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Stylized facts in internal rates of return on stock index and its derivative transactions

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Abstract

Universal features in stock markets and their derivative markets are studied by means of probability distributions in internal rates of return on buy and sell transaction pairs. Unlike the stylized facts in normalized log returns, the probability distributions for such single asset encounters incorporate the time factor by means of the internal rate of return, defined as the continuous compound interest. Resulting stylized facts are shown in the probability distributions derived from the daily series of TOPIX, S & P 500 and FTSE 100 index close values. The application of the above analysis to minute-tick data of NIKKEI 225 and its futures market, respectively, reveals an interesting difference in the behavior of the two probability distributions, in case a threshold on the minimal duration of the long position is imposed. It is therefore suggested that the probability distributions of the internal rates of return could be used for causality mining between the underlying and derivative stock markets. The highly specific discrete spectrum, which results from noise trader strategies as opposed to the smooth distributions observed for fundamentalist strategies in single encounter transactions may be useful in deducing the type of investment strategy from trading revenues of small portfolio investors.

Keywords: Universal features; Stylized facts; Returns on strategy; Trend investment; Technical analysis; NIKKEI 225 derivative market

1. Introduction

Stylized facts in market indicator data [1–7] have been reported, up to date, especially on the session-to-session basis, such as the fat tail law in the distribution of log-normalized returns $R_t \equiv \log(P_i/P_{i-1})$ on various time scales and aggregation levels for numerical data P_i at time ticks $i = 1, \ldots, N$. The reasons for introducing this form of statistics are well known: the values P_{i-1} and P_i should a priori correlate most strongly; the relative factor P_i/P_{i-1} is suitable for the analysis of market trends; and the application of the log function brings additional symmetry between the cases of indicator increase and decrease, while preserving the original value of the relative factor for small market changes, $\log(1+x) \sim x$ for $|x| \ll 1$. Finally, it is well established that the histograms of R values are practically symmetric for long-term data series with respect to the inversion of $R \to -R$, and may have fat tails (relative to the normal distribution), which are known as the stylized facts. The above factors constitute a strong case for studying the indicator data in terms of probability distributions

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of normalized log returns. Last but not least, the ratio $P_i/P_{i-1} - 1$ is the profit (or loss) from buying the asset at session (or tick) $i-1$ and selling it at i without transaction costs.

In this paper, we follow and extend the interpretation of R_t as an elementary transaction (i.e., a function on tick data) as a means of market data analysis. While the standard form of this function, i.e., $P_i/P_j - 1$ is based only on $i-j=1$ correlations and can be both positive and negative, our intent is to develop a more general criterion which can allow for complex correlations between all ticks, i.e., $j<i$ within the entire data sample. In particular, the size of the tick segment $i-j$ in our framework *depends* on the analyzed data, and is defined by taking j as the minimum of $i'>i$, for which a required *internal rate of return* (IRR) is achieved on the tick data pair R_i and R_j. In broader terms, by using a probability density of returns on fundamental transactions, it may be possible to relate market stylized facts (data) to the ultimate paradigm of market investment, the optimal transaction (strategy). The problem of optimal investment [8] is also central in mathematical finance.

Both the standard distribution of normalized log returns, and the above motivated distribution of internal rates of return (IRR transform) result from elementary transactions on a risky asset, hereafter referred to as a single encounter. The optimal transaction may involve a two-component portfolio (money and a stock index in this paper) or a multi-component portfolio with particular stock titles. Depending on whether the transaction profit is reinvested or not, the geometric mean or the arithmetic mean of the expected return of single encounter investment should be optimized [9]. The former case of geometric mean is known as the Kelly criterion [10], which specifies the optimal single encounter ratio x for one risky asset with return b and probability p by $x = p - (1-p)/b$, in order to maximize the expectation value of the logarithm of the single encounter outcome, i.e., the economics' utility function on one hand, which is identical with the entropy of this system on the other hand. That is where the present, probabilistic formulation of single asset encounter naturally relates to the Shannon's information entropy, and its generalized models.

In general, the present approach should complement a variety of econometric, econophysics, statistical and artificial intelligence approaches to time series analysis [11–14].

The paper is organized as follows. Section 2 explains the theoretical rationale of the generalized stylized features as a transform in the internal rates of return. The single encounter investment in the continuous interest model is applied to the time series of TOPIX, S & P 500 and FTSE 100 data, for which the resulting IRR transforms are derived and their stylized features are discussed. In Section 3, the characteristic spectrum of one particular single encounter strategy based on the comparison of short-term and long-term trends is shown. Section 4 demonstrates both the standard and the present formulations of the stylized facts on the minute-tick data from NIKKEI 225 index, and its futures market, and discusses their causal relation in terms of the IRR transform. In Section 5, the stylized facts in the IRR transform are distinguished from the standard formulation in terms of normalized log returns. We conclude with final remarks in Section 6.

