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Contagion effects in a chartist–fundamentalist model with time delays

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Abstract

In this paper two models of speculative markets are developed to study the effects of feedback mechanisms in financial markets. In the first model, a crash market model couples a linear chartist–fundamentalist model with time delays with a log-periodic market index $I(t)$ through direct coupling. Numerical solutions to the model show that asset prices exhibit significant persistence as a result of the coupling to the log-periodic market index. An extension to include endogenous wealth dynamics shows that the chartists benefit from the persistent dynamics induced by the coupling. The second model is a two-asset model represented by a 2-dimensional delay-differential equation. Asset one price exhibits limit cycle dynamics while in the second market asset prices follow stable damped oscillations. The markets are coupled through a diffusive coupling term. Solutions to the coupled model show that the dynamics of asset two changes fundamentally with the price now exhibiting a limit cycle. The stable converging dynamics is replaced with limit cycle oscillations around the fundamental.

Keywords: Delay-differential equations; Financial crises; Speculative markets

1. Introduction

The correlation of asset prices across financial markets is one of the stylized facts of financial market dynamics [1–3]. Empirically, asset cross-correlations have been uncovered across assets in single financial markets and across major financial market indexes globally. Such asset correlations are a source of contagion dynamics across assets and markets leading to the coupling of financial cycles or in extreme events to the spread of crash dynamics across markets. Such "extreme event" possibility has been experienced in the financial crises of Russia in 1998 and the East Asian crisis in 1997. Such events with propagation dynamics can have ramifications on risk measurement in financial markets. Contagion dynamics can cause standard risk analysis models (such as the value at risk (VaR model) to fail in predicting portfolio values during crash times [4]. In practical endeavors, such correlations of asset prices represent important challenges for portfolio construction as the central model of finance (CAPM) exploits such correlations for portfolio choice. Given this theoretical, empirical and practical relevance of asset price correlations, it is important to construct

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theoretical models that explain the dynamics of such contagion phenomena. Current theoretical models are mainly probabilistic models with the underlying assumption that financial markets are efficient [5,6]. However, financial markets have been shown to deviate form the efficiency hypothesis and persistent deviations of asset prices from "fundamental" values implied by efficiency have been detected in many empirical studies [7]. This lends support to the existence of speculative dynamics in financial markets generated by herd behavior and speculative investors. The models in this paper are an extension of the fundamentalist–chartist model presented in Ref. [8]. The paper is divided as follows. Section 2 presents the basic time-delay chartist–fundamentalist model with direct coupling to a crashing market index represented by log-periodic dynamics. Section 3 extends the basic time-delay model into two markets whose independent dynamics are governed by damped oscillations and limit cycles, respectively. Section 4 concludes.

2. A chartist–fundamentalist model with coupling to market index

In this section, the chartist–fundamentalist models of speculative markets are extended into two-asset model with feedback mechanisms and time delays in reaction times. Such theoretical models have been developed to explain synchronization phenomena in biological and physical systems and are at the forefront of research in dynamical systems [9]. On the other hand, the chartist–fundamentalist model of speculative markets has been successful in explaining financial market dynamics such as market cycles, speculative bubbles and crashes [8,10,11]. The models combine herding behavior, trend following and feedback effects, all important for the generation of market fluctuations, bubbles and crashes [12]. The models are deterministic two-agent models (a fundamentalist and a speculator) that represent financial markets as differential, difference or delay-differential equations. All market models are governed by an excess demand for the modeled asset [7]. In chartist–fundamentalist models, the asset price is driven by excess demand $D(s,p)$ according to the following:

$$\frac{\dot{p}}{p} = \sigma D(s,p), \tag{1}$$

where p is the asset price, σ, the elasticity of asset price to excess demand and s, a measure of price trend. The total market demand is equal to the sum of demands by two market participants

$$D(s,p) = D^{\mathrm{c}}(s) + D^{\mathrm{f}}(p). \tag{2}$$

The demand function of the fundamentalists can then be written as

$$D^{\mathrm{f}}(p) = -m(p - v), \tag{3}$$

where v is the fundamental price of the asset and m the fundamentalists' share of wealth. Since fundamentalists believe that asset prices must converge instantaneously to v, then they will sell (buy) the asset when $p > (<) v$. On the other hand, chartists believe that future prices depend on past movement of the asset price. Hence, the demand function of the chartists can be written as

$$D^{\mathrm{c}}(p) = \mu(1 - m) \sum_{i=0}^{n} \phi_{i+1}(p(t - \tau_i) - p(t - \tau_{i+1})), \tag{4}$$

