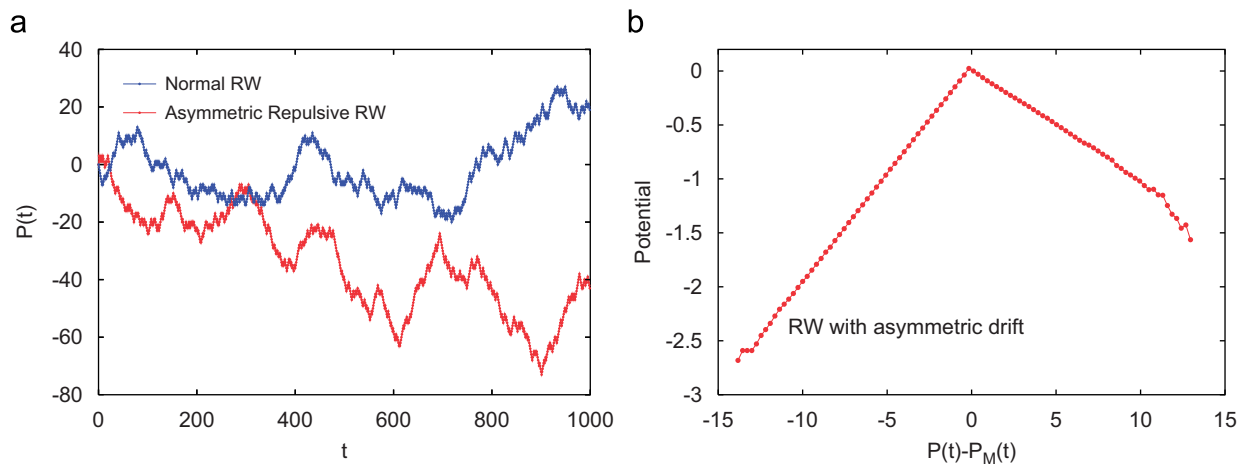


Fig. 5. In figure (a) is plotted the time evolution the price whose dynamics is described by Eqs. (5) and (6), compared with the time evolution of a simple RW. In figure (b) is plotted the shape of the potential obtained analyzing the data plotted in (a). We can see a piecewise linear shape because the relative model depends only on the sign of the difference $P(t) - P_M(t)$. The slope of the two lines of the potential depends on the parameters ε_1 and ε_2 .

be considered. For example one can suppose that the price dynamics could depend only on the sign of the difference $P(t) - P_M(t)$. In this case an interesting model is represented by the following dynamics for a RW with only up and down steps [3].

$$\begin{cases} p(\uparrow) = 1/2 + \varepsilon_1 & \text{for } P(t) - P_M(t) > 0, \\ p(\downarrow) = 1/2 - \varepsilon_1, \end{cases} \quad (5)$$

$$\begin{cases} p(\uparrow) = 1/2 - \varepsilon_2 & \text{for } P(t) - P_M(t) < 0, \\ p(\downarrow) = 1/2 + \varepsilon_2. \end{cases} \quad (6)$$

This model implies a tendency of destabilization (or stabilization) depending on the signs of ε_1 and ε_2 . The potential analysis for this case leads to a piecewise linear potential in which the slopes are related to ε_1 and ε_2 . The potential will be asymmetric if $\varepsilon_1 \neq \varepsilon_2$. In Fig. 5(a) and (b) are shown the time evolutions of a price whose

dynamical equations is given by Eqs. (5) and (6) in the case of asymmetric repulsive potential, and the relative shape of the potential.

In order to test this model, we have considered an agent based model and we have performed the potential analysis on the time series of the theoretical price that comes from the simulations. In the next section we are going to describe the specific rules of the agent based model that we have considered.

3. Application and test on an agent-based model

It is instructive to analyze the effective potential scenario in agent-based models, where the price process is not defined explicitly but only through the aggregate choices of a group of traders. The simplest and most studied framework from a statistical physics perspective is that of minority games [6,7], in which each of N agents must decide at every (discrete) time step whether to buy ($a_i(t) = 1$) or sell ($a_i(t) = -1$) an asset. The resulting price process is determined by the decisions of all agents through the “excess demand” $A(t) = \sum_{i=1}^N a_i(t)$. In particular, neglecting liquidity effects for the sake of simplicity, one can write that

$$P(t + 1) - P(t) = A(t), \tag{7}$$

which amounts to defining the (log-)price as $P(t) = \sum_{t' < t} A(t')$. It is clear that an agent’s trading behavior will depend on his expectations about the future price increment $A(t)$, denoted by $\mathbb{E}_i[A(t)]$. For example, it has been argued [8] that if

$$\mathbb{E}_i[A(t)] = \psi_i A(t - 1) + (1 - \psi_i) A(t - 2), \tag{8}$$

agent i behaves as a trend-follower for $\psi_i > 1$ (correspondingly he perceives the market as a majority game with payoff $\pi_i(t) = a_i(t)A(t)$), while he behaves as a fundamentalist for $0 < \psi_i < 1$ and plays a minority game with payoff $\pi_i(t) = -a_i(t)A(t)$. Let us now consider an agent who forms expectations according to (2). It is easy to see that such an agent is described by a generalization of (8). Indeed, a direct calculation shows that (2) corresponds to

$$\mathbb{E}_i[A(t)] \propto \sum_{\tau=1}^{M-1} \frac{M - \tau}{M} A(t - \tau). \tag{9}$$