2. Internal rates of return—market transform

As mentioned above, the stylized facts in indicator time series [3–7] are usually studied by means of log-normalized returns, $\log(P_i/P_{i-1})$. Let us consider an investment on stock market index (which itself is one of the fundamental types of portfolios) for a variable period of t sessions (or ticks) with a minimal required profit rate r per session. In other words,

$$0 = C_0 - \frac{C_t}{(1+r)^t} \tag{1}$$

with an unknown period t, cash flow C_0 equal to the buying price, and cash flow C_t equal to the selling price. The termination condition for the investment thus requires

$$P_t \geqslant P_0(1+r)^t \tag{2}$$

at the smallest possible time t for which the above inequality holds. In economics terms, this can correspond to the maximal implicit interest rate r of reverse repo operation in the interval $\langle 0, t \rangle$. The fixed value of r will be the basis of the IRR transform.[1]

[1] Although the fixed r-value is a mathematical concept here, it also corresponds to the fundamentalist strategy.

By using the compound discounting across market sessions, and the standard identity

$$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = \mathrm{e}^a,$$

the termination condition in Eq. (2) is equivalent to

$$P_t \geqslant P_0 \mathrm{e}^{\rho t},$$

where ρ is the IRR density in the continuous compound interest model, hereafter the independent variable of the IRR transform. Let us note that in case of $t = 1$, the ρ value approximately reduces to the log-normalized return. For any fixed value of ρ, there is a certain probability $p(\rho)$ to realize this value in the market, which can be computed for instance by Monte Carlo simulation or a full scan of all data points, $i = 1, \ldots, N$, depending on the size of data sample, and the required accuracy of the result. The boundary conditions for $p(\rho)$ in standard data samples are $\rho \to 0 \Rightarrow p \to 1$, and $\rho \to \infty \Rightarrow p \to 0$. For most actual market data, larger values of ρ imply a very rapid decay of $p(\rho)$, because of the exponential explosion in the compound interest function, which cannot match the limited rates of return achievable in the real markets.

The quantity $\rho p(\rho)$ is the expected IRR, and must have a non-trivial local maximum at some point ρ^*. Since in general $t \neq 1$, the stylized facts in $I(\rho) \equiv \rho p(\rho)$ are different from the stylized facts in the log-normalized returns, which would be $R(\rho)p(\rho)$. More than 30% of the actual distributions dealt with here are found to correspond to $\Delta t \geqslant 2$ events (cf. Section 5).

There is a significant difference in data processing between the standard approach and the IRR transform. In case of the normalized log-return distributions, the computed values $\log(P_i/P_{i-1})$ are partitioned into histogram bins in order to obtain the probability distribution; in case of the IRR transform, the probability distribution is obtained directly as the ratio of j's, $j < N$, for which a required value of ρ was achieved. This defines a transform over an interval of fixed internal rates of returns. Because of the exponential behavior of compound interesting, the size of the data sample has no effect on the distributions of IRR even for moderate values of N. Let us also note that in both cases, the probability values are obtained ex post from the index data, and therefore their economic interpretation implicitly assumes no feedback between the single encounter virtual transactor and the market.

Fig. 1 shows such probability distributions for 2458 daily values of three stock indices, TOPIX, S &P 500 and FTSE 100 for a 10 year period. The functional form of all three distributions is remarkably similar; in addition, the optimal point ρ^* coincides to high accuracy for all three distributions, $\rho^* \sim 0.007$, an unexpected fact even in case when the asymptotic power law exponents were similar. Note that for asymmetric markets with a strong downward trend, for instance, the value of $p(0)$ may be less than the a priori normalization (there may exist data points at which no positive ρ could be achieved). Thus, in spite of the absence of a priori expectations on the magnitude and shape of curves in Fig. 1, the similarity of their functional form, and their coinciding maxima (although representing a less stringent test) document the stylized similarity among the

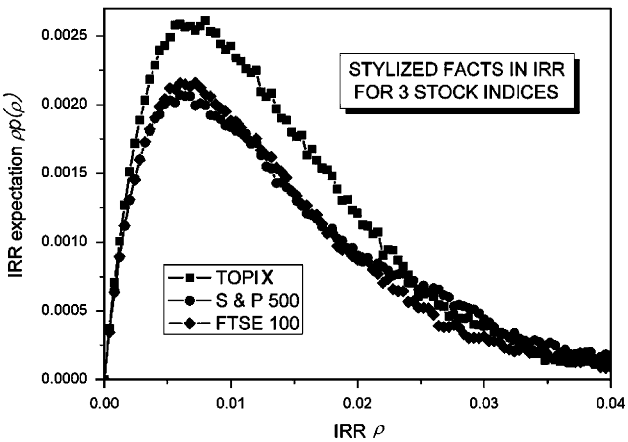

Fig. 1. Stylized facts in the IRR transform for three stock indices in the period of 1995/09/01 to 2005/08/30 (daily quotes).

three markets represented by the stock index time series. Let us also note that the close similarity of the S & P 500 and FTSE 100 distributions may be based on the strong economic and financial relations between the US and the UK.

