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Extracting the exponential behaviors in the market data

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Abstract

We introduce a mathematical criterion defining the bubbles or the crashes in financial market price fluctuations by considering exponential fitting of the given data. By applying this criterion we can automatically extract the periods in which bubbles and crashes are identified. From stock market data of so-called the Internet bubbles it is found that the characteristic length of bubble period is about 100 days.

Keywords: Exponential behaviors; Bubble; Crash; Financial market price fluctuation

1. Introduction

Recently, the analysis of bubbles and crashes in financial markets is attracting much attention in the field of econophysics [1–4]. One of the pioneering studies has been done by Sornette et al. [5–9]. They introduced a specific functional form to describe bubbles and crashes by considering the end of bubble as spontaneous singularity occurring at a certain critical time. According to this method, the end of bubble can be predictable, however, so far, there is no mathematical definition or criterion formula for the onset of these phenomena. With the least numbers of parameters we would like to define the period of bubbles or crashes automatically.

In order to tackle this problem we pay attention to an empirical fact that an exponential curve fits well to bubble or crash data than the popular linear trend lines. Here, we try to propose a mathematical definition of bubbles and crashes, so that we can automatically specify the period of a bubble or a crush.

In this paper we analyze the high-frequency NASDAQ data focusing on the Internet bubble or the dot-com bubble appeared at the end of 20th century. We calculate an average of prices every thirty seconds in the tick data. Regular trading time in NASDAQ is from 9:30 to 16:00, so the number of data points in a day is 780.

2. Extraction of the exponential behaviors

We introduce the following formula for extracting the exponential behaviors in the financial time series

$$P(t) - P_0(i;T_i) = \omega_1(i;T_i)\{P(t-1) - P_0(i;T_i)\} + F(t). \tag{1}$$

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This formula has an autoregressive (in short AR) form where the current state is given by the past states. In this formula, $P(t)$ is a price at time t and $\omega_1(i; T_i)$ is the parameter characterizing the exponential behaviors in the ith period of length T_i. If $\omega_1(i; T_i)$ is larger than 1.0 then the time series is either exponentially increasing or decreasing, and $P_0(i; T_i)$ gives the base line of these exponential divergence. If $\omega_1(i; T_i)$ is less or equal to 1.0, it means that there is no bubble-like trend or the time series is convergent, and $P_0(i; T_i)$ shows the asymptotic value of convergence. $F(t)$ is the residual noise term. The parameters $\omega_1(i; T_i)$ and $P_0(i; T_i)$ can be determined uniquely under the condition that minimizes the errors, which is calculated by the sum of squares of $F(t)$.

3. Estimation of the optimal period

For applying Eq. (1) to the time series, we estimate the length of the period T_i that can be fitted by an exponential function. We introduce a minimum period of T_i by using the following AR model for the price difference time series.

$$\Delta P(t) = \sum_{j=1}^{j=N-1} b_j \Delta P(t-j) + f(t), \tag{2}$$

$$\Delta P(t) = P(t) - P(t-1). \tag{3}$$

Here, b_j gives the AR parameters that make the residue, $f(t)$, almost an independent random noise [10–12]. In this equation we tune the parameters of AR so that the standard deviation of $f(t)$ as similar as possible to the real stock price data such as Yahoo! Inc.(ticker symbol YHOO). Now we define the time scale of T_i by the minimum time scale that satisfies the condition where $\omega_1(i; T_i)$ is always less or equal to 1.0 when the time series is created by the Eqs. (2) and (3) with $N = 5$. By changing T_i from 1 day to 100 days, the frequency of finding $\omega_1(i; T_i)$ larger than 1.0 decreases. When we set T_i to be longer than 100 days, we cannot observe $\omega_1(i; T_i)$ to take a value larger than 1.0 in practical sense. Therefore, we fix the optimal time scale T_i for observing the exponential behaviors to be 100 days for the stock price of Yahoo!. On the basis of this AR analysis, if we observe $\omega_1(i; T_i)$ larger than 1.0 in the time range of 100 days in real data, we can say that the real time series of that range is statistically different from the AR model. In this case we need to introduce a non-stationary description.

