

Stock price fluctuations and the mimetic behaviors of traders

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Abstract

We give a stochastic microscopic modelling of stock markets driven by continuous double auction. If we take into account the mimetic behavior of traders, when they place limit order, our virtual market shows the power-law tail of the distribution of returns with the exponent outside the Levy stable region, the short memory of returns and the long memory of volatilities. The Hurst exponent of our model is asymptotically $\frac{1}{2}$. An explanation is also given for the profile of the autocorrelation function, which is responsible for the value of the Hurst exponent.

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1. Introduction

In financial markets, it seems very natural to believe that large price changes are caused by large transaction volumes [1]. Farmer et al., have, however, proposed an entirely different story of large price changes in the markets driven by continuous double auctions [2]. They have argued that large returns are not caused by large orders, while the large gaps between the occupied price levels in the orderbook lead to large price changes in each transaction, and actually showed that the gap distribution closely matches the return distribution, based on the analysis of the orderbook as well as the transaction records on the London Stock Exchange. They have also shown that the virtual market orders of a constant size reproduce the actual distribution of returns.

Then, we arrive at the next question, that is, what really causes large gaps. Maslov has introduced and studied a simple model of markets driven by continuous double auctions [3]. In his model, traders choose one from the two types of orders at random. One is a market order which is an order to sell or buy a fixed amount of shares immediately at the best available price on the market on the time. The other is a limit order which is an order to buy or sell a fixed amount of shares with the specification of the limit price (the worst allowable price). The relative price of a new limit order to the most recent market price is a stochastic variable drawn from a uniform distribution in a given interval. Numerical simulations of the model show that it has such realistic features to a certain extent as the power-law tail of the distribution of returns, and the long range correlation of the volatility. However, the price evolution has some essentially different statistical properties from the actual one on the market of large investors. First the exponent α of the power-law tail of the distribution of returns is inside the Levy stable region ($0 < \alpha \leq 2$), while the actual value is close to 3 [4,5]. Second the Hurst

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exponent $H = \frac{1}{4}$ is unrealistic in wide enough time windows. In actual market, we have $H = \frac{1}{2}$ for long-term, which is the value for free diffusion. Challet and Stinchcombe have proposed a model with non-constant occurrence rates of various types of orders as a model in the same class as Maslov's [6]. The price of their model is over diffusive ($H > \frac{1}{2}$) for short-term and diffusive ($H = \frac{1}{2}$) for long-term.

In this paper, we propose a stochastic model with a novel feature in the same class as the above models. We take into account the mimetic behavior of traders, when they place limit order, then our virtual markets show the behaviors which are widely recognized as the stylized facts in financial markets, that is, the power-law tail of the distribution of returns with the exponent outside the Levy stable region [4,5], the short memory of returns and the long memory of volatilities [7]. The Hurst exponent of our model is asymptotically $\frac{1}{2}$. An explanation is also given for the profile of the autocorrelation function, which is responsible for the value of the Hurst exponent.

2. Model

We give the definition of our model here. We introduce the cancellation of orders to maintain the number of orders stored in orderbook as in the model of Smith et al. [8] and in other models [6]. We have, therefore, three types of orders in total on both sides of trade, namely sell/buy market order, sell/buy limit order and the cancellation of sell/buy limit order. The conservation law indicates $\alpha_i - \mu_i - \delta_i = 0$ for $i = \text{sell or buy}$, where the parameters α_i , μ_i and δ_i denote the occurrence rate of limit order, market order and cancellation, respectively. For simplicity, we also assume that traders always place a fixed size of order.

The prices at which new limit orders are placed reflect traders' strategies. In Maslov's paper [3], they are determined by offsetting the most recent market price by a random amount drawn from a uniform distribution in a given interval. Challet and Stinchcombe assume the Gaussian distribution from which random variables for the relative prices of sell/buy limit order to ask/bid are drawn, and put the exclusion hypothesis that if there is already an order at the price, the price is not available [6]. In the model of Smith et al. [8], traders place the sell(buy) limit order at any price in the semi-infinite range above bid (below ask) with uniform probability.

In this paper, we take a little bit more strategic order placement than previous works. We assume the mimetic behavior of traders, when they place limit orders. Traders in our model sometimes (or a part of traders always) leave the decision of the limit price to the others. They believe the majority, and should be so patient that they can wait in line the execution of their orders for long time. The assumption is implemented here as follows: with probability p , a limit price of sell(buy) order is chosen with the probability proportional to the size of orders stored on the price, and with probability $1 - p$, a price is randomly chosen between $[\text{bid} + 1, \text{ask} + 1][\text{bid} - 1, \text{ask} - 1]$ with uniform probability. Parameter p is crucial for the model. The choice of the limit price in our model follows the preferential attachment dynamics in the growth model of scale free networks [9,10]. "Rich gets richer" is a concept common to both models. However, our model is not a growth model, but the total amount of limit orders is loosely fixed, because the number of coming limit orders are balanced with market orders and cancellations. Instead, the result of simulations will show that the distribution of the fluctuation of the gaps between occupied price levels has a power-law tail owing to the unequal attractive powers of each of the prices.

