Utilizing Evolutionary Algorithms for Problems with Vast Infeasible Regions and Expensive Function Evaluations

Scott Zimmer, Chris Ranieri, Jason Anderson,

Matt Ferringer, Tim Thompson, Todd Cerven, Marc DiPrinzio, David Garza, Wayne Hallman, Rob Markin

June 27, 2008

Outline

- Evolutionary Algorithm Overview
- GTOC3 Description
- Overview of GTOC3 Difficulties
- Grid Search Mitigation
- Inner Loop Mitigation
- Multi Fidelity Search
- Future Research
- Conclusions

Evolutionary Algorithm (GRIPS)

- Aerospace Corporation tool based on Deb's NSGA2 algorithm
  - Limitations of NSGA2
    - Random seed variability of solutions
    - Potential for long duration run times
    - Trial and error analysis for parameterization
  - GRIPS improvements to NSGA2
    - Epsilon dominance archiving allows the user to control solution precision
    - Auto-adaptive population sizing helps prevent local convergence
    - Time continuation reinvigorates the search when it stagnates
    - Parallelized using the master-slave model
    - Utilizes method of generational synchronization that insures nearly 100% processor utilization

Initial Capabilities

- GRIPS had an API for C code (no existing trajectory optimization capability)
- Indirect optimization code for rendezvous and intercept (Ranieri grad school)
- Lambert targeting code (Aerospace corporate tool)
- Code to calculate delta v at a flyby (Zimmer grad school)
- Lacked software to model a flyby on the first leg
  - Solution---no flyby before asteroid 1 encounter
- Lacked software to string together multiple flybys in any optimal manner
  - Solution---One flyby per leg
- All existing code in Fortran

Initial Goals

- Submit a solution
- Don’t embarrass Aerospace Corporation
- Don’t finish last
- Travel to Italy to present results (personal goal)

GTOC 3 Description

- Near Earth Asteroid rendezvous tour
  - Mission begins with hyberbolic excess velocity relative to Earth
  - Must rendezvous with 3 out of 140 asteroids for at least 60 days each
  - Mission ends with Earth rendezvous
  - 10 year launch window
  - 10 year mission duration
- System model
  - Initial mass 2000 kg, Thrust=0.15 N, Isp = 3000 s
  - Asteroids and Earth in Keplarian orbits about Sun
  - Central body gravity only
    - Patched conic flybys of Earth allowed at any time during mission

GTOC 3 Difficulties

- High fidelity search of entire space is computational intractable
  - 2.5 million asteroid tour combinations
  - Numerous permutations of gravity assists
  - Long mission duration and large launch window
  - Solution due one month after problem announcement
    - Much of the simulation code must be written or modified for use with GTOC3
- Vast regions of the search space are infeasible
  - For many selections of event dates (asteroid departure/arrival, etc), engine is not capable of completing the transfer
  - Including selection of thrust parameters causes nearly all solutions to fail to meet rendezvous conditions

GTOC3 Optimization Problem

- Cost function $$J = \frac{m_f}{m_i} + 0.02\min\left(\tau_j\right)$$
- Decision variables
  - Discrete variables (Lambert Grid Search)
    - Asteroid sequence
    - Flyby sequence
  - Continuous variables (GRIPS)
    - Hyberbolic excess velocity
    - Event dates
    - Flyby parameters
  - Other variables (Inner Loop Optimizer)
    - Optimal spacecraft thrust throughout trajectory

Lambert Grid Search

- Flybys only allowed between asteroid encounters
- Perform a Lambert solution for each possible transfer
  - 1 day grid for earth to asteroid or asteroid to asteroid transfers
  - 10 day grid for asteroid to flyby to asteroid transfers
  - Retain the best single leg solution for 0-1 rev, 1-2 revs, and 2-3 revs
- Examine all permutations of best single leg solutions (>500 million) to construct complete impulsive solutions
  - Use branch and bound to eliminate permutations where total Δv exceeds 8 km/s or where constraints are violated
- Technique is well suited to parallel processing
  - Performed over 5 trillion permutations
  - Computational time required: 768 days

