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Physica A 382 (2007) 340–346
PHYSICA A
www.elsevier.com/locate/physa

Characterization of foreign exchange market using the threshold-dealer-model

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Available online 1 March 2007

Abstract

We introduce a deterministic dealer model which implements most of the empirical laws, such as fat tails in the price change distributions, autocorrelation of price change and non-Poissonian intervals. We also clarify the causality between microscopic dealers' dynamics and macroscopic market's empirical laws.

Keywords: Artificial market; Threshold dynamics; Deterministic process; Nonlinear dynamics and chaos

1. Introduction

Mathematical models of open markets can be categorized into two types. In one type, the market price time series are directly modeled by formulation such as a random walk model, ARCH and GARCH models [1,2], and the potential model [3–5]. The other type is the agent-based model which creates an artificial market by computer programs [6–9]. The agent-based model is able to clarify the relationship between dealers' actions and market price properties. Just like the simple ideal gas model reproducing basic properties of real gas, we can expect that simple dealers' actions can reproduce the empirical laws of market prices.

In this paper we systematically introduce four deterministic dealer models in order to clarify the minimal actions of dealers to satisfy the empirical laws of markets. These are revised models of so-called the threshold model which is originally introduced by one of the authors (H.T.) and coworkers [9] in order to demonstrate that dealers' simple actions can cause deterministic chaos resulting the market price apparently random even the dealers' actions are completely deterministic. We revise the model step-by-step to reproduce most of the empirical laws.

2. Construction of the dealer model

In this section, we introduce four models, from the model-1 to the model-4. In each subsection, we describe dealers' bid price dynamics by differential equations. Firstly, we construct the simplest model named the

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2007.02.027

model-1 and we compare the statistical nature of the model-1 to the real market to clarify the difference between the model and the real market. Then in Section 2.2 (model-2) and Section 2.3 (model-3) we add new effects to the model-1 supported by real data. Finally, we combine the model-2 and the model-3 to construct the model-4 which largely satisfies known empirical laws.

2.1. The model-1

Following the original dealer model [6] we assume an artificial Yen–Dollar market consisted of N dealers, who are offering limit prices of bid and ask. The dynamics of the ith dealer's bid price at time s is given by the following differential equation:

$$\frac{\mathrm{d}b_i}{\mathrm{d}s} = \sigma_i c_i, \tag{1}$$

where c_i is a positive number and σ_i indicates the dealer's strategy at time s,

$$\sigma_i = \begin{cases} +1 & \text{buyer,} \\ -1 & \text{seller.} \end{cases}$$

Namely, if he wants to buy Dollars he raises his bid price monotonically at rate c_i until he can trade, and if he is a seller he decreases the bid price. For simplicity we assume that the value of spread is a constant, L, so the ask price, a_i, is given by $a_i = b_i + L$. Large c_i means that the ith dealer is quick-tempered. The values of c_i are given randomly as an initial condition representing that each one's disposition is different. This heterogeneity is important for realizing transaction.

A trade takes place if the model's parameters satisfy,

$$\max\{b_i\} - \min\{b_i + L\} \geqslant 0. \tag{2}$$

A deal is done between the two dealers who give the maximum and minimum values. The market price is quoted by the mean value, $[\max\{b_i\} + \min\{b_i + L\}]/2$. We assume that dealers in this market have a minimum unit of Yen or Dollar, and after a trade the seller and the buyer change the position.

Although the dynamics is completely deterministic, apparently the market prices move up and down following a random walk (Fig. 1(a)). This randomness is caused by the threshold dynamics. Actually, the autocorrelation of the price change vanishes quickly (Fig. 1(b)). Similarly, both the autocorrelation of volatility defined by the absolute values of price changes and that of time intervals of transaction decay quickly (Fig. 1(c,d)). No volatility clustering can be observed in this model and the occurrence of transaction is fairly modeled by a Poisson process. Namely, the model-1 can be understood as a uniform random noise generator. However, the real market data are characterized by fat tails in the price change distributions, non-Poissonian intervals, and the autocorrelation of price change having larger negative value at one tick than model-1. So, we need to add new effects in the following model-2 and model-3.

2.2. The model-2

We now focus on transaction intervals to make our model closer to the real market. For comparison with real data we analyze a set of tick-by-tick data of the Yen–Dollar exchange rates for 3 months from January 1999 to March 1999. We pay attention to the periods where the frequency of trading is high, such as $6{:}00-11{:}00$ on New York time. The time intervals of transactions tend to make clusters as typically shown in Fig. 2, namely, shorter intervals tend to follow after shorter intervals, and longer intervals follow longer intervals. This effect is called as the self-modulation effect and the distribution of transaction time intervals generally has a longer tail than an exponential distribution [10]. It is known that there exists an optimal moving average and by normalizing the transaction intervals by its optimal moving average the normalized transaction intervals follow nearly the Poisson process with the mean value being unity. This result means that we can construct realistic time interval sequence from a Poisson process.