3. Numerical simulations of the model

The models of continuous double auctions such as Maslov's and ours have a difficulty to solve analytically due to the existence of free boundaries: sell (buy) limit prices acceptable for traders who place the orders are bounded on the best price of the opposite side, namely the bid (ask) price on the time. Slanina [11] and Smith et al. [8] have formulated a mean field approximation of the master equations to have good results. We adopt, however, a numerical method, and leave the analytical examination for the coming papers.

We generate each type of orders in the ratio of $\alpha_i = 0.25$ (limit order), $\mu_i = 0.125$ (market order) and $\delta_i = 0.125$ (cancellation) for i either equal to sell or buy. For the several values of p , we perform 1000 times runs of 10,000 step iterations with different initial conditions. We place a unit of shares at each price 1, 2, -1, -2 as the ask, the second best sell limit price, the bid and the second best buy limit price, respectively, and also

198 units of sell (buy) limit orders at random in the range between $1(-1)$ and the random integer drawn from a uniform distribution in the interval $[2, 201]([-201, -2])$ as an initial condition.

Here we present the most interesting results obtained through the numerical simulations. Fig. 1 shows the cumulative distribution functions of price shifts, the gaps between ask and the second best sell limit price and spreads. Price shifts and the gaps are sampled after every buy market order. The results for sell market order are omitted here in order to avoid a redundancy, because we take the symmetric values of the parameters and the initial conditions. Spreads are sampled after every sell and buy market orders. All the three distributions become broader when the parameter p becomes larger. The power-law tails appear in all the graphs when p beyond 0.4. The distributions for the parameter p beyond 0.5 are very broad, but steep falls are observed in the tail. We pick up some points from the interval $0.4 < p < 0.5$, and roughly estimate that the power-law exponent