Agents thus tend to discount events further back in time and give larger weight to recent price changes when estimating the future returns. Clearly, such an agent has a more complicated reaction pattern than a pure Minority or Majority Game player and will be described by a payoff function that accounts for the possibility of behaving differently in different market regimes.

Models of this type have been introduced recently and appear to be an ideal testing ground to verify the emergence of the effective potential scenario in a microscopic setting. Specifically, we have tested it on a model in which agents may switch from a trend-following to a fundamentalist attitude (and vice versa) depending on the market conditions they perceive, which was introduced in Ref. [5]. We refer the reader to the literature for a detailed account of the model’s definition and properties. In a nutshell, it describes agents who strive to maximize the payoff

$$\pi_i(t) = a_i(t)[\mathcal{A}(t) - \varepsilon \mathcal{A}(t)^3], \tag{10}$$

where $\mathcal{A}(t) = A(t)/\sqrt{N}$ is the normalized excess demand. The idea is that for small price movements ($A(t) \simeq 0$) agents perceive the game as a majority game as they try to identify profitable trends. However, when price movements become too large, the game is perceived as a minority game, i.e., agents expect the price to revert to its fundamental value. As in most minority games, agents have fixed schemes (“strategies”) to react to the receipt of one of P possible external information patterns and learn from experience to select the strategy and, in turn, the action $a_i(t)$ that is more likely to deliver a positive payoff. A realistic dynamical phenomenology is obtained in a whole range of values of the model parameter ε when the number N of players is large compared to the amount of information available to them P (this is measured by a parameter $\alpha = P/N$, see Ref. [5] for details). In Fig. 6 is shown the time evolution of $P(t)$ for a game with parameters $\alpha = 0.05$ and $\varepsilon = 1$. This choice of parameters corresponds to be in the range in which the competition between trend followers and contrarians is stronger. In fact, in Fig. 6, we can observe some “ordered” periods, where

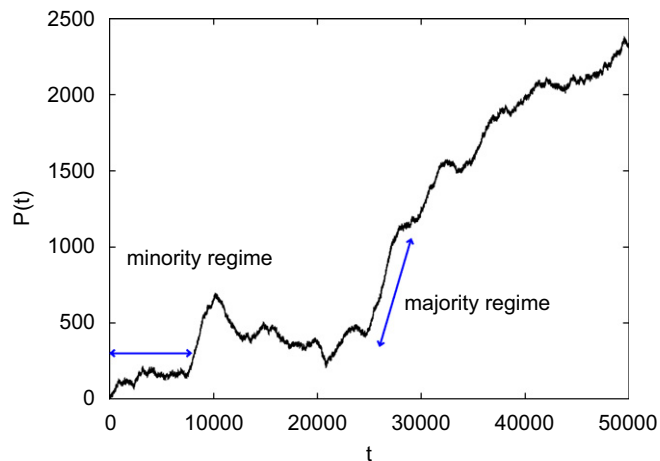


Fig. 6. The time evolution of $P(t)$ for a minority–majority game with $\varepsilon = 1$ and $\alpha = 0.05$ is shown. We can observe the alternation of different regimes. In the graph are indicated two periods by means of arrows. In the minority regime the price remains near to its fundamental value, while in the majority regime appears a well defined trend.