Candidate Cases

	Leg #1					Leg #2							Leg #3							Leg #4						
Total Delta V	Departure JD	Departure Body	Arrival Body	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	Time to Earth Flyby	# Revs to Earth Flyby	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	Time to Earth Flyby	# Revs to Earth Flyby	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	TOF to Earth	# Revs to Earth	Total TOF	Total Delta V
5.9363	58376	0	76	904	2	59478	76	88	0	0	212	0	59798	88	49	0	0	200	0	60698	49	0	905	2	3227	5.9363
6.0341	58092	0	88	999	2	59778	88	19	0	0	504	1	60428	19	49	0	0	203	0	60698	49	0	905	2	3511	6.0341
6.1982	58450	0	96	254	0	59188	96	88	0	0	327	0	59588	88	49	0	0	584	1	60698	49	0	905	2	3153	6.1982
6.2029	59189	0	88	1100	2	60948	88	11	0	0	887	2	61968	11	49	0	0	281	0	62318	49	0	500	1	3629	6.2029
5.3918	59401	0	96	691	1	60318	96	76	0	0	579	1	60958	76	49	920	2	230	0	62318	49	0	594	1	3511	5.3918
5.3948	58463	0	88	1100	2	59778	88	19	0	0	504	1	60418	19	49	520	1	790	2	61788	49	0	200	0	3525	5.3948
5.4407	58464	0	88	731	1	59458	88	76	0	0	252	0	59848	76	49	950	2	710	1	61768	49	0	204	0	3508	5.4407
5.6549	58767	0	37	831	2	59998	37	85	0	0	477	1	60568	85	49	910	2	250	0	61788	49	0	200	0	3221	5.6549
5.7477	58104	0	88	921	2	59088	88	11	0	0	237	0	59418	11	49	290	0	800	2	60698	49	0	905	2	3499	5.7477
4.8755	60720	0	49	904	2	62118	49	37	650	1	200	0	63128	37	85	0	0	263	0	63848	85	0	293	0	3421	4.8755
4.9672	58084	0	88	209	0	58418	88	96	590	1	300	0	59468	96	49	810	2	680	1	61038	49	0	577	1	3531	4.9672
5.3369	58449	0	96	255	0	58888	96	19	230	0	930	3	60418	19	49	520	1	790	2	61788	49	0	200	0	3539	5.3369
5.3787	58450	0	96	254	0	58878	96	37	260	0	1040	3	60308	37	49	990	3	410	1	61768	49	0	204	0	3522	5.3787
5.4160	58450	0	96	254	0	58858	96	30	270	0	1090	3	60488	30	49	520	1	480	1	61768	49	0	204	0	3522	5.4160
5.4360	58450	0	96	254	0	58798	96	111	870	2	700	1	60448	111	49	670	1	610	1	61788	49	0	200	0	3538	5.4360
5.5544	58447	0	96	257	0	58848	96	64	290	0	920	2	60228	64	49	940	2	360	0	61768	49	0	204	0	3525	5.5544
5.6003	60968	0	49	998	2	62118	49	37	650	1	200	0	63028	37	85	250	0	650	1	64208	85	0	286	0	3526	5.6003

Inner Loop Optimization

- Allow the outer loop to provide the sequence including event dates
  - Initial states, final states, and transfer time are inputs from outer loop
  - Most selections of thrust direction and magnitude will yield infeasible solution
- Optimization problem is reduced to determining a series of optimal intercept (flyby) or rendezvous fixed time transfers
  - Find optimal thrust pointing history and thrust durations to achieve transfer with minimum fuel
  - Optimizer is ideally closed-loop: starts with a generic initial guess and converges with no user intervention
    - Allows local optimizer to be plugged into outer optimizer (EA) which adjusts the input parameters sent to the local optimizer (Initial and final times and states)