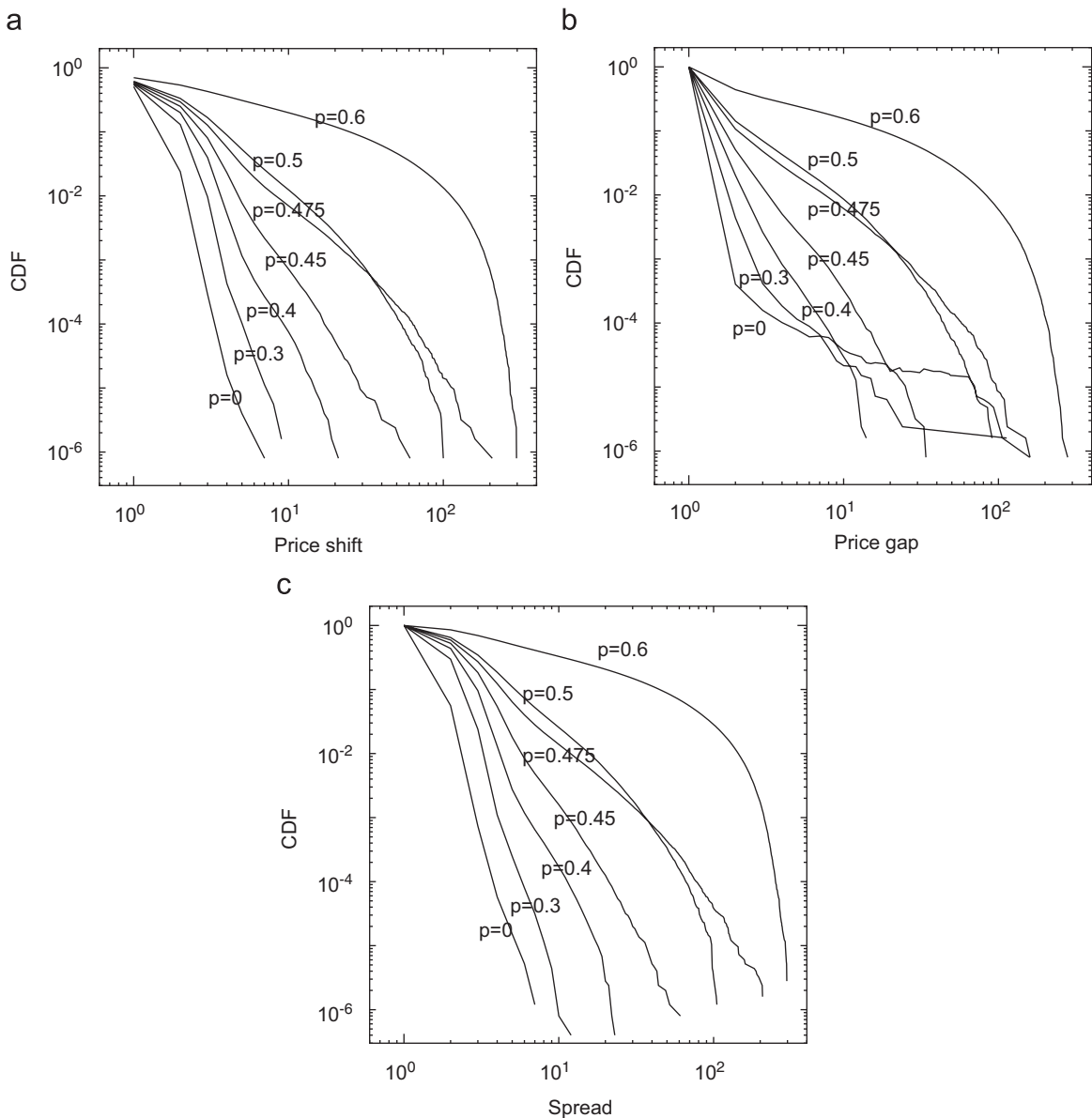


Fig. 1. Cumulative distribution functions of price shifts, the gaps between ask and the second best sell limit price and spreads. The power law exponents of price shifts (the gaps, spreads) are 3.97 ± 0.11 (4.27 ± 0.12 , 4.49 ± 0.11), 2.72 ± 0.08 (2.97 ± 0.11 , 3.09 ± 0.08) and 3.78 ± 0.11 (3.80 ± 0.11 , 4.14 ± 0.10) for $p = 0.45, 0.475$ and 0.5 , respectively.

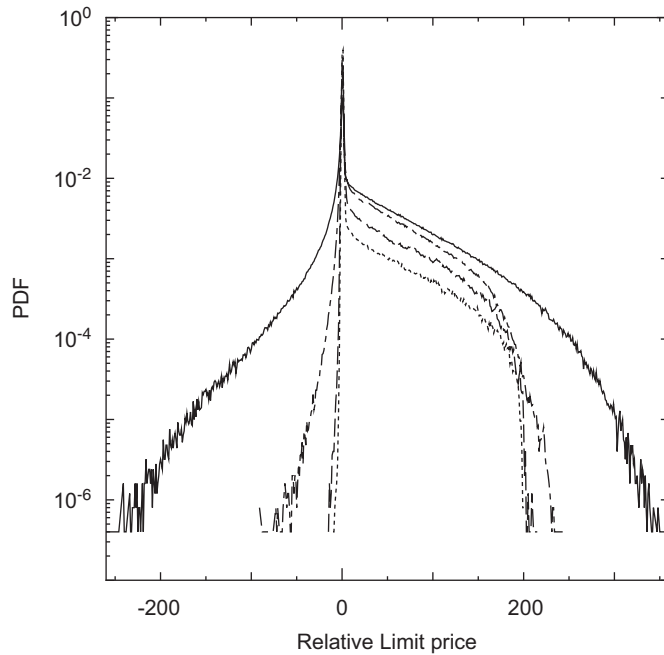


Fig. 2. Probability distribution function of the relative limit price. The results are shown for the three cases with $p = 0.3$ (dotted line), $p = 0.4$ (dashed line), $p = 0.5$ (dot-dash line) and $p = 0.6$ (solid line).