4. Assignment of bubble, crash and convergence

We now assign each time step (one time step is thirty seconds) either exponential or convergent. If the observing box of 100 days is judged as exponentially diverging, namely $\omega_1(i; T_i)$ is larger than 1.0 in the box, we assign all time steps in the box as exponential. Then, we shift the box by one time step, and calculate $\omega_1(i; T_i)$ for the new box. If the value of $\omega_1(i; T_i)$ is less than 1.0, then only the latest time step is assigned as convergent (Fig. 1). Repeating this process to cover all the data we can separate the exponential periods and convergent periods as shown in Fig. 2. At this stage the length of each period takes any value independent of the observing time scale T_i. Note that we can detect the start of slow exponential behavior before the extreme price fluctuations where $\omega_1(i; T_i) > 1$ as shown in Fig. 3.

Next, we calculate the parameters $\omega_1(i; T_i)$ and $P_0(i; T_i)$ to each period by applying Eq. (1) again. Then, we can draw an exponential trend curve for each exponential period as shown in Fig. 2 by using the following equation:

$$P_{trend}(t) = \omega_1(i; T_i) P_{trend}(t-1) + (1 - \omega_1(i; T_i)) P_0(i; T_i), \tag{4}$$

where $P_{trend}(t)$ is the exponential trend price at time t. When the exponential trend is diverging upward we call it as ''bubble'', when it is diverging downward it is called as ''crash''. When the trend curve is converging we call it as ''convergence''. From Fig. 2 we find that the exponential behaviors continue for more than 250 days in the period A. A subtle case in the results of our analysis may be the period D. As this period starts with a sharp drop, it may intuitively look like a kind of ''crash'', however, our analysis gives the trend line showing a convergence to a lower price.

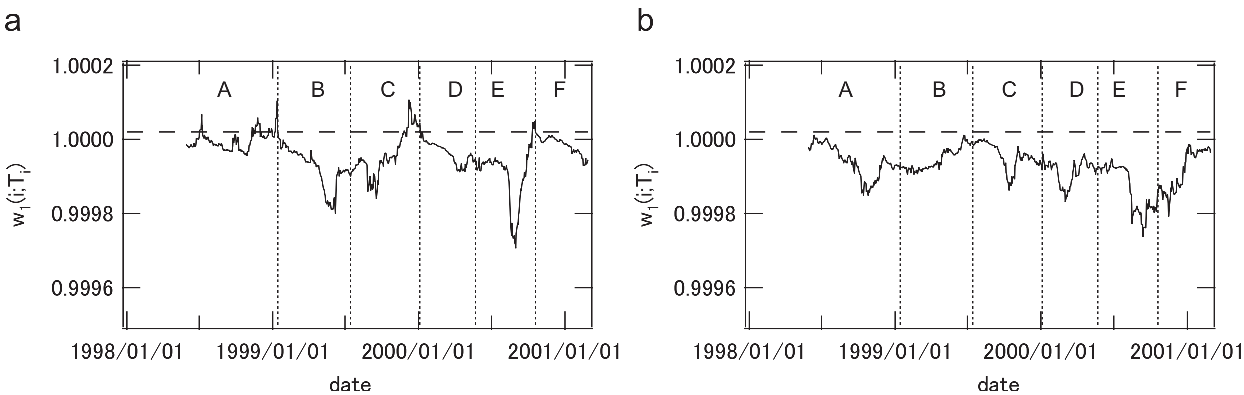

Fig. 1. The time series of $\omega_1(i; T_i)$ with $T_i = 100$ days. (a) The case of YHOO (b) Random walk based on AR(5).