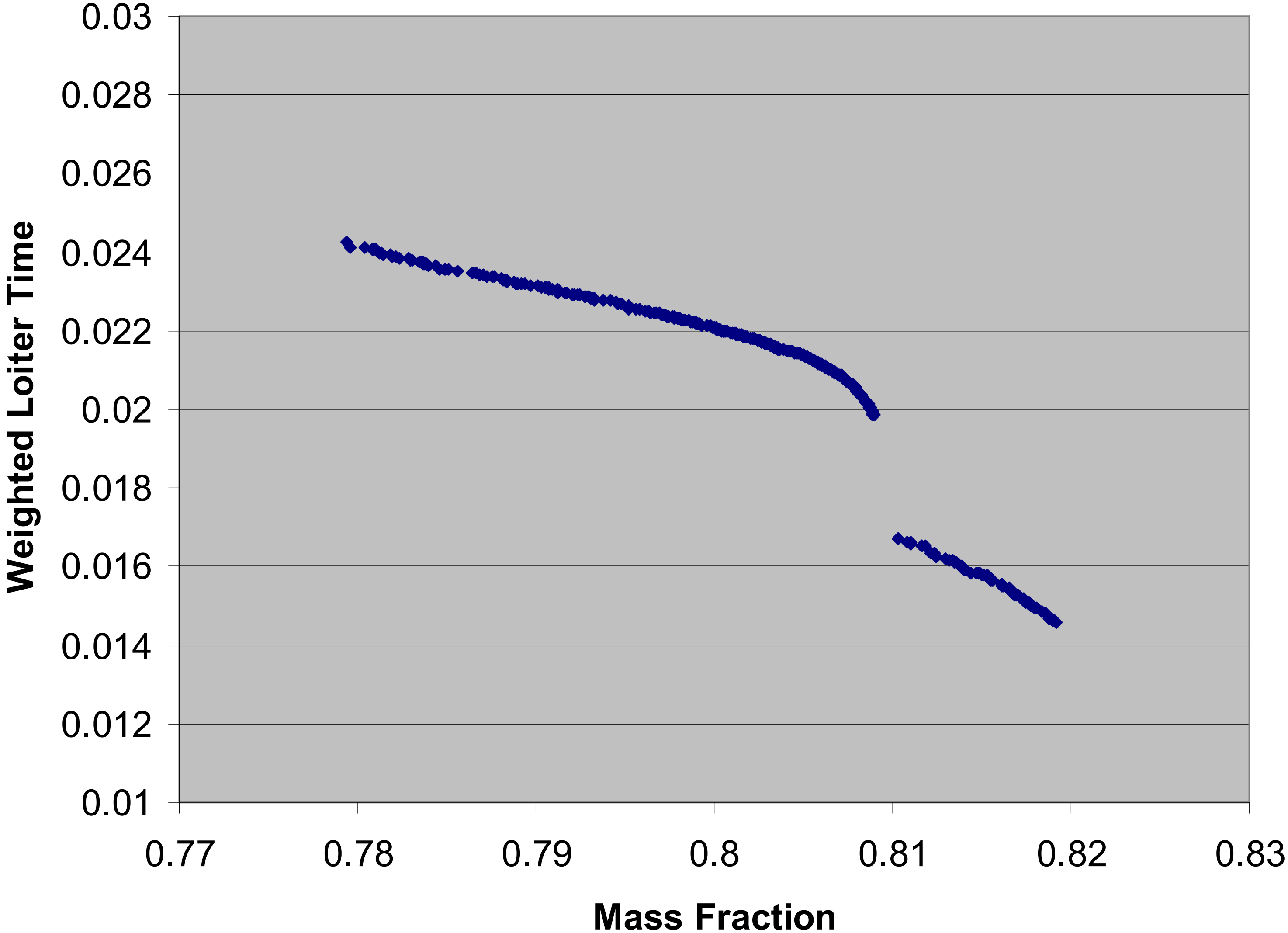
Inner Loop Technique

- Indirect optimization technique utilizing adjoint control transformation
  - Two-point boundary value problem: equal number of initial unknowns (adjoints and problem parameters) and final optimality and physical problem constraints
  - Cartesian coordinates used for this application due to time constraints on solution
  - Differential equations describe infinite dimensional optimal control history
  - Solved with FORTRAN subroutine using a boundary value solver from Harwell Subroutine Library Archives (Broyden's Method combined with Steepest Descent)
- Difficult to find generic initial guess that converges for all feasible transfers
  - Start with a simpler problem and morph it into the actual problem dynamics
    - Initial thrust set to 10*Actual Thrust (1.5 N)
    - Shorter TOF for initial trajectory of 270 days
- Incrementally increase TOF to desired value while decreasing thrust after converging each simplified trajectory
  - If simplified trajectories do not converge, transfer is infeasible and inner loop returns an error message to the EA