From the artificial time axis of model-1 we construct a new time axis for the model-2 which satisfies the empirical law of the transaction intervals, so that the time axis of model-2 can be recognized as the real time.

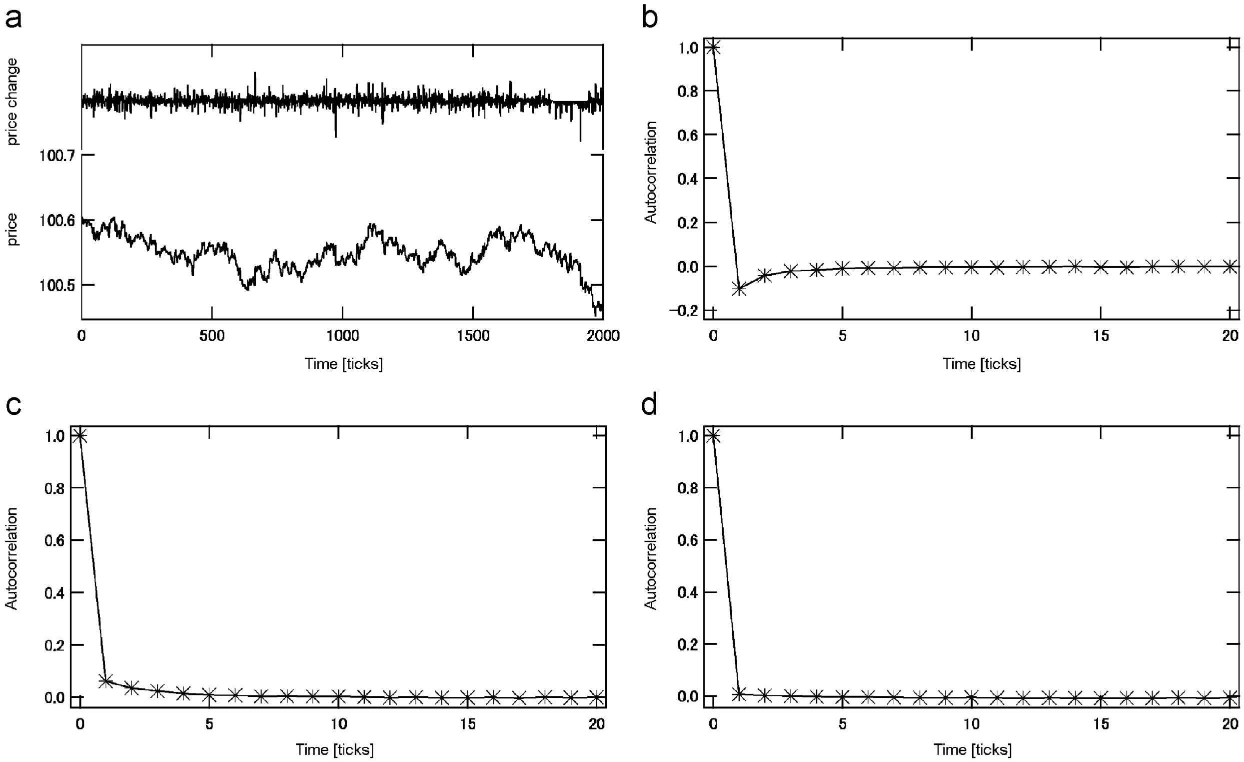

Fig. 1. Results of the model-1. We set parameters: N(number of dealers) = 300, L(spread) = 1.0, $dt = 0.01$, $c_i = [0.001, 0.010]$. (a) Time series of market price (lower line) and price changes (upper line). (b) The autocorrelation of price changes. (c) The autocorrelation of volatility. (d) The autocorrelation of time intervals of transaction.