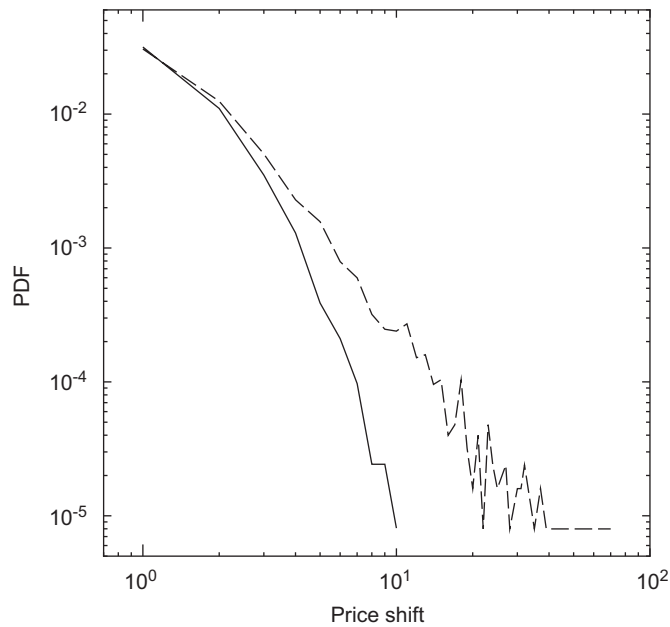


Fig. 3. Probability distribution function of price shift of the surrogate data. The original data is generated by 100 times runs of 10,000 step iterations with $p = 0.5$. The comparison with that of the original data (dashed line) is also given.

of the tails have minima in the interval $0.45 < p < 0.5$, and the values are near 3 at $p = 0.475$ as given in the caption of Fig. 1. We use the Hill estimator of the largest \sqrt{n} data, where n is the size of sample.

We see from Fig. 2 that the relative limit price, namely the distance at which new limit orders are placed away from the current best price, is broadly distributed when p becomes large. We want to demonstrate,

however, that the broadness itself of the distribution of the relative limit price does not create the fat tail of the price shift distribution. First of all, for the purpose, we collect the data of the limit order by a numerical simulation of the model. Then we shuffle the order of arrivals of limit orders, and perform a simulation using the surrogate data instead of the limit price generated by the original rule of our model. The comparison of the resultant probability distribution of price shift of the surrogate data with that of the original data is shown in Fig. 3. The tail of the distribution does not show a power-law behavior, though the original data do. This experiment reveals that the information of orderbook plays an essential role in the decision of the price at which new orders are placed in our model. A similar role of the orderbook will be expected even in real markets, though the style of the reference to the orderbook is possibly different from that assumed here.

4. Autocorrelation functions of the model

We derive the autocorrelation functions of price shift and of the absolute value of the price shift obtained by the numerical simulations of the model. The results are given in the panels of Fig. 4 including comparison with the autocorrelation functions of the surrogate data mentioned in the previous section. In those panels, the unit of time increment corresponds to a market order.

The autocorrelation function of price shift almost vanishes except the value of time lag $\tau = 1$ for both data. The values of the autocorrelation at time lag $\tau = 1$ are -0.41 and -0.46 , respectively. Those values are close to -0.5 , and are explainable by the mean field approximation of the autocorrelation function as follows: let δ_1 and δ_2 denote the mean square root of price shift normalized by the standard deviation. The value δ_1 measures the price shift across the spread, corresponding to the case that the side of the trade changes from bid to ask or from ask to bid. The value δ_2 corresponds to the case that the side of the trade remains the same. If we assume that the four cases occurs with the same probability $\frac{1}{4}$, the mean field approximation of autocorrelation functions gives the equation $\rho_i = \langle dp_t dp_{t+i} \rangle / \sigma^2 = 1/4(\delta_2^2 - \delta_1^2)\delta_{1i}$. From the normalization condition $\delta_1^2 + \delta_2^2 = 2$ and the inequality $\delta_1 \geq \delta_2$ (because spread always exists, while the trade successively occurred on the same side do not necessarily move the price), we have the result $\rho_i \approx -0.5\delta_{1i}$. The profile of the autocorrelation

