Resistively and capacitively shunted Josephson junctions model for unconventional superconductors

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***Abstract*—An array of resistively and capacitively shunted Josephson junctions with nonsinusoidal current-phase relation is considered for modelling the resistive transition in high-T$_c$ superconductors. The emergence of higher harmonics, besides the sinusoid $I_c \sin\phi$, is expected for dominant *d*-wave symmetry of the Cooper pairs, random distribution of potential drops and *dirty* grains, or in nonstationary conditions. We show that additional cosine term acts by modulating the global resistance, due to the weak-links whose transition occurs through mixed superconductive-normal states.**

I. Introduction

Arrays of Josephson junctions are under intensive investigation for their potential implementation as superconductor quantum bits and ability to model several fundamental phenomena in disordered superconductive films [1]–[12]. In particular, the resistively and capacitively shunted Josephson Junction model (RCSJ model) has been adopted to describe the resistive transition in granular superconductors [13]–[19]. In the conventional RCSJ model, the Josephson current is the simple sinusoid $I_{\rm S}(\phi) = I_c \sin\phi$, where I_c is the critical current and $\phi = \theta_2 - \theta_1$ the phase difference of the superconductor order parameters $\Delta_1 \exp(i\theta_1)$ and $\Delta_2 \exp(i\theta_2)$ [20].

Sign and magnitude of I_c are affected by the gap function symmetry and relative orientation of the superconductor electrodes. According to the microscopic approach, the current-phase relation can be expressed as:

$$I_S(\phi) \propto \int_{-\infty}^{+\infty} [1 - 2f(E)]\mathrm{Im}[I_E(\phi)]dE \ , \qquad (1)$$

with $f(E)$ the electron energy distribution and $\mathrm{Im}[I_E(\phi)]$ the spectral current, which depend on material, geometry and nonequilibrium conditions. The current-phase relation (1) can be written as an n-order Fourier series [21], [22]:

$$I_{\rm S}(\phi) = \sum_{n\geq 1} \left[\tilde{I}_n \sin(n\phi) + \tilde{J}_n \cos(n\phi)\right] \ . \qquad (2)$$

When the sum is restricted to the 1^{st} order, $\tilde{I}_n \sin(n\phi)$ reduces to the familiar sinusoidal Josephson current $I_c \sin\phi$. The term $\tilde{J}_n \cos(n\phi)$ is the quasi-particle-pair-interference current (QPIC). Deviations from the sinusoidal shape have been experimentally observed at temperatures below T_c because, in general, these effects are of the second order. In the vicinity of T_c, they have been theoretically predicted and observed in normal-metal weak-links, as a consequence of the depairing either by proximity effect by supercurrent or in long junctions or in far-from-equilibrium conditions [21]. A disordered polycrystalline superconductor is a nonhomogeneous system with wide variability of the physical and chemical properties of the grains. For current $I \sim I_c$ and voltage $0 < V < V_c$ in the vicinity of the transition, nonequilibrium effects arise in the weak-links making their relevant properties spatially and temporally dependent on the external drive [16]–[19], [23]–[29]. When a polycrystalline superconductor undergoes the transition, far-from-equilibrium condition, due to the abrupt voltage drops across the grains, may result in the emergence of higher harmonics according to the local voltage values, geometry and material composition of the grains. In the presence of evolution equations which are nonlinear -such as those of Josephson Junctions- intrinsic localized modes (ILM) are obtained as solutions of sine-Gordon equations. Theses solutions are characterized by being time-dependent and spatially localized as opposed to translationally invariant lattices, in the absence of disorder or defects, where an initially localized excitation

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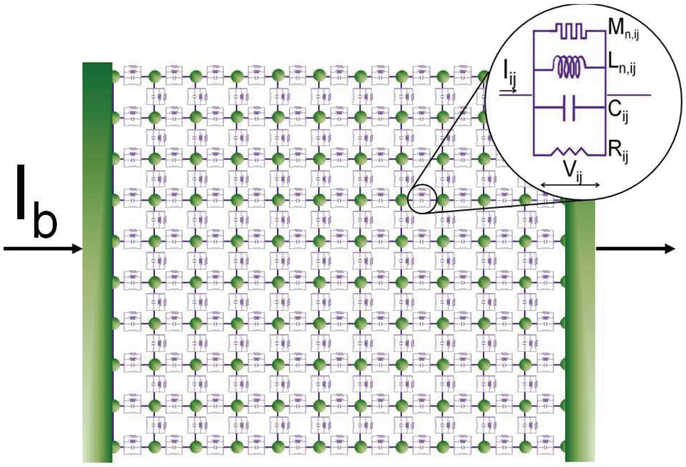