where the chartists base their estimation of the asset price trend on an adaptive process that takes into account past values of the price trend where τ_i is the time delay, μ the strength of response of chartists' demand to price trends. $\tau_0 = 0$ and ϕ_i is the weight assigned to various past trend estimations. Adding (3) and (4), we get

$$D(p) = \mu(1 - m) \sum_{i=0}^{n} \phi_{i+1}(p(t - \tau_i) - p(t - \tau_{i+1})) - m(p - v) \tag{5}$$

and the asset price dynamics is governed by the following given $\sigma = 1$:

$$\dot{p} = \mu(1 - m) \sum_{i=0}^{n} \phi_{i+1}(p(t - \tau_i) - p(t - \tau_{i+1}))p(t) - m(p - v)p(t). \tag{6}$$

The model (6) has been solved in Ref. [8] and asset prices are shown to have explosive or damped oscillations depending on the parameters of the delay-differential equation. Hence the above chartist–fundamentalist model cannot explain speculative bubbles and crashes in stock markets as the asset price converges to the fundamental price in finite time. On the other hand, explosive oscillations cannot be considered realistic solutions since no asset exhibits boundless price movements.

In this respect, empirical evidence has shown that the ‘‘market’’ tends to drag assets up or down with it [7]. A crashing market can be modeled as a log-periodic process [13]. Empirical evidence for such log-periodic motion has been uncovered for many international stock, bond, real estate and foreign exchange markets. In Ref. [13], the stock market is considered as an example of self-organizing cooperative system. The stock market crash of 1987 is modeled empirically using a log-periodic function for 2 years before the crash in similarity with critical phenomenon. The log-periodic function used is

$$I(t) = A + B(t_c - t)^{\delta}[1 + C\cos(\omega \log(t_c - t) - \phi)], \tag{7}$$

which represents a power-law behavior plus a general log-periodic correction of the stock market which exhibits a singularity at the critical time of the crash. The expression (7) explains two important structures of stock market dynamics before a crash. First, the accelerated increase in the stock index value around 2 years before the crash. Second, the superimposition on this increase of fluctuations that increase in frequency as time approaches critical time, t_c. Assuming that the asset prices are ‘‘dragged’’ down during a market crash by the market index, then the model represented by Eq. (6) can be modified to include such a forcing effect of the market on the asset price. If the asset price is forced with the market index $I(t)$ given by Eq. (7) the model then becomes

$$\dot{p} = \mu(1 - m)\sum_{i=0}^{n} \phi_{i+1}(p(t - \tau_i) - p(t - \tau_{i+1}))p(t) - m(p - v)p(t) + \lambda I(t), \tag{8}$$

where λ is the strength of coupling. Model (8) is solved numerically using the program NDELAYDSOLVE in MATHEMATICA.

The model is solved for the following parameters $m = 0.5, v = 5, \mu = 0.9, \phi_1 = 0.5, \phi_2 = 0.3, \phi_3 = 0.2, \lambda = 0.001, A = 412, B = -165, C = 0.07, \tau_1 = 2, \tau_2 = 3, \tau_3 = 4, t_c = 87.74, \omega = 7.4, \phi = 0.7, \delta = 0.33$. The coupled model shows more persistent dynamics than the uncoupled model (Fig. 1). The asset price does not converge to the fundamental value and hence the divergence of the asset price from the fundamental value is persistent. This divergence should have an effect on speculative activity and the gains and losses made by different types of agents in the market. In this respect, an interesting investigation would be the effect of the coupling to crashing market index on the wealth dynamics of chartists and fundamentalists. The model is hence extended to include endogenous wealth dynamics. Wealth dynamics is introduced into the model through an equation for the share of the fundamentalists wealth as a function of the deviation of the asset price from the fundamental. In speculative markets, the farther away the price is from the fundamental value, the more speculative momentum there is in the market and the fundamentalists lose out market, share to the chartists. The fundamentalists' wealth $m(t)$ becomes then endogenous and can be

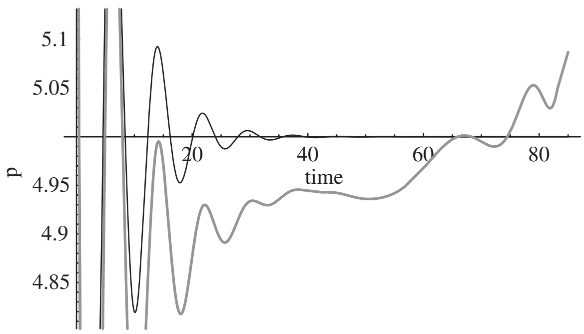

Fig. 1. Asset price dynamics for uncoupled (solid) and coupled models (gray).