The empirical distribution $I^*(\rho)$ for TOPIX in Fig. 1 serves as a reference case for a transform of discrete strategies discussed in the next section, and also the generalizations of IRR transform discussed in Section 4.

3. Internal rates of return—strategy transform

The above proposed IRR formulation of the stylized facts in market indicator time series has one important motivational point. While the log-normalized returns build on the $[i-1, i]$ transaction pairs, the IRR is defined for any $[i, i']$ transaction, $0 < i < i' < N$, not necessarily restricting to the preset fixed value in the IRR transform above. Thus, given a market and any sort of single encounter framed strategy, one always obtains an IRR probability spectrum, which can characterize both the market, and the strategy.

Since the log-normalized returns do not allow for evaluation of single encounter strategies for non-adjacent ticks, it is interesting to study the spectral projection of such strategies, i.e., their I(ρ) distributions, within the IRR transform, namely the difference from a reference spectrum $I^*(\rho)$ (cf. e.g. the curve for TOPIX in Fig. 1). This section discusses the possible benefits of such an approach, and uses the moving average (MA) Dead Cross/Golden Cross trend strategy for illustrations.

To address the important case of noise trading, we adopt a buy signal/sell signal strategy, which is based on the comparison of short- and middle-term trends (often used as a reference by online brokers and elementary investment handbooks). This approach is based on the comparison of MAs,

$$P^{MA}(t) = \frac{1}{n_{max}} \sum_{n=0}^{n_{max}-1} P_{t-n},$$

in the short-term ($n_{max} = S = 5$) and middle-term ($n_{max} = L = 25, 75, 200$) periods.

According to the model, a strong bullish market trend is signaled by an intersection of the rising short-term and middle-term curves, a signal to buy. A strong bearish market trend is signaled by an intersection of the falling short- and middle-term market curves, a signal to sell. These two turning points are routinely referred to as the Golden Cross and the Dead Cross and are usually associated with analysts' recommendations to buy and sell, respectively. Although the Golden Cross point could be used in connection with some required IRR value, for instance, we assume the investment lasts from the buy to the sell signal, which is consistent e.g. with the period from market bubble formation to its burst. Thus, provided the two signals originate on the opposite sides of a local maximum in the market, an investment between the two turning points may turn profitable. Since both cross points are based on relative criteria, there is no a priori guarantee that the resulting IRR density $I(\rho)$ from such transactions will in fact be positive, in contrast to the case of IRR market transform.

The strategy is formalized as follows. Let us denote the daily short-term and middle-term MA time series as θ_i and λ_i, respectively, and their difference $\delta_i = \theta_i - \lambda_i$. The investment policy is then determined by the buy and sells signals arising in the market:

- Buy: $\delta_i \times \delta_{i-1} < 0$ (cross), $\lambda_i - \lambda_{i-1} > 0$ (bullish long-term trend), $\theta_i - \theta_{i-1} > 0$ (bullish short-term trend) and $(\theta_i - \theta_{i-1}) - (\lambda_i - \lambda_{i-1}) > 0$ (acceleration of the long-term bullish trend).
- Sell: $\delta_i \times \delta_{i-1} < 0$ (cross), $\lambda_i - \lambda_{i-1} < 0$ (bearish long-term trend), $\theta_i - \theta_{i-1} < 0$ (bearish short-term trend) and $(\theta_i - \theta_{i-1}) - (\lambda_i - \lambda_{i-1}) < 0$ (acceleration of the long-term bearish trend).

By using the Monte Carlo computer simulation method, the initial day of investment i_0 is decided at random between the values $i = 1$ and N, the size of the data set. Then the data are scanned for $i_0 < i \leqslant N$ until the buy signal is found at some $i = i_b$. The day i_b corresponds to the start of the investment; the data are scanned again, $i_b < i \leqslant N$, for the reverse signal to terminate the investment at some $i = i_s$. The termination condition defines the respective ρ resulting from each transaction k as

$$P_{i_s^{(k)}}/P_{i_b^{(k)}} = \exp(\rho^{(k)}(i_s^{(k)} - i_b^{(k)})),$$

and its relative weight in the contribution to the IRR spectrum is $\rho^{(k)}$.