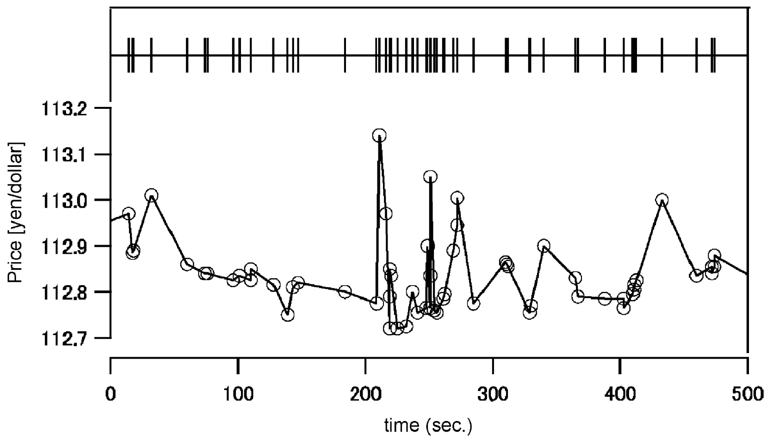

Fig. 2. Time series of real Yen/Dollar rate (bottom) and the transaction intervals (top).

In the model-1 the time intervals denoted as S_k are nearly independent as already shown in Fig. 1, however, the distribution of the intervals does not follow an exponential law. In order to get a pseudo-Poisson process we introduce a modified quantity, $(S_k)^\alpha$, then it is confirmed that the distribution of this new quantity becomes close to the exponential distribution when $\alpha \sim 1.21$. By reversing the tick interval analysis we construct a real time axis using the normalized pseudo-Poisson process $(S_k)^\alpha / \overline{S^\alpha}$, where $\overline{\cdots}$ denotes an average taken over all realizations. The new time axis t and bid price dynamics in the model-2 are defined by the following relation:

$$\begin{cases} \dfrac{\mathrm{d}b_i}{\mathrm{d}t} = \dfrac{1}{G_k \langle T_k \rangle_\tau} \sigma_i c_i, \\ \mathrm{d}t = G_k \langle T_k \rangle_\tau \mathrm{d}s. \end{cases} \tag{3}$$

Here, $G_k = (S_k)^{\alpha-1}/\overline{S^\alpha}$. And T_k corresponds to the kth "real" transaction interval and $\langle T_k \rangle_\tau$ is a moving average of intervals for the latest τ seconds. In order to keep the value of $\langle T \rangle_k$ finite, we restrict the range of $\langle T \rangle_k$ as [6,30]. By this way the "self-modulation" effect is taken into account in this new time axis. Other than this change of time axis the dealer's dynamics in model-2 is equivalent to the model-1. The distribution of transaction intervals of model-2 has fat tails close to the real data as expected (see Fig. 3).

2.3. The model-3

In the model-3 we cast a spotlight on the volatility. In the model-1 the distribution of volatility is exponential as shown in Fig. 4. Now we add a new effect to the model-1, namely the feedback effect of price changes to the dealers. "Trend-follow" is a tendency of dealers' action that they predict price rise when the

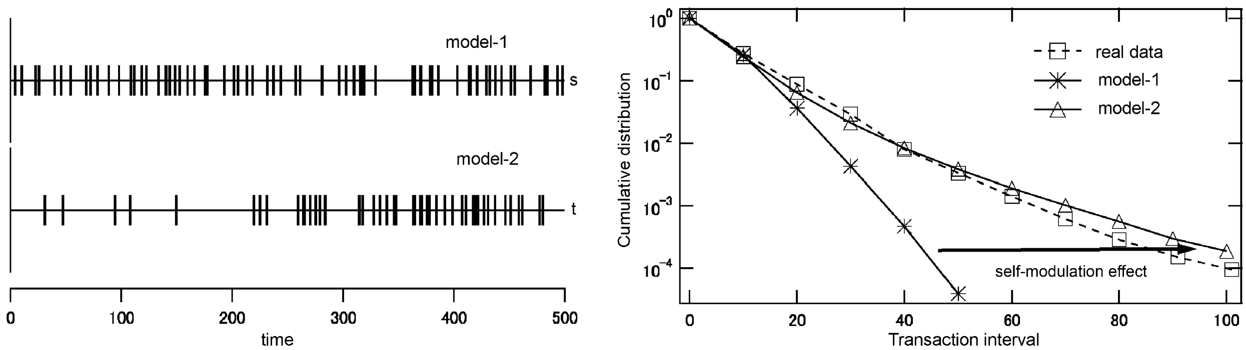

Fig. 3. Comparison of transaction intervals between the model-1 and model-2. The left figure represents the relation of the two time axes and transaction intervals. The right figure shows cumulative distributions of transaction intervals normalized by the average of real transaction intervals. In the model-2 the model parameters are given as follows: N(number of dealers) = 300, L(spread) = 1.0, dt = 0.01, c_i = [0.001, 0.010] and τ = 150.