represented by

$$m(t) = \frac{1}{1 + \beta + \Theta\sqrt{(|p(t) - v|)/v}}. \tag{9}$$

Eq. (9) is adapted from Ref. [11] where β is the basic influence of the chartists and Θ the popularity coefficient of chartists. The endogenous wealth model is then solved numerically. Figs. 2 and 3 show the evolution of the chartists wealth for the noncoupled and coupled cases, respectively. In the noncoupled market, the independent dynamics of the speculative market causes the chartists share of wealth to go to zero as time evolves. This is a result of the convergence of the asset price to the fundamental for large times. In the coupled model, the coupling to the crashing market index causes the chartists share of wealth to follow a more complex path. The chartists clearly benefit from the persistent dynamics induced by the coupling. The chartists share of wealth experiences large periods where the market share is maintained at high levels. This lends theoretical support to the empirical fact that speculators are drawn into speculative markets that exhibit persistent dynamics.

3. Coupled asset price limit cycles

Price feedback mechanisms in one asset models have been shown to be responsible for large market fluctuations, bubbles and crashes [12]. In order to further study the effects of such contagion effects, the model is now extended into a two-asset market with an interaction term: a position feedback mechanism formed by the price differential between the asset prices. The feedback mechanisms in this model will be investigated to see their role in synchronization and contagion effects between different markets. The independent asset

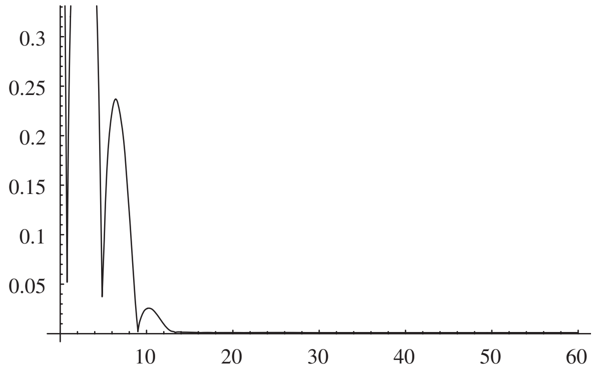

Fig. 2. Chartists wealth share as a function of time for non-coupled model. Parameters are $\beta = 0.001, \Theta = 3, v = 5, \phi_1 = 0.5, \phi_2 = 0.3, \phi_3 = 0.2, \tau_1 = 2, \tau_2 = 3, \tau_3 = 4, \lambda = 0, p_0 = 6$.

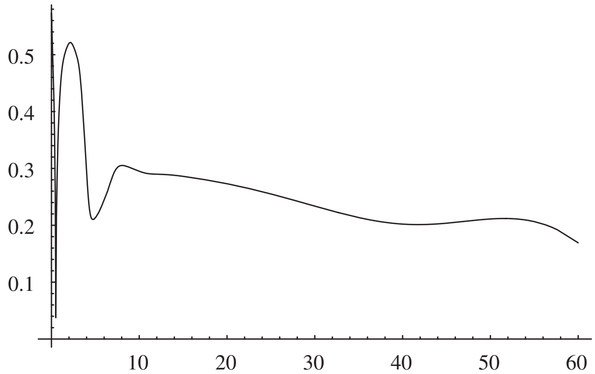

Fig. 3. Chartists wealth share as a function of time for coupled model. $\beta = 0.001, \Theta = 3, v = 5, \phi_1 = 0.5, \phi_2 = 0.3, \phi_3 = 0.2, \tau_1 = 2, \tau_2 = 3, \tau_3 = 4, \lambda = 0.001, p_0 = 6$.

dynamics (without feedback mechanisms) will be the benchmark against which the synchronization effects are seen. In this section, the nonlinear chartist–fundamentalists model from Ref. [8] is extended to two markets. In this model, the expectations function of the chartists is nonlinear which represents the chartists' belief that prices cannot increase indefinitely and hence introduces a "saturation effect" into the demand function of the chartists. The nonlinearity is represented by the hyperbolic tangent function. The model then becomes

$$\dot{p} = (1-m)\tanh(p(t) - p(t-\tau))p(t) - m(p - v)p(t). \tag{10}$$

In a two-asset market, with coupling between the two markets, the model then can be written as

$$\dot{p}_1 = (1-m)\tanh(p_1(t) - p_1(t-\tau_1))p_1(t) - m(p_1 - v_1)p_1(t) + \lambda(p_1 - p_2), \tag{11}$$

$$\dot{p}_2 = (1-n)\tanh(p_2(t) - p_2(t-\tau_2))p_2(t) - n(p_2 - v_2)p_2(t) + \eta(p_2 - p_1), \tag{12}$$

where λ and η represent the coupling strengths between the two markets and n, the market share of fundamentalists in market 2. Fig. 4 presents the solution to the independent asset market with no interaction terms ($\lambda = \eta = 0$). The solution shows the unsynchronized benchmark state for the market for the following parameters: $m = 0.5, \tau_1 = 2, v_1 = v_2 = 5, \lambda = \eta = 0, n = 0.65, \tau_2 = 2$.