Fig. 2 shows the discrete spectrum for the 2458 daily values of the TOPIX index for three periods L defining the middle-term trend. As the length of the middle term increases, the frequency of cross events decreases, and the distribution becomes more singular. The spectrum $I(\rho)$ extends to the negative region of ρ. Although the data for the S & P 500 and FTSE 100 indices are not shown in Fig. 2, their corresponding spectra are also

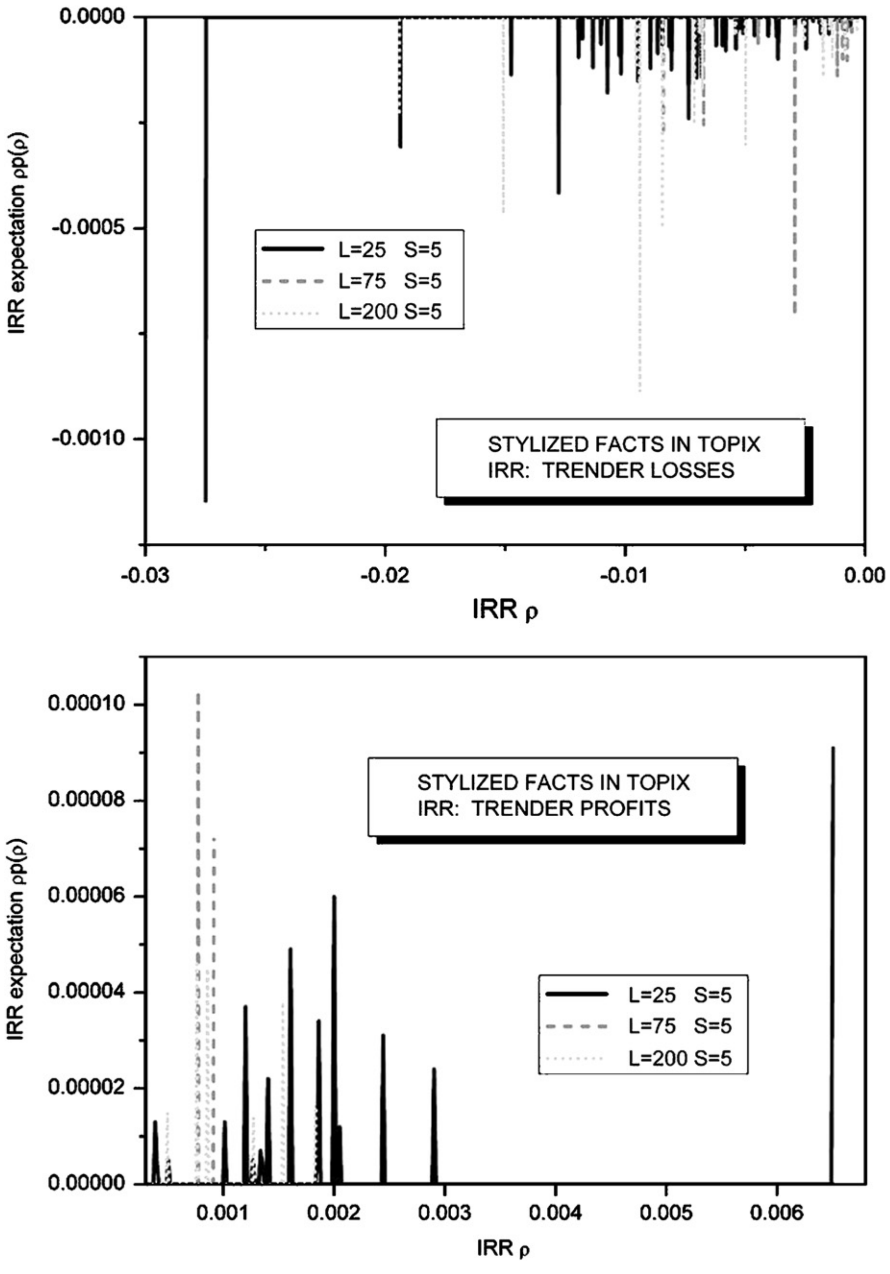

Fig. 2. Discrete spectrum of the Golden Cross/Dead Cross strategy for the TOPIX data between 1995/09/01 and 2005/08/30 (cf. Fig. 1 for the IRR transform). Upper panel: losses, lower panel: profits. The short-term trend is taken as one week floating average ($S = 5$); the curves show the variance in the spectrum for middle- to long-term trends defined by 25, 75 and 200 data point averages.

discrete, and specific for each market. However, because of the relatively infrequent cross point events in the data set, the statistics in Fig. 2 is rather qualitative. It is, however, sufficient to support the discrete spectrum findings.