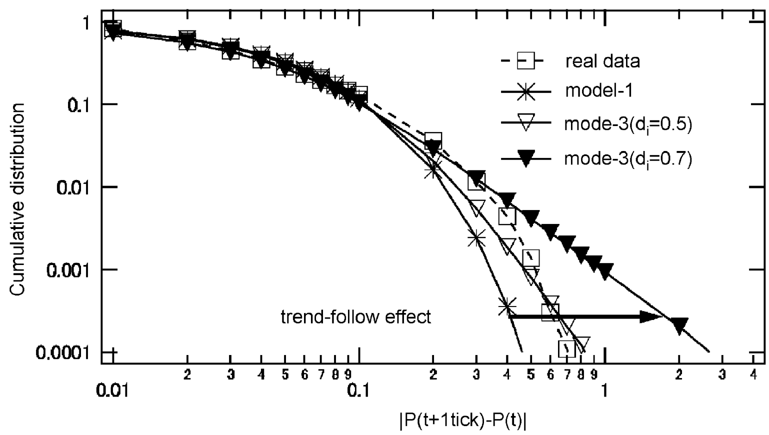

Fig. 4. Cumulative distribution of volatility. The values are normalized by average of real volatility. We can observe that the tails are shifted by the "trend-follow" effect. In the model-3 we use the parameters, N(number of dealers) = 300, L(spread) = 1.0, dt = 0.01, c_i = [0.001, 0.010], the value of d_i is expressed in the figure, and $\langle \mathrm{d}P \rangle$ is the latest price change.

market price has risen. Mathematically we add a term that is proportional to the latest averaged price change, $\langle \mathrm{d}P \rangle_\eta$.

$$\frac{\mathrm{d}b_i}{\mathrm{d}s} = \sigma_i c_i + d_i \langle \mathrm{d}P \rangle_\eta \tag{4}$$

Here $\langle \mathrm{d}P \rangle$ is

$$\langle \mathrm{d}P \rangle_\eta = \sum_{t=0}^{\eta} \frac{w_j}{W} \mathrm{d}P_t. \tag{5}$$