EA Decision Variable Ranges

Asteroid Sequence	88-96-49
Earth Departure Date	57568 to 57968
Departure V∞	0 to 0.5
Departure RA	0 to 360
Departure Dec	0 to 180
TOF to Asteroid 1	187 to 587
Stay Time Asteroid 1	60 to 443
TOF to	436 to 836
Flyby Perigee Radius	6871 to 40000
Flyby Angle	0 to 360
TOF to Asteroid 2	187 to 587
Stay Time Asteroid 2	152 to 552
TOF to Asteroid 3	372 to 572
Stay Time Asteroid 3	113 to 513
TOF to Earth	494 to 894

Non-Dominated Front



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GRIPS Optimal Values

Asteroid Sequence	88-96-49
Earth Departure Date	57738
Departure V∞	0.5
Departure RA	228
Departure Dec	70
TOF to Asteroid 1	377
Stay Time Asteroid 1	267
TOF to Flyby	627
Flyby Perigee Radius	6871
Flyby Angle	331
TOF to Asteroid 2	362
Stay Time Asteroid 2	425
TOF to Asteroid 3	492
Stay Time Asteroid 3	353
TOF to Earth	676
Objective Value	0.83345

Low Fidelity to High Fidelity Transition Problems

- It is not feasible to perform all simulations in high fidelity
- Low fidelity search may eliminate optimal high fidelity solutions
  - Not allowing flybys before the first gravity assist eliminated best solutions
  - Mapping from low fidelity to high fidelity is not perfect
    - If mapping is perfect, high fidelity is unneeded
- Feedback from high fidelity simulations is needed for low fidelity decision making

Mixed Fidelity Search

- First perform a low fidelity simulation on a candidate
  - Quickly determine if candidate is near the optimal
  - Low fidelity search can be approximate
    - Exact ranking of these solutions in unnecessary
  - Goal is to save as many high fidelity searches as possible
- Examine near optimal solutions in higher fidelity
  - Provide an exact ranking for near optimal solutions
  - Desire to perform a high fidelity simulation for any low fidelity solution that could be optimal in high fidelity
    - The number of high fidelity simulations is dependent on the accuracy of the low fidelity simulation
      - Low fidelity simulation will trade accuracy for simulation time

Multi Fidelity Example

- MOP2 test problem from Huband et. al.
  - Separable, unimodal minimization problem where the solution is not at the extremal or medial values of the decision variables
- Variables range from -4 to 4
- High fidelity simulation evaluates function using double precision
- Low fidelity simulation rounds each decision variable to nearest 0.1 and then perturbs the decision variable by a value between -0.1 and 0.1