Fig. 1. Two-dimensional Josephson junction array representing a granular superconductor. Circles represent superconducting grains connected by weak-links. The bias current I_b is injected to the left electrode and collected from the right electrode. Equivalent circuit of the weak-link between the grains i and j is shown in the zoom. The linear resistor R_{ij}, the linear capacitor C_{ij}, the nonlinear inductor $L_{n,ij}$ and memristor $M_{n,ij}$ are connected in parallel. The current I_{ij} flows from grain i to grain j. V_{ij} is the voltage drop across the weak-link.

distributes its energy over the entire system. Deviations from the simple sinusoidal shape in the I-V characteristics of single Josephson junctions and arrays as an effect of the formation of intrinsic localized modes have been reported in [28], [29]. The-pair-interference current $\tilde{J}_n \cos(n\phi)$ emerges when the pair-symmetry is broken and it is expected to come into play when the junctions are partly dissipative. This may occur in the mixed state, i.e. in the vicinity of T_c, for current $I \sim I_c$ and voltage $0 < V < V_c$.

In this work, we put forward a model of the superconductive-resistive transition where a network of resistively and capacitively shunted nonsinusoidal Josephson junctions are considered. The network of weak-links, modeled as nonsinusoidal Josephson junctions, should be particularly relevant when the effect of nonequilibrium in the presence of disorder and nonlinearity should be taken into account in the transition of granular superconductors.

II. Model

A two-dimensional array of Josephson junctions is sketched in Fig. 1. The bias current I_b is injected to the left electrode and collected from the right electrode. Circles represent superconducting grains connected by weak-links. According to the RCSJ model, the current I_{ij} flowing through each junction is:

$$I_{ij} = C_{ij}\frac{dV_{ij}}{dt} + \frac{V_{ij}}{R} + I_{S,ij}(\phi_{ij}) + \delta I_{L,ij} \ . \quad (3)$$

where C_{ij} and R_{ij} are the shunt capacitance and resistance between grains i and j, $I_{S,ij}(\phi_{ij})$ is the Josephson current, $\delta I_{L,ij}$ is the Langevin fluctuation source. The voltage drop across the junction is given by:

$$V_{ij} = V_i - V_j = \frac{\hbar}{2e}\frac{d\phi_{ij}}{dt} \ , \quad (4)$$

with ϕ_{ij} the phase difference of the order parameters in the grains i and j. In the usual RCSJ model, $I_{S,ij}(\phi_{ij})$ is a simple sinusoid, whereas in the present work the nonsinusoidal form given by Eq. (2) is considered. Therefore, the current I_{ij} flowing through each junction connecting the grains i and j writes as:

$$I_{ij} = C_{ij}\frac{dV_{ij}}{dt} + \frac{V_{ij}}{R_{ij}} + \\ + \sum_{n\geq 1}[\tilde{I}_{n,ij}\sin(n\phi_{ij}) + \tilde{J}_{n,ij}\cos(n\phi_{ij})] + \delta I_{L,ij} \ . \quad (5)$$

I_{ij} is given by the sum of the following contributions: the charging current through the shunt capacitance C_{ij}, the Ohmic current through the shunt resistance R_{ij}, the n Josephson current sources $\tilde{I}_{n,ij}\sin(n\phi_{ij})$ and $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ and the Langevin current.
The equivalent circuit of each junction is highlighted in the circle of Fig. 1. It corresponds to a parallel connection of a linear capacitor C_{ij}, a linear resistor R_{ij}, a parallel of n inductors $L_{n,ij}$ (related to the $\tilde{I}_{n,ij}\sin(n\phi_{ij})$ terms) and a parallel of n memristors $M_{n,ij}$ related to the $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ terms (we use the notation memristor after [30]). Eq. (5) can be written more compactly as:

$$I_{ij} = C_{ij}\frac{dV_{ij}}{dt} + \frac{V_{ij}}{R} + \\ + \sum_{n\geq 1} I_{c,n,ij}\sin(n\phi_{ij} + \phi_{o,n,ij}) + \delta I_{L,ij} \ , \quad (6)$$

with:

$$I_{c,n,ij} = \sqrt{\tilde{I}^2_{n,ij} + \tilde{J}^2_{n,ij}} \quad (7)$$

and:

$$\phi_{o,n,ij} = \arctan(\frac{\tilde{J}_{n,ij}}{\tilde{I}_{n,ij}}) \quad (8)$$

Josephson junctions are usually classified in terms of the Stewart-McCumber parameter $\beta_c = \tau_{RC}/\tau_J$ with $\tau_{RC} = RC$ and $\tau_J = \phi_o/2\pi I_c R_o$, as overdamped ($\beta_c \ll 1$), general ($\beta_c \simeq 1$) and underdamped ($\beta_c \gg 1$).