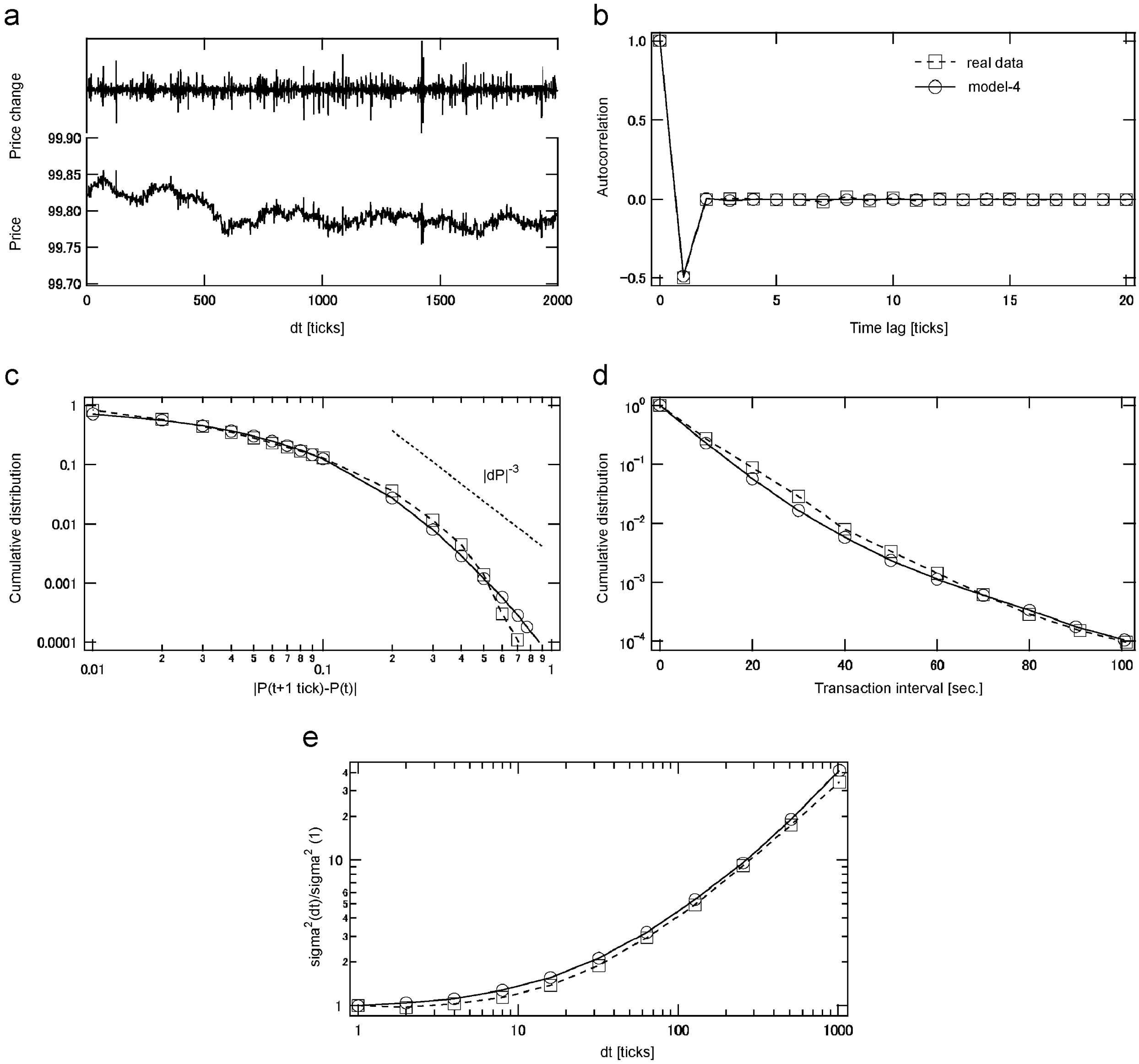

Fig. 5. Comparison of real data (dotted lines) and the model-4 (bold lines). The values of the model-4 are calculated by using the parameters, N(number of dealers) $= 300$, L(spread) $= 1.0$, $\mathrm{d}t = 0.01$, $c_i = [0.001, 0.010]$, $\tau = 150$, $\eta = 150$ and $d_i = [-1.5, -3.5]$. (a) Examples of prices (lower line) and price changes (upper line). (b) Autocorrelations of price changes. (c) Cumulative distributions of price change which normalized by average of real data. (d) Cumulative distributions of transaction intervals. (e) Variances as function of time difference.

Table 1
Results of each model

	Model-1	Model-2	Model-3	Model-4
Correlation of price change	–	–	–	S
Distribution of volatility	–	–	S	S
Distribution of intervals	–	S	–	S
Diffusion of price	–	–	–	S

–: Not satisfy; S: satisfy.

The term w_j is the jth weight and W is a normalization factor that is, $W = \sum w_j$. Here, we assume that dealers' response to the trends are different, and initially the coefficients d_i are randomly assigned. If d_i is positive, the ith dealer is a follower, in contrast if d_i is negative the ith dealer is a contrarian. This causes the tails of distribution of volatility being stretched (Fig. 4). The functional form of this distribution depends on the values of d_i [11].

2.4. The model-4

Finally, we combine the model-2 and model-3 to make the model-4. Mathematical expression is given as follows:

$$\begin{cases} \dfrac{\mathrm{d}b_i}{\mathrm{d}t} = \dfrac{1}{G_k\langle T_k\rangle_\tau}(\sigma_i c_i + d_i\langle \mathrm{d}P\rangle_\eta), \\ \mathrm{d}t = G_k\langle T_k\rangle_\tau \mathrm{d}s. \end{cases} \tag{6}$$

In the model-4 we set $w_j = \exp(-0.3j)$ and $\eta = 150$, as we know that the dealer refers to the latest 150 s and the weights decay exponentially [12]. As expected we can largely reproduce the empirical laws established in real markets. The autocorrelation of price change decays quickly after a negative value at 1 tick (Fig. 5(b)). And the distribution of volatility and transaction intervals are both close to the real market data (Fig. 5(c,d)). Moreover, the diffusion property, the variance as a function of time, is very close to that of real data (Fig. 5(e)).

3. Conclusion

We started with the model-1, the basic dealer model. Considering why the model-1 differs from real market data, we added two effects to the model-1, both are feedback effects. One is the "self-modulation" introduced in the model-2, and the other is the "trend-follow" applied in the model-3. The model-4, applying both of these effects, satisfies most of the basic empirical laws. It should be noted that each dealer has only three parameters describing his character. Finally, we summarize our results in Table 1.

Sato and Takayasu already showed that the dealer model's price fluctuations can be approximated by ARCH model in some conditions [13]. Such approach of connecting the stochastic description of market models to dealer-based artificial market models will be fruitful study in the near future.

Acknowledgement

This work is partly supported by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research ♯16540346 (M.T.).

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