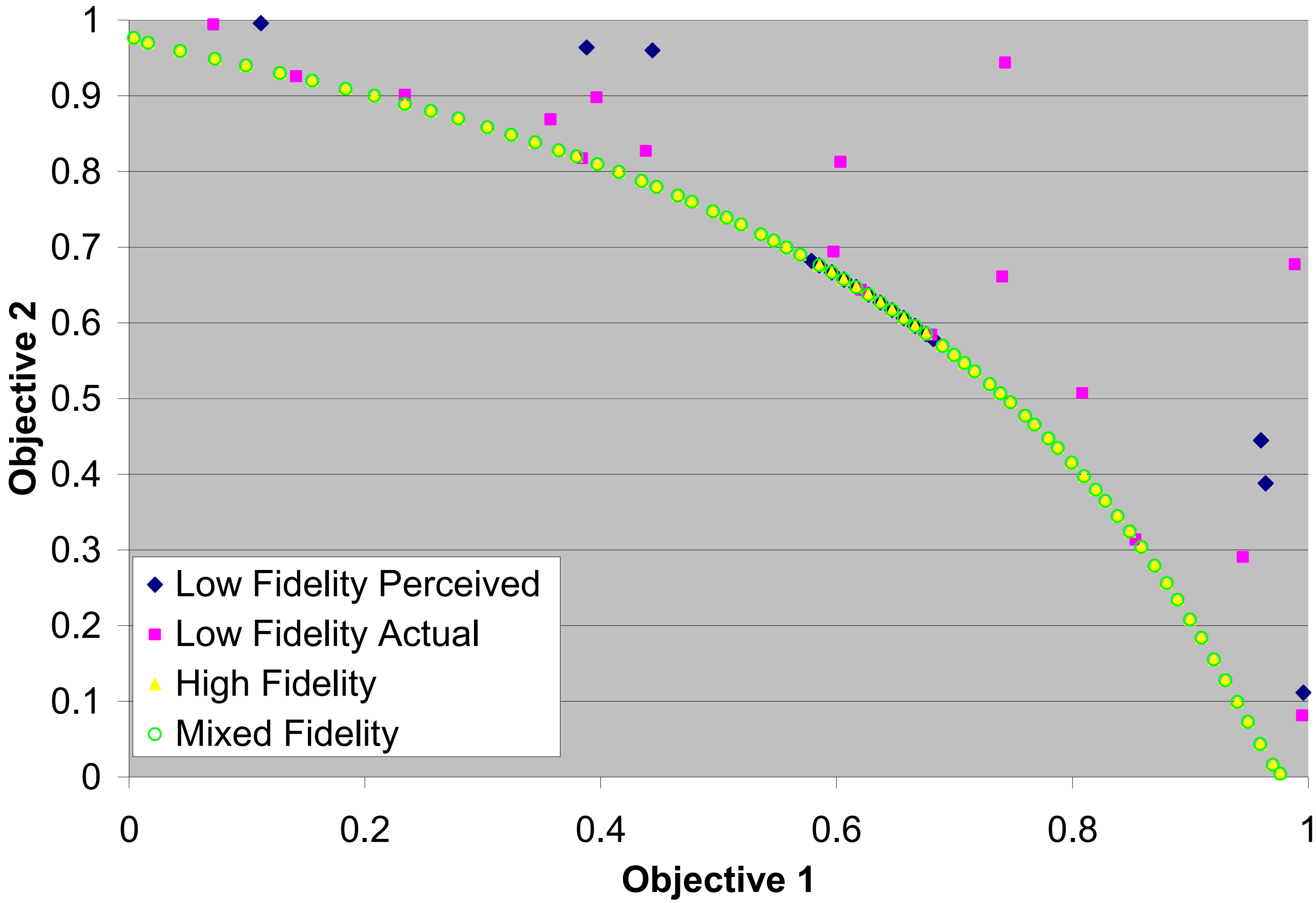
$$f_1 = 1 - \exp\left[-\left(x_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(x_2 - \frac{1}{\sqrt{2}} \right)^2 \right]$$

$$f_2 = 1 - \exp\left[-\left(x_1 + \frac{1}{\sqrt{2}} \right)^2 - \left(x_2 + \frac{1}{\sqrt{2}} \right)^2 \right]$$

Solution Technique

- Need all high fidelity objective values to be more optimal than low fidelity objectives
  - Add one to the any low fidelity objective
- Determine when to perform a high fidelity simulation
  - Utilize a brief run completely in low fidelity and determine the non-dominated front
  - Fit a curve to the front (quadratic)
  - Any low fidelity simulation that is better than the curve is performed in high fidelity