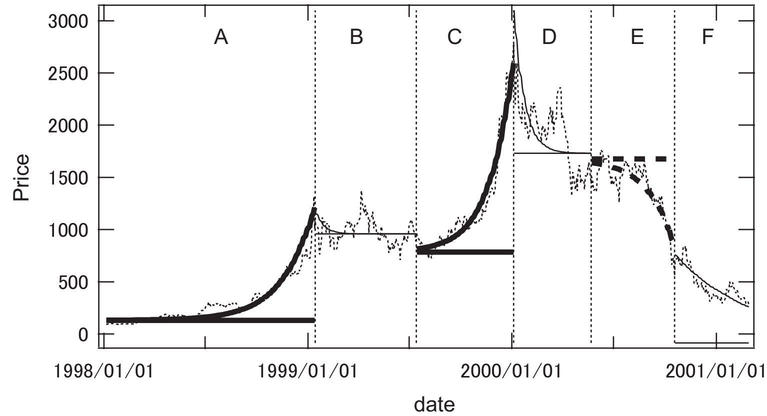

Fig. 2. The exponential trend curves and $P_0(i; T_i)$ in each period. Time series of YHOO (dotted line). The bubble periods (heavy line) are A and C. The crash period (heavy dashed line) is E. The convergent period (line) are B, D and F.

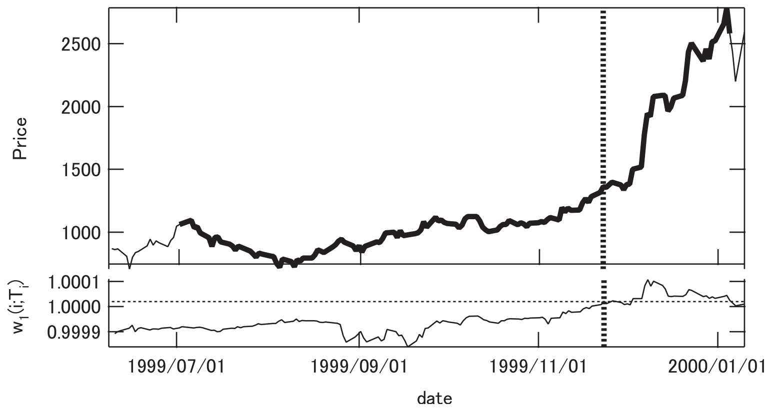

Fig. 3. The time series of a price and $\omega_1(i; T_i)$ in the scale from the end of the period B to the beginning of the period D. The vertical dotted line shows the first point of detecting the bubble.

In Fig. 4 we compare the error estimation between our exponential approximation and the usual linear approximation in each period. The errors $E(i)$ are calculated by the following equation:

$$E(i) = \sqrt{\langle (P(t) - P_{trend}(t))^2 \rangle}. \quad (5)$$

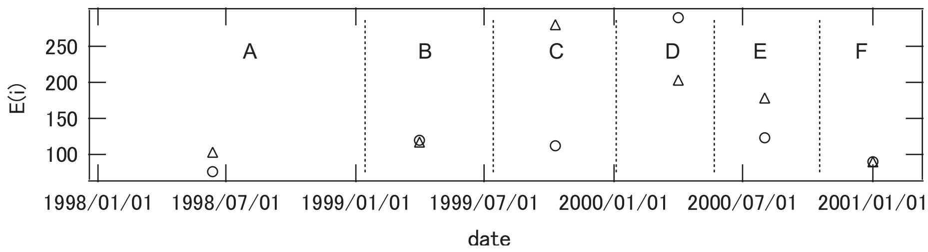

Fig. 4. Comparison of the errors $E(i)$ in each periods. Exponential approximations (circles) and linear approximations (triangles).

The linear approximation is determined by the least-square-method. We can find that errors become smaller for exponential approximation compared to the linear approximation in cases of the bubbles or crashes (the periods A,C and E).

5. Conclusion

In this paper we have mathematically defined bubbles and crashes by the exponential behaviors. As we have discussed in the preceding section this method can be used for prediction of large price changes in macroscopic scale, however, we cannot tell when a bubble will stop. In the theory of "finite-time singularity signature" [5–8], time series of financial indices grow faster than exponential function, and the end of bubble is specified by the diverging point. In our formulation this super-exponential behavior can be represented by growing value of $\omega_1(i;T_i)$. Therefore, if we can observe the tendency of divergence of $\omega_1(i;T_i)$, we will be able to estimate the end of bubble. We are also planning to link the present method with the potential theory of market price in which an exponential growth of price realizes when the potential function is given by a negative quadratic function [13–15]. By adding higher order potential terms we expect that such super-exponential behaviors can be represented in the potential formulation.

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