The spectrum of the IRR transform can be a priori mathematically formulated for any rational single encounter policy which solely uses regressive data to generate buy and sell signals, although the IRR transform itself is not restricted to such assumptions. Let $\{P_i\}$ be the set of data points, I the set of ticks $i = 1, \ldots, N$, $B \in I$ the set of buy signal ticks B_i $(i = 1, \ldots, i_b)$, $S \in I$ the set of sell signal ticks S_i $(i = 1, \ldots i_s)$; remove the points B_i for which there exists no $S_j > B_i$, and the points S_j for which there exists no $B_i < S_j$ (thus decreasing i_s to i'_s, and i_b to i'_b). By denoting $\theta(B_i) \equiv \min(S_j > B_i)$ for $B_i \in B$, the IRR transform has a discrete spectrum of i'_s values $\lambda_i \log(P_{\theta(B_i)}/P_{B_i})/(\theta(B_i) - B_i)$ with intensities proportional to $|S_i - S_{i-1}|$ $(S_0 \equiv 0)$. Note that there can also arise random degeneracies between the spectral values λ_i and λ_j for $i \neq j$. The above analytical spectrum can serve as a reference point for analyzing discrete and quasi-continuous IRR spectra of more complex (irrational, stochastic, etc.) strategies.

4. IRR transform and causality

In this section, the advantages of the normalized log return and the IRR transform of minute-tick data from NIKKEI 225 and its futures market are examined. The data sample is taken between 9:01 on January 4th to 15:05 on June 8th, 2000, which covers most of the first two quartal NIKKEI 225 futures emissions in 2000. The log-normalized return distribution for the NIKKEI 225 index, and the differences of the futures market from the underlying index are shown in the upper and lower panel of Fig. 3, respectively. Within the statistic

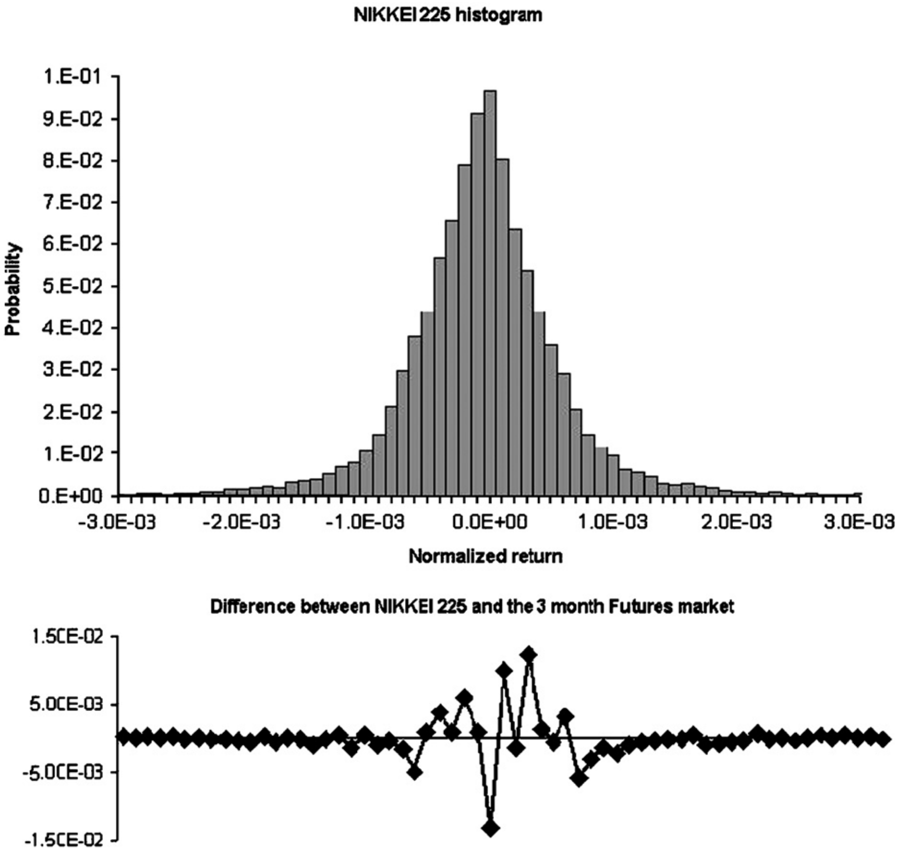

Fig. 3. The histogram of log-normalized returns of NIKKEI 225 (upper panel) and the difference from the probability distribution in the futures market (lower panel).