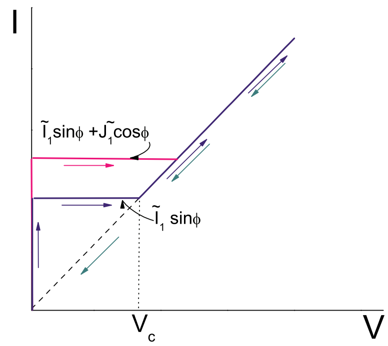

Fig. 2. Josephson junction characteristics of a weak-link with current-phase relation $I_{\rm S}(\phi) = I_c \sin(\phi)$ (the blue line), $I_{\rm S}(\phi) = \tilde{I}_1 \sin(\phi) + \tilde{J}_1 \cos(\phi)$ with $\tilde{I}_1 = 1{\rm mA}$ and $\tilde{J}_1 = 0.5{\rm mA}$ (the pink line). The generalized Stewart-McCumber parameter is $\beta_c^* = 45$.

For the nonsinusoidal junction described by Eq. (6), the generalized Stewart-McCumber parameter can be defined as $\beta_c^* = \tau_{\rm RC}/\tau_{\rm J}^*$, with $\tau_{\rm J}^* = \phi_o/(2\pi \sum_n I_{c,n,ij} R_o)$. Eq. (6) can be numerically solved for an arbitrary number n of harmonics. Nonetheless, we restrict our discussion to the following case relevant to the physics of superconductors:

$$I_{{\rm S},ij}(\phi_{ij}) = \tilde{I}_{1,ij} \sin(\phi_{ij}) + \tilde{J}_{1,ij} \cos(\phi_{ij}), \qquad (9)$$

The scheme of the current-voltage characteristics of an underdamped ($\beta_c^* \gg 1$) Josephson junction obtained by solving Eq.(6) is shown in Fig. 2. In particular, the blue line in Fig. 2 refers to the simple sinusoid, the pink line to $I_{{\rm S},ij}(\phi_{ij})$ given by Eq.(9). The intermediate states are characterized by voltage drops in the range $0 < V_{ij} < V_{c,ij}$ and current $I_{ij} = I_{c,n,ij}$. Upon current (voltage) decrease starting from the normal state, the behavior is always resistive, implying that the system reaches the superconductive ground state without exploring the intermediate states.

For overdamped junctions ($\beta_c^* \ll 1$), the intermediate states are characterized by voltage drop and current respectively in the range $0 < V_{ij} < 2V_{c,ij}$ and $I_{c,n,ij} < I_{ij} < I_{c,n,ij}[2V_{c,ij}]$. Upon increasing and decreasing the external drive, the current-voltage behavior is the same, hence no hysteresis is observed.

In the general case ($\beta_c^* \approx 1$), the $I - V$ curve is partly hysteretic. Upon increasing the external drive, the intermediate states are characterized by a voltage drop in the range $0 < V_{ij} < V_{c,ij}$ and current equal to $I_{c,n,ij}$. As the external drive decreases, the backward current lies slightly below the forward current. It is worthy of remarks that with the nonsinusoidal current phase relation the capacitive effect is reduced in comparison to the simple sinusoidal case.

III. Results and discussion

As stated above, the resistive transition is modeled by using a network of weak-links, with Josephson junction characteristics given by Eq. (6). The solution of the network is obtained by a system of Kirchhoff equations that has been already used for the simple sinusoidal Josephson current characteristics in [**?**]. We have routinely solved the Kirchhoff equations of the networks by using the generalized RCSJ model Eq. (6) with weak-links with nonsinusoidal current-phase relation given by Eq. (2) in the temperature range just below T_c. The network is biased by constant current I_b. The superconductor-insulator transition is simulated by solving the system of Kirchhoff equations at varying temperature. The critical currents $\tilde{I}_{n,ij}$ and $\tilde{J}_{n,ij}$ are assumed to vary on temperature according to the linearized equations $\tilde{I}_{n,ij} = \tilde{I}_{o,n,ij}\,(1 - T/T_c)$ and $\tilde{J}_{n,ij} = \tilde{J}_{o,n,ij}\,(1 - T/T_c)$, where $\tilde{I}_{o,n,ij}$ and $\tilde{J}_{o,n,ij}$ are the lowest temperature values of $\tilde{I}_{n,ij}$ and $\tilde{J}_{n,ij}$. Hence, the critical current $I_{c,n,ij}$ depends on temperature according to $I_{c,n,ij} = I_{c_o,n,ij}\,(1 - T/T_c)$, with $I_{c_o,n,ij} = \sqrt{\tilde{I}^2_{o,n,ij} + \tilde{J}^2_{o,n,ij}}$. In order to take into account the disorder of the array, $\tilde{I}_{n,ij}$ and $\tilde{J}_{n,ij}$ are taken as random variables, distributed according to Gaussian functions with mean values $\tilde{I}_{o,n}$ and $\tilde{J}_{o,n}$ and standard deviations $\Delta\tilde{I}_{o,n} = \Delta\tilde{J}_{o,n}$.