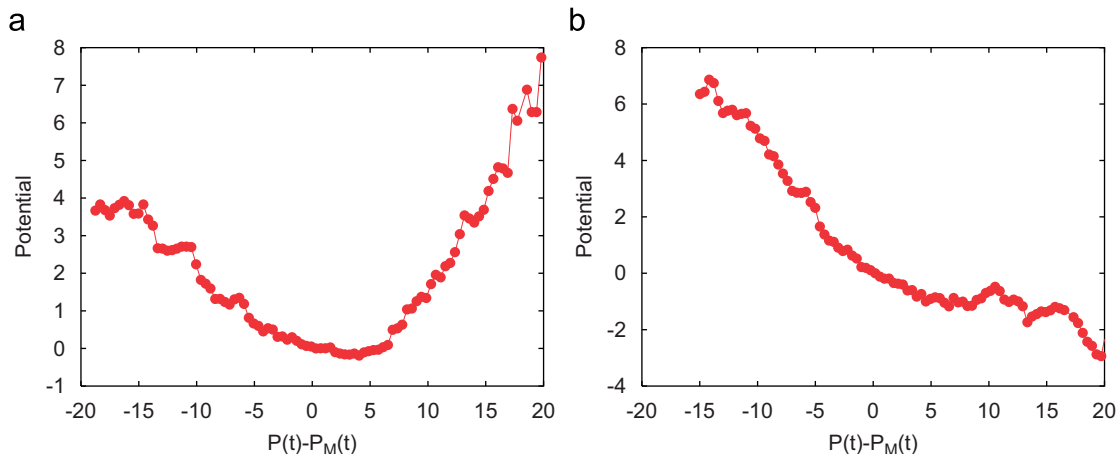


Fig. 7. The potential analysis with $M = 20$ for the two periods indicated in Fig. 6 is shown. The analysis for the minority region leads to an attractive (even though not really quadratic) potential. Instead, analyzing the majority region we found a repulsive potential.

$A(t)$ is small and well defined trends in the price dynamics appear, but also “chaotic” periods where the dynamics of the price is dominated by the contrarians. In Fig. 6 we have identified two periods in which the different behaviors of the agents are well defined and we have used these periods as data set for our potential analysis. In Fig. 7 are plotted the potentials obtained performing the effective potential analysis with $M = 20$. We can observe that, when the market is dominated by contrarians, we obtain an attractive potential. This shape of the potential reproduces the agents’ tendency to keep the price near its “fundamental” value. We can also note that this potential is not perfectly quadratic as in the model described in Eqs. (3) and (4). In fact, plotting different potentials with various values of M we cannot obtain a data collapse scaling the potentials with the factor $(M - 1)$. In case of market dominated by trend followers, we can observe the presence of well defined trends (bubbles and crashes). In this case the agents try to follow the trends and the price tends to go away from its fundamental value. In this case we obtain a repulsive potential. Therefore, the potential analysis is able to detect the agents’ behavior based on microscopic rules only analyzing the data of a macroscopic variable, $P(t)$.

From the viewpoint of modeling real markets, it will be very interesting to introduce an agent based model in which agents perform their decision (buy/sell) by considering their expectations about the next price

increment using Eq. (9), with different constants of proportionality and different values of the ‘memory’ M and not on the basis of a set of given strategies (as in the minority game framework). Work along these lines is currently in progress.

4. Results for real stock prices from NYSE

For our potential analysis we consider as database the price time series of all the transactions of a selection of 20 NYSE stocks. These have been selected to be representative and with intermediate volatility. This corresponds to volumes of 10^5 – 10^6 stocks exchanged per day. We consider 80 days from October 2004 to February 2005. The time series we consider are by a sequential order tick by tick. This is not identical to the price value as a function of physical time but we have tested that the results are rather insensitive to this choice. The statistical properties of these kind of data are relatively homogeneous within the time scale of a trading day but the large jumps of the prices between different days prevent the extension of the analysis to large times [9]. So we focus our potential analysis considering the stock-prices fluctuations within a trading day. In Fig. 8 are plotted the time evolutions of three stock indexes in a trading day.

If we perform the effective potential method for a trading day of a given stock, we found shapes of the effective potential that are very irregular and often asymmetric. In Fig. 9 are plotted the results obtained for the data plotted in Fig. 8. We can see that the shapes of the potentials are not always quadratic. The potential in Fig. 9(a) seems rather quadratic and attractive while in Fig. 9(b) has a piecewise linear shape similar to the potential plotted in Fig. 5(b). The potential in Fig. 9(c) seems flat as one expects from a simple RW model.

Instead, if we consider some averages over a long period (80 trading days) of the potentials obtained for a single day, the resulting potentials seems to be quadratic as in Ref. [4]. In Fig. 10 are shown the average shape

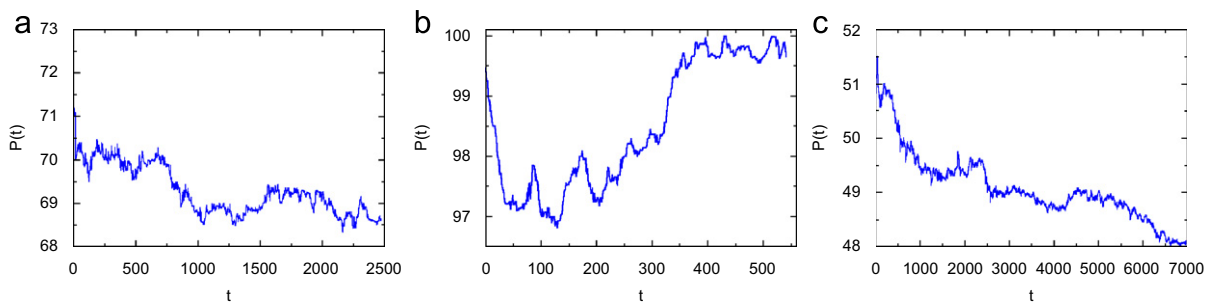


Fig. 8. The time evolution of three stock index (CLF, ITU and TOL) is shown. The time is expressed in ticks and corresponds to one trading day.