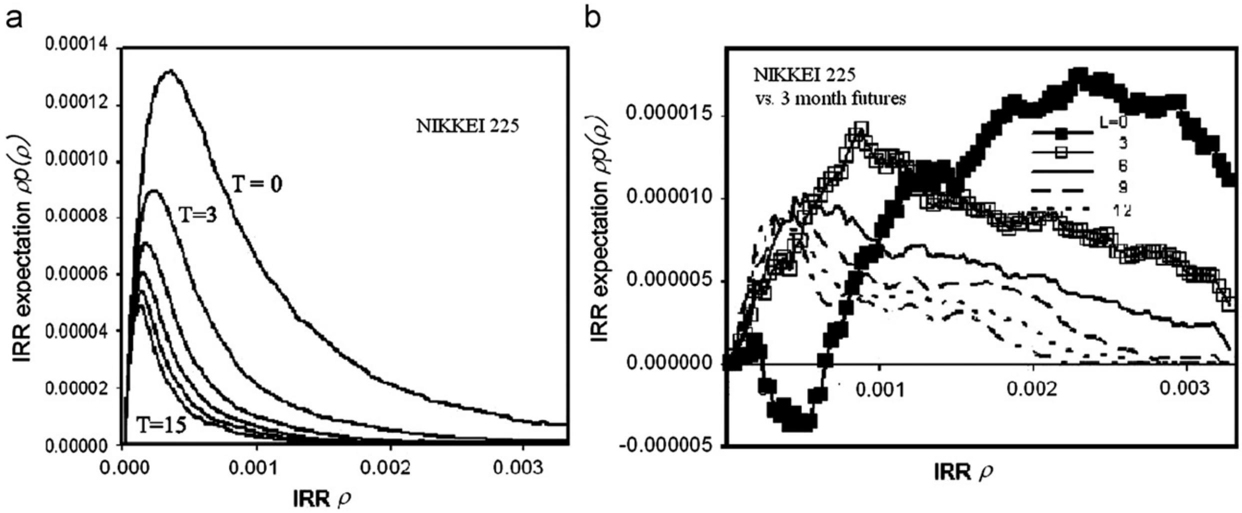

Fig. 4. The IRR transform: (a) the $I(\rho)$ distributions of NIKKEI 225 index for minimal investment periods of $T = 0, 3, 6, 9, 12$, and 15 sessions (the magnitude of the distributions decreases monotonously with T). (b) The difference of $I(\rho)$ distributions between the NIKKEI 225 market and its derivative market.

of this 29,810 point data sample, the difference of the two R-distributions has four confirmed nodes within the FWHM region, sized in the order of up to 10% of NIKKEI 225 histogram. Note that the statistical fluctuations in the node region shown should not exceed 4% according to the standard square root rule, i.e., 10^{-3} on the absolute scale of Fig. 3. Although the nodes are certainly an interesting result, there is no straightforward way how to interpret their number and their meaning in the differential histogram. Summarizing, the differential histogram is not a tool that could reveal a correlation or causal relation of the two distributions.

Next, the IRR transform is applied to the differential analysis of the same data samples. In Fig. 4(a), the $I(\rho)$ distributions obtained with an imposed minimal investment period T are shown. Because of the compound interest, it becomes increasingly difficult to maintain a constant IRR density ρ over a prolonged period; therefore the height of the distributions monotonously decreases with T increasing in the series of $0, 3, 6, 9, 12$ and 15 min. More interestingly, Fig. 4(b) shows the differences of $I^{(T)}(\rho)$ between the index and its futures market for the two probability distribution T-series. Only in the case of $T = 0$, there appear two nodes, with the maximum value in the negative direction approximately corresponding to the peak of the $I(\rho)$ distributions. For $T \geqslant 2$, all nodes disappear. Therefore the IRR transform provides a means to discriminate short and longer term tick data correlations between the two markets, which was impossible with the differential histogram, and differentiated the time series data from the two markets on a more systematic basis (cf. Figs. 3 and 4b). In a different study, we have proposed an optimized symbolic analysis method for the alignment of the indicator time series [15]. Based on the alignment of the underlying index with the futures data series, we suggest that the nodes for $T = 0$ correspond to the lead–lag relation of the two markets within this region, which is approximately $\Delta t \sim 1$. Together with the finding in Fig. 4(b), it therefore appears that a series of the $I^{(T)}(\rho)$ for various values T may be useful in lead–lag causality analysis of closely related financial markets.

5. The root of IRR stylized facts

There is an important question on the nature of the stylized facts in the IRR transform defined in Section 2 and generalized in Section 4, namely what is their difference from stylized facts in terms of normalized log returns (e.g. the power law exponent of the fat tails). To that aim, we bootstrap the high frequency NIKKEI 225 minute-tick series data (sample from Section 4) by computing a histogram of the normalized log returns, from which artificial time series are created as a corresponding random walk, and compare the resulting IRR transform with that of the original data (cf. Fig. 5).