By effect of the temperature increase and consequent reduction of the critical current, the weak-link with the lowest value of the critical current $I_{c,n,ij} = I_{c,min}$ switches to the intermediate state and, then, becomes resistive when $V_{ij} > V_c$. The resistive transition of the first weak-link has the effect to set the value of the voltage drop across the other weak-links in the same layer. The result is the formation of a layer of weak-links either in the resistive or in the intermediate state. As temperature further increases, the critical current $I_{c,n,ij}$ further decreases. More and more weak-links gradually switch from the superconductive to the intermediate state and then to the resistive state. The term $\tilde{J}_{n,ij}$ acts by increasing the critical current value of the weak-link in the intermediate state in the layers undergoing the transition. It is worthy to remark that the increase of critical current is relative to the fraction of normal electrons in the mixed states. The onset of $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ is indeed triggered by the elementary resistive transition of the weak-link

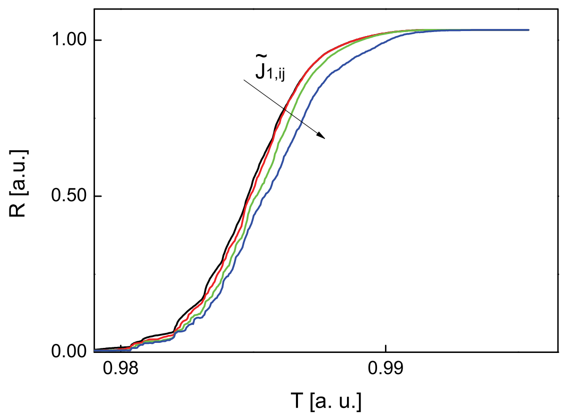

Fig. 3. Resistive transition of a two-dimensional network with current-phase relation of the form $I_{\mathrm{S},ij}(\phi_{ij}) = \tilde{I}_{1,ij}\sin(\phi_{ij}) + \tilde{J}_{1,ij}\cos(\phi_{ij})$. The average value of the critical current $\tilde{I}_{1,ij}$ is 1mA. The curves correspond to different average values of the critical current $\tilde{J}_{1,ij}$, namely $\tilde{J}_{1,ij} = 0\mathrm{mA}$, $\tilde{J}_{1,ij} = 0.5\mathrm{mA}$, $\tilde{J}_{1,ij} = 0.75\mathrm{mA}$ and $\tilde{J}_{1,ij} = 1\mathrm{mA}$. The normal resistance R_o is 1Ω equal for all the junctions.

with the lowest critical current, since it is related to the partly broken pair-symmetry of the weak-links in the intermediate state. It has no effect on the links in the superconductive state, neither on those in the fully resistive state.

Fig. 3 shows the curves of the resistive transitions obtained with current-phase relation $I_{\mathrm{S},ij}(\phi_{ij}) = \tilde{I}_{1,ij}\sin(\phi_{ij}) + \tilde{J}_{1,ij}\cos(\phi_{ij})$ for a two-dimensional 30×30 network. The curves correspond to different values of the term $\tilde{J}_{1,ij}$. The values of the critical currents are $\tilde{I}_{1,ij} = 1\mathrm{mA}$ and $\tilde{J}_{1,ij}$ ranging from 0 to 1mA. The standard deviation of the critical currents is $\Delta I_{o,n} = 0.5\mathrm{mA}$. Initially, the weak-links are in the superconductive state, thus the network resistance is negligible. As temperature increases, the weak-link with the lowest critical current switches to the intermediate state and then to the resistive state with the consequent onset of the term $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ and redistribution of the currents. One can notice that the curves overlap at the beginning of the transition, whereas become more separated when $T \rightarrow T_c$, implying that the effect of the term $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ is more relevant as the transition approaches its end. The amplification of the $\tilde{J}_{n,ij}\cos(n\phi_{ij})$ effect, as the resistance increases, means that $\tilde{J}_{n,ij}$ acts as modulation of the resistance. The modulation effect due to $\tilde{J}_{n,ij}$ can be noted at the level of each elementary transition step.

IV. Conclusions

The nonsinusoidal current-phase relation has been considered in the resistively shunted Josephson junction model for describing the superconductive transition. By solving a system of Kirchhoff equations for the array of nonsinusoidal Josephson junctions, it is found that additional cosine and sine terms modify the transition curves by changing resistance and Josephson coupling. The model might be relevant for Cooper pairs with *d*-wave dominant over *s*-wave symmetry.

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