Objective Space Results



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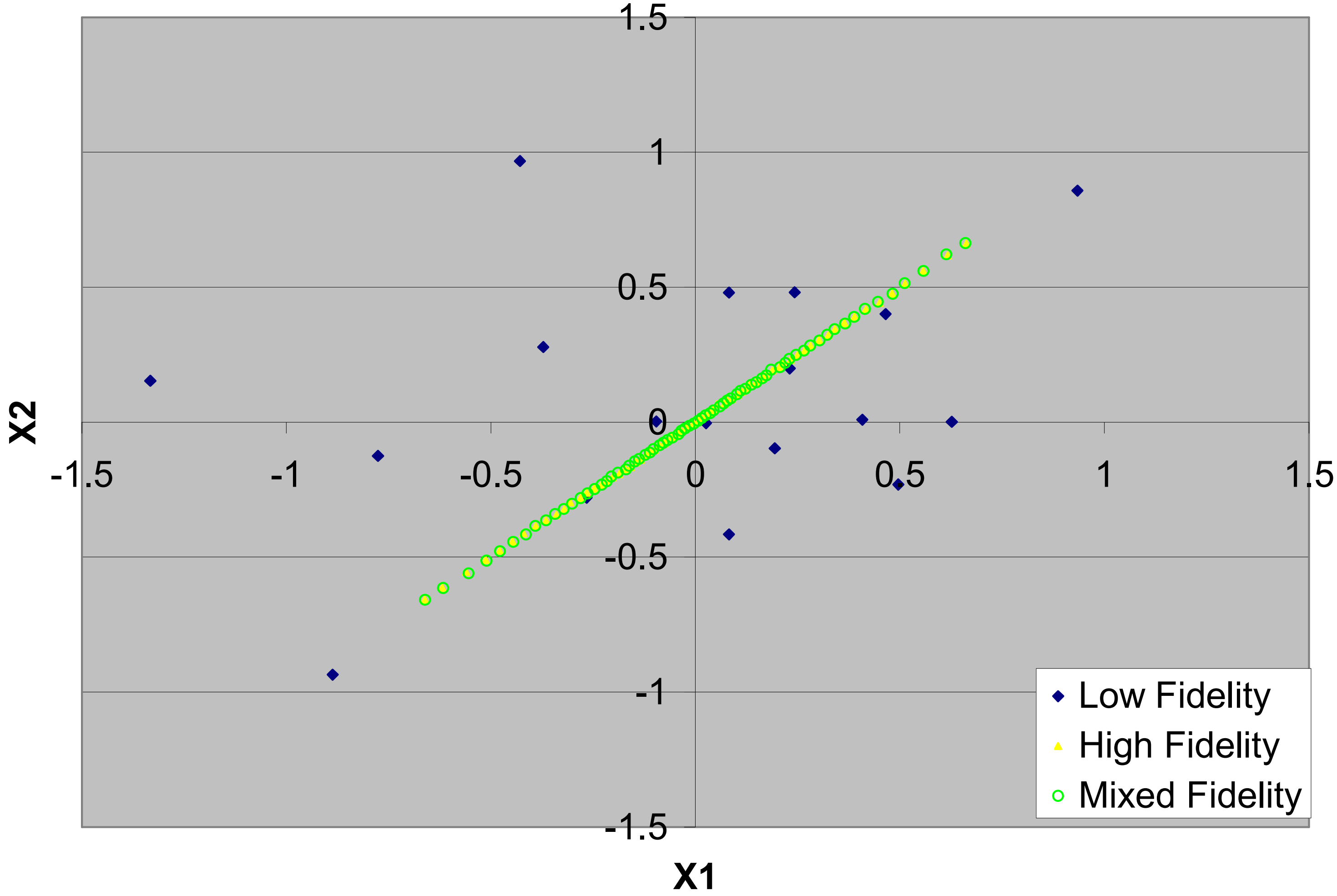
Decision Variable Results



Multi Fidelity Future Work

- Determining when to perform high fidelity simulations
  - Static decisions
    - Top percentage of low fidelity simulations
    - Low fidelity simulation better than pre determined value
  - Dynamic decisions
    - Maintain an evolving low fidelity front
    - Periodically examine all solutions in high fidelity
      - Determine appropriate percentage to examine in high fidelity for future generations
- Efficient parallel computer utilization
  - Low fidelity solutions may run orders of magnitude more quickly
    - Low fidelity simulations will achieve numerous generations during a single high fidelity simulation
    - Fast low fidelity simulations may require packets of chromosomes be passed to a single computer node

Conclusions

- Evolutionary algorithm successfully selected event dates and flyby parameters
  - Inner loop indirect optimization routine operated robustly and autonomously
- Initial assumptions and limited success in mapping from low fidelity to high fidelity solutions eliminated many of the optimal solutions
- Mixed fidelity simulation offers the potential to examine more of the space without falsely eliminating as many potential solutions
- Significant work is needed to efficiently utilize mixed fidelity simulations for real world problems

Questions