Fig. 5(a) shows the IRR distributions for the original and bootstrapped data. The functional form of the original distributions is again similar to that of daily data in Fig. 1. The differences in the IRR transforms are beyond the statistical uncertainty corresponding to the size of the data sample, and therefore indicate the existence of longer term correlations. The inset in Fig. 5(a) illustrates the difference of the real and random data, focusing on correlations exceeding three ticks ($T = 3$). A differential plot for the IRR distributions from Fig. 5(a) is shown in Fig. 5(b). Since the IRR transform is always locked to zero for $\rho = 0$, the differential $p(\rho)$ distributions are also shown in Fig. 5(c) for $T = 0$, and in Fig. 5(d) for $T = 3$, along with their statistical uncertainties $\pm\Delta_{stat}$. Both Figs. 5(c,d) show that the distributions differ beyond the statistical error, and therefore the IRR transform reproduces causality contents not captured by the normalized log returns. Similarly to Section 4, there is also an interesting conditioning of the differential distributions in $p(\rho)$ by the minimal values of T (Figs. 5(b,c,d)).

Let us note that the IRR-formulated stylized facts in Fig. 5 are presented for high-frequency tick data drawn from a high-liquidity market; the original and the bootstrapped IRR distributions are therefore relatively similar, and demonstrate that the Efficient Market Hypothesis is indeed a good approximation for the 225 components of the NIKKEI index on short-time scales. Longer time scale data, e.g. those including dividend streams, or more substantially correlated data such as those from the emerging markets, will further differentiate