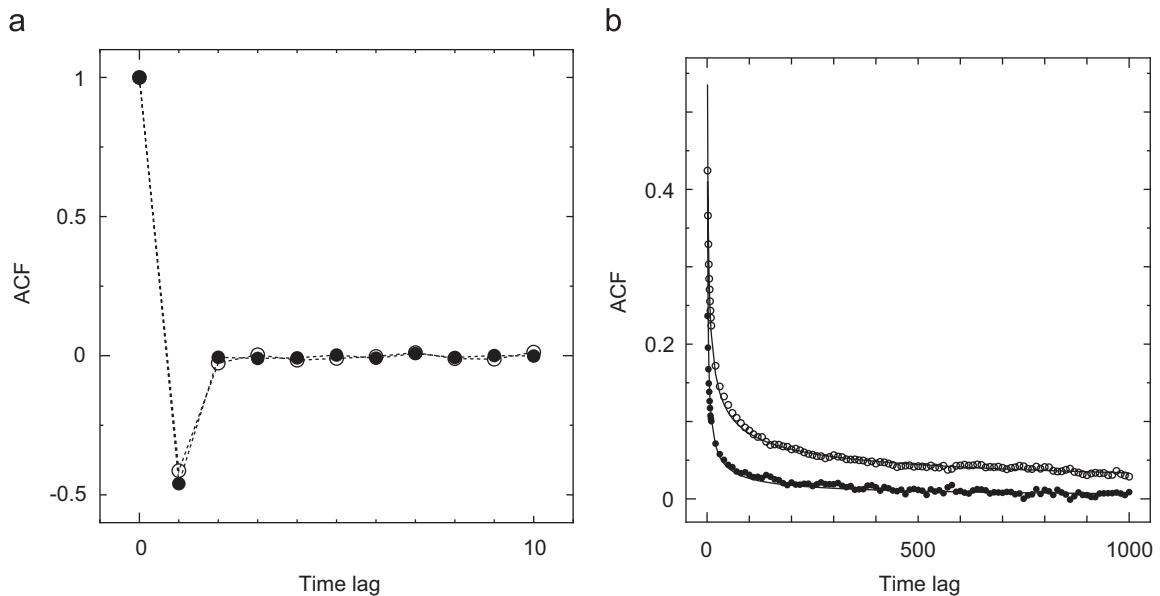


Fig. 4. Autocorrelation functions of price shift and of the absolute value of price shift obtained by the numerical simulations of the model. In both panels, the unit of time increment corresponds to a market order. Empty circle (\circ) represents the results for the original data, and filled circle (\bullet) for the surrogate data mentioned in the previous section. (a) The autocorrelation function of price shift. (b) The autocorrelation of the absolute value of price shift with the power law fittings (solid lines). The exponents of the power law fittings are estimated by linear regression of the data plotted in log–log plain. The result is -0.40 ($R^2 = 0.99$) for the original data, and -0.60 ($R^2 = 0.79$) for the surrogate data.

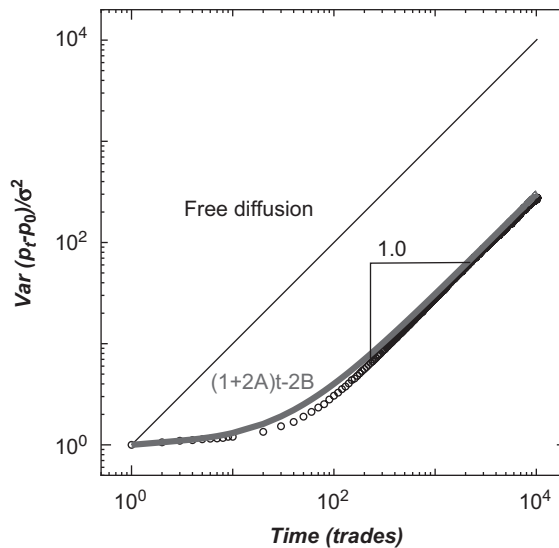


Fig. 5. Empirical study of the price diffusion. We analyzed about 45 millions transaction data from November 1999 through October 2000 of active 5 IT or e-commerce companies (Intel, Microsoft, Amazon, Oracle, Cisco) listed on Nasdaq using TAQ Database. The theoretical line is also given, where $A = \sum_{i=1}^t \rho_i$ and $B = \sum_{i=1}^t i\rho_i$.

function is responsible for the value of the Hurst exponent H through the equation $Var(p_t - p_0)/\sigma^2 = Var(\sum_{i=1}^t dp_i)/\sigma^2 = t + \sum_{i=1}^t (t-i)\rho_i \sim t^{2H}$. In such case of short memory as our model, we have the equation $Var(p_t - p_0)/\sigma^2 = Dt \sim t^{2H}$ for $t \gg 1$. In our case, the diffusion constant $D = 1 + 2\sum_{i=1}^t \rho_i$ is quite small owing to the equation $\rho_i \approx -0.5\delta_{1i}$, and $H = \frac{1}{2}$ for large t . An empirical study of the price diffusion is presented in Fig. 5.

We see from the panel (b) of Fig. 4 that the autocorrelation functions of the absolute value of price shift (empirical volatility) have long memory. Both data plotted there are well fitted by power laws. The original data, however, hold the memory of volatility stronger than the surrogate data do.

5. Conclusions

Taking the strategy leaving the decision of the limit price to the others in the stochastic model of financial markets driven by continuous double auction, the virtual market shows the power-law tail of the distribution of returns with the exponent near 3 according to the parameter which determines the ratio of the mimetic limit order. The short memory of returns and the long memory of volatilities are also reproduced by the model. The Hurst exponent H of our model is asymptotically $\frac{1}{2}$. The mean field approximation explains the profile of the autocorrelation function, which is responsible for the value of the Hurst exponent H . The strategy assumed here are effective in holding the memory of market volatility strong.

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