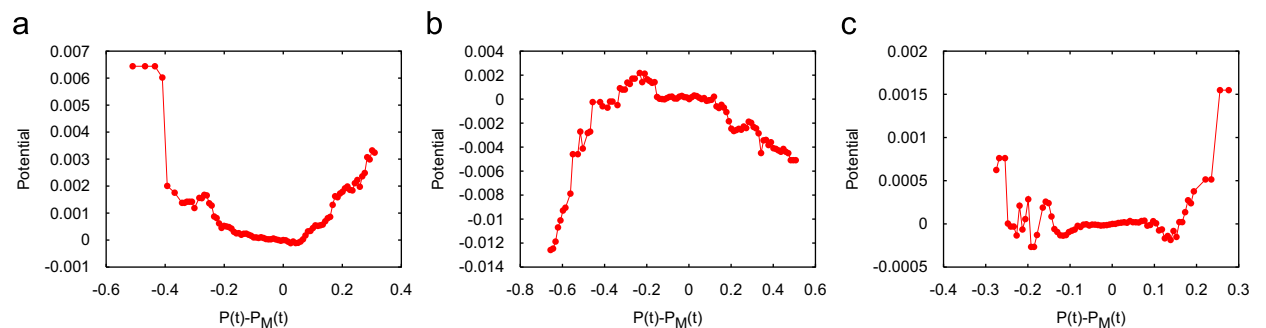


Fig. 9. The effective potential method has been applied to the price evolution of the three stocks plotted in Fig. 8. We can see that the effective potentials are rather asymmetric and not all quadratic. The potential in (a) seems quadratic and attractive in the central part but has asymmetric tails. The potential in (b) and (c) are not quadratic. In particular the potential in b is asymmetric and looks like piecewise linear as in Fig. 5(b). The potential in (c) is flat like a simple RW potential.

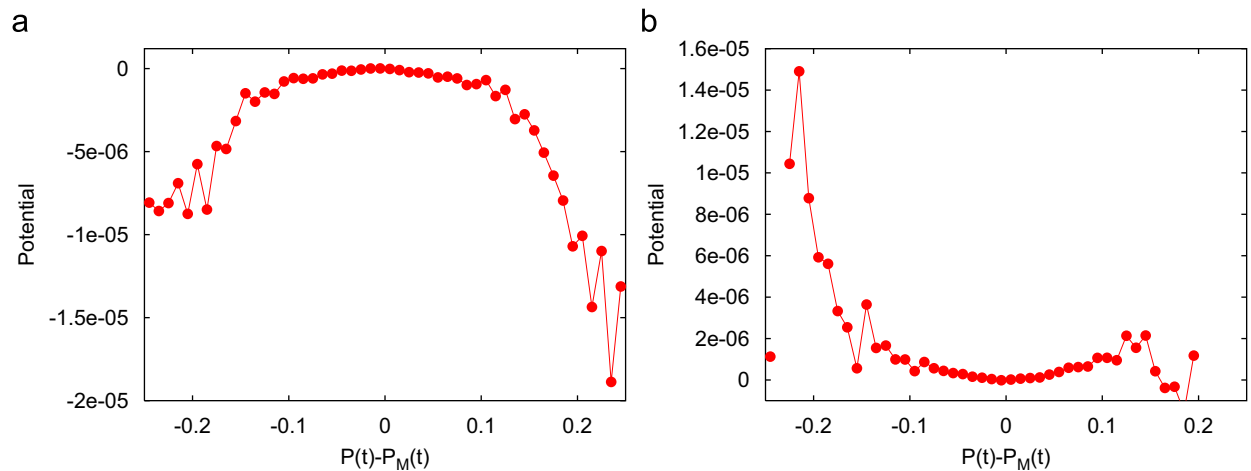


Fig. 10. This plot shows the average shape of the potential over 80 trading days for two different stock indexes (BRO and PG). The shape is quite quadratic and symmetric, and in (a) is repulsive while in (b) attractive.

of the potential for two stock indexes (BRO and PG). We can observe a rather quadratic shape of the potentials. In Fig. 10(a) the potential is quadratic and repulsive while for the index PG the potential is attractive, as we can see in Fig. 10(b).

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