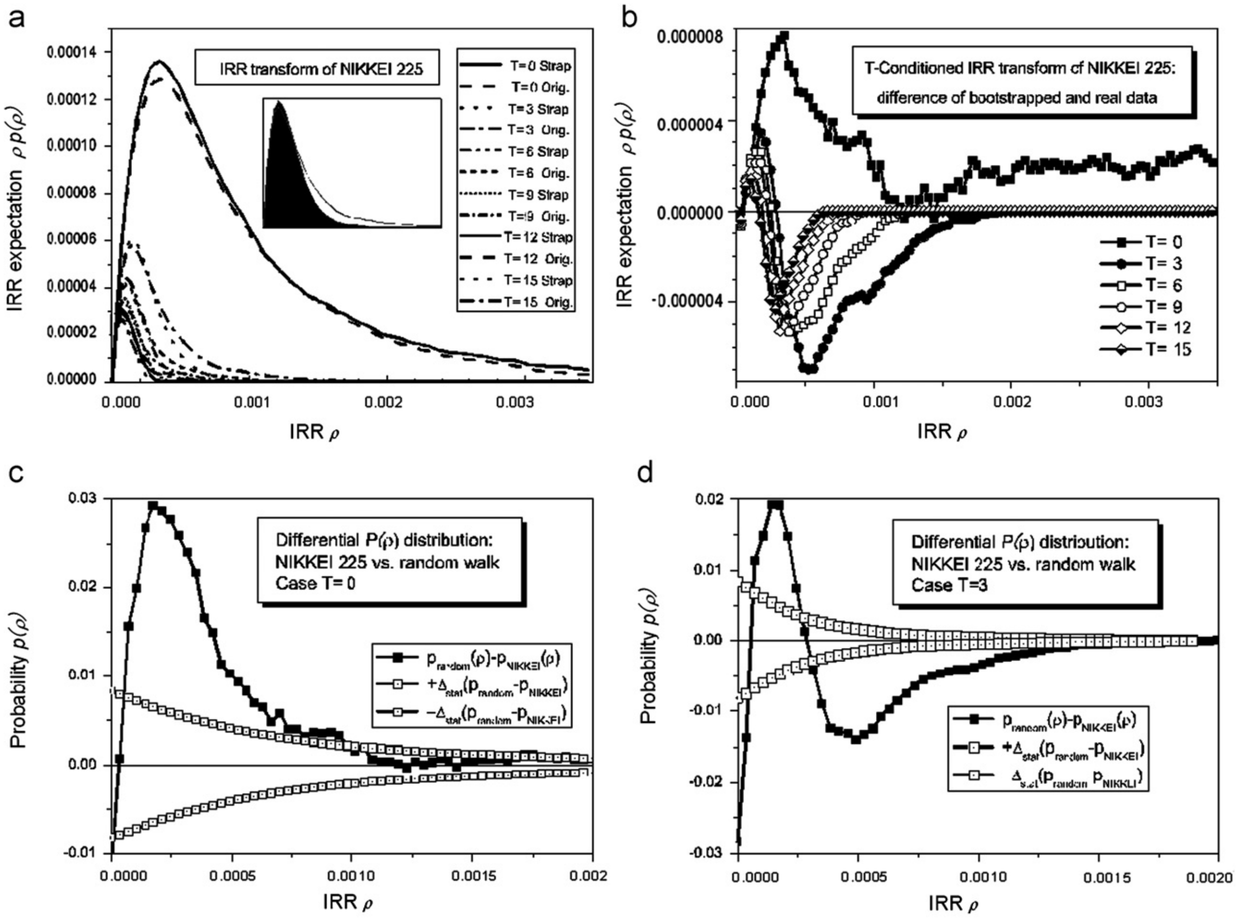

Fig. 5. Comparison of the IRR transform on a 29,810 minute-tick data sample of NIKKEI 225 with its bootstrapped data: (a) the original and bootstrapped T-conditioned IRR transforms (the inset depicts the difference for $T = 3$), (b) difference of the original and bootstrapped curves in (a), (c) difference of $P(\rho)$ curves for $T = 0$, (d) difference of $P(\rho)$ curves for $T = 3$.

the IRR distribution from the bootstrapped one, revealing long-term correlations. A further investigation would be in place to decide upon the form and origin of stylized facts in such a case. In case of irregular market events, such as bubble formation, external shocks or crashes, the IRR transform may be a useful tool for the decomposition of overall market trend into the frequency components *rho*. In addition, its formulation in terms of the inception and termination investment dates could further be related to general trading strategies (cf. Section 3), thus adding a bias to the a priori neutral Monte Carlo screening of the market data.

6. Conclusion

A framework for studying market statistics and its stylized facts based on the internal rate of return from single encounter transactions has been proposed and compared with the common log-normalized return approach. Based on the present formulation, an interesting stylized fact was observed in the coinciding functional form $I(\rho)$ and the location of the optimal density of internal rate of return, ρ^*, in the probability distributions of TOPIX, S & P 500 and FTSE 100 index data series on a 10 year scale. The IRR transform allows, in addition, for a spectral characteristics of single encounter strategies, which was demonstrated on the TOPIX data series for several cases of trend-following strategies. The discrete $p(\rho)$ spectrum may serve as a fingerprint of particular market strategy; it can also possibly allow for determining the type of investment strategy from the series of its returns on individual transactions. The discrete spectrum feature may also present an interesting point of view on the possible role of moving averages in the stylized facts in numerical time series. The IRR transform has been further generalized by introducing a minimal interval T on compound interest time period, which screens the $I(\rho)$ spectrum in a monotonous way. This generalization allowed for indirect identification of the lead–lag relation in the minute-tick series of NIKKEI 225 and its derivative market in the first two quartals of the year 2000.

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References

[1] H.E. Stanley, V. Plerou, Scaling and universality in economics: empirical results and theoretical interpretation, Quant. Finance 1 (2001) 563–567.
[2] S.-H. Chen, T. Lux, M. Marchesi, Testing for non-linear structure in an artificial financial market, J. Econ. Behav. Organ. 46 (3) (2001) 327–342.
[3] T. Kaizoji, M. Kaizoji, Exponential laws of a stock price index and a stochastic model, Adv. Complex Systems 6 (3) (2003) 1–10.
[4] A. Carbone, G. Castelli, H.E. Stanley, Time-dependent Hurst exponent in financial time series, Physica A 344 (2004) 267–271.
[5] F. Wang, et al., Scaling and memory of intraday volatility return intervals in stock market, Phys. Rev. E 73 (2005) 026117.
[6] J. McCauley, Thermodynamic analogies in economics and finance: instability of markets, Physica A 329 (2003) 199–212.
[7] Y. Chen, E. Lopez, S. Havlin, H.E. Stanley, Universal behavior of optimal paths in weighted networks with general disorder, Phys. Rev. Lett. 96 (2006) 068702.
[8] F. Slanina, On the possibility of optimal investment, Physica A 269 (1999) 554–563.
[9] D. Bernoulli, Exposition of a new theory on the measurement of risk (1738), Econometrica 22 (1954) 23–36.
[10] J.L. Kelly Jr., A new interpretation of the information rate, Bell Syst. Tech. J. 35 (1956) 917.
[11] J.D. Farmer, D.E. Smith, M. Shubik, Is economics the next physical science?, Phys. Today 58 (9) (2005) 37–42.
[12] T. Lux, S. Schornstein, Genetic learning as an explanation of stylized facts of foreign exchange markets, J. Math. Econ. 41 (1–2) (2005) 169–196.
[13] P. Sen, B.K. Chakrabarti, Small-world phenomena and the statistics of linear polymers, J. Phys. A 34 (2001) 7749–7755.
[14] H. Takayasu, A. Sato, M. Takayasu, Stable infinite variance fluctuations in randomly amplified Langevin systems, Phys. Rev. Lett. 79 (1997) 966–969.
[15] L. Pichl, T. Yamano, T. Kaizoji, On the symbolic analysis of market indicators with the dynamic programming approach, Lecture Notes in Computer Science 3973 (2006) 432–441.