The solution shows that in the absence of coupling, asset 1 exhibits limit cycle dynamics. Price fluctuations are persistent and the price oscillates around the fundamental prices. In market 2, however, price dynamics follow stable damped oscillations. The price of asset 2 converges to the fundamental price as time evolves. Hence, we a have a cyclical market and a stable market. If the coupling strengths are not zero, the dynamics of asset 2 changes fundamentally. Figs. 5 and 6 show the dynamics of the two markets under coupling $\lambda = \eta = 0.1$. The price of asset 2 now follows a limit cycle. The stable converging dynamics is replaced with limit cycle oscillations around the fundamental. This qualitative change in dynamical behavior shows

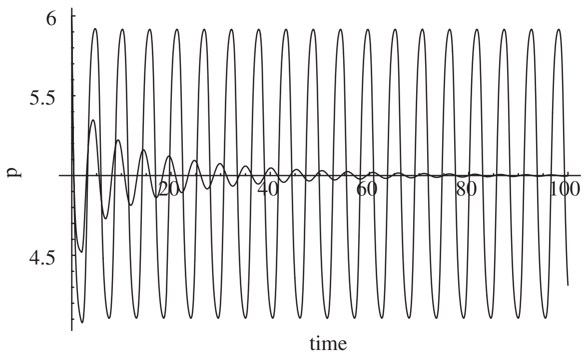

Fig. 4. The dynamics of the asset prices for uncoupled model.

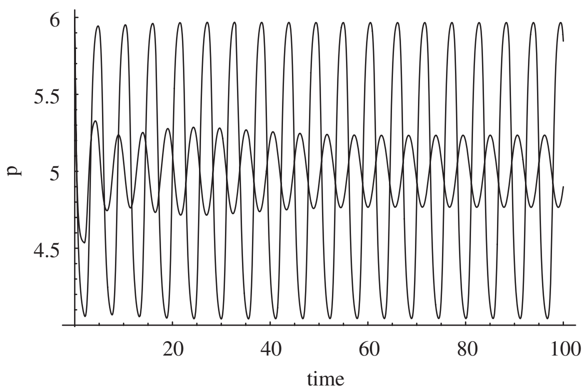

Fig. 5. The dynamics of asset prices in the coupled model.

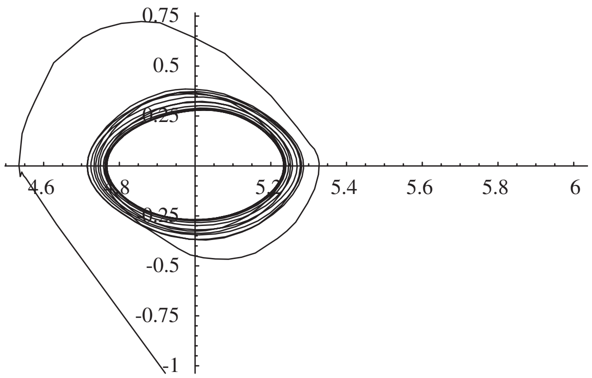

Fig. 6. Phase space dynamics of asset two in the coupled model.

that coupling and contagion between different markets lead to persistent cyclical motion being transmitted across markets.

4. Conclusions

The inclusion of a log-periodic market index $I(t)$ through direct coupling in a chartist–fundamentalist model paved the way for the understanding of the contagion effects of a large financial crash on individual asset prices specifically on market dynamics and speculators' wealth evolution. Moreover, the chartist–fundamentalist model extension into two-asset market with linear feedback mechanisms was found useful in explaining synchronization and high correlation between different assets' prices. The model shows that asset price correlations can be a source of contagion dynamics across markets leading to the coupling of financial cycles and to the spread of crash dynamics across markets. The second model showed that coupled speculative markets with chartists and fundamentalists in each but with different dynamics can be synchronized through diffusive coupling. The stable converging dynamics is replaced with limit cycle oscillations around the fundamental. This qualitative change in dynamical behavior shows that coupling and contagion between different markets can lead to the transmission of fluctuations across financial markets. The models presented in this paper can be made more realistic by further extensions to markets with noise and N-assets. Such extensions can model actual financial markets that include many assets and where noise plays a fundamental role in financial market dynamics.

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