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Constraints for solar neutrinos fluxes

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Constraints on the neutrino fluxes Φ_p, Φ_B, Φ_{CNO} and Φ_{Be} are derived from the available experimental results. Physical implications are discussed by means of non-extensive statistics which takes into account many-body effects and long-range interactions. The different reduction factors for ^7Be and ^8B fluxes with respect to their standard solar model values are physically understood, with proper shapes of the shape of the electron and proton distribution functions.

Long-range interactions and many-body effects involving large numbers of particles influence the density of low-energy pairs in the central solar plasma. Stationary energy distributions different from the Maxwellian one, with depleted tails or vanishing at energies of some tenths of k_BT, can be justified on theoretical grounds by the Tsallis statistics by considering the sun as a stellar polytrope of index n. Nuclear rates, Gamow peaks and consequently neutrino fluxes can then be expressed as products of their standard solar model (SSM) values by a function of the parameter δ (or the q Tsallis parameter) characterizing the depletion of the Maxwellian tail.

We set four equations (constraints) for the four fluxes (variables) $\Phi_p(pp + pep)$, Φ_B, Φ_{CNO}, Φ_{Be} following the approach described, for instance, by Calabresu et al.[1].
These equations are relationships among the fluxes based on the well known value of the solar luminosity, on the Gallium signal, on the Chlorine experiment and on the result of the Kamiokande experiment, assumed by us to be correct, which gives the Boron neutrino flux $\Phi_B = (2.9 \pm 0.4)\, 10^6\, cm^{-2}\, s^{-1}$. We use here the more recent experimental results [2–5] and the theoretical calculations of cross sections on Ga and Cl by Bahcall et al. [6].

The SSM predictions are well known and we may choose the values obtained by Bahcall et al.[7] (see also Dar and Shaviv [8]).

The several coefficients of the four equations depend on different quantities, among which the position of the Gamow peak of the different reactions and the maximum neutrino energy [9]. These quantities are functions of the parameter δ. However the corrections to be introduced for this dependence on δ are small and can be neglected.

After algebraic manipulation of the equations we find that the following values are consistent with the four constraints: $\Phi_p = 60\, 10^9\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}$, $\Phi_{Be} < 0.65\, 10^9\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}$, $\Phi_{CNO} < 0.4\, 10^9\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}$, $\Phi_B = (2.9 \pm 0.4)\, 10^6\, \mathrm{cm}^{-2}\, \mathrm{s}^{-1}$.
These quantities are not too different from the constraints indicated by Castellani et al.

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[10]. Therefore we must expect that the SSM values be reduced for some physical reasons of a factor $\simeq 8$ for Φ_{Be}, of $\simeq 3$ for Φ_{CNO}, 2.35 ± 0.35 for Φ_B; on the contrary for Φ_p there is no reduction, rather we must impose a very small enhancement. The Φ_p can weakly differ from the SSM value increasing its value, and still satisfying the imposed constraints.

An explanation of the behavior, just described, of the different fluxes by means of the approach recently introduced by us [11], based on the non-extensive Tsallis statistics [12,13] will be here proposed.
By means of it we can handle easily the long-range gravitational interaction and the many-body effects among the particles of the different systems (H, He, Z and e^-) constituting the solar core. Moreover by using the Clayton and coll.[14] intuition, the physical meaning of the depletion of the Maxwellian tail of the ion and electron distribution functions can be understood.

The internal structure of the sun can be considered polytropic with index $n = 3.5$ [15]. The Clayton et al. distribution belongs to Tsallis class family and is certainly due to long-range gravitational interactions, many-body effects and, in particular for the electrons, to strong electromagnetic fields. We obtained a relation between n and the Tsallis parameter q linked to the Clayton parameter δ ($\delta = (1-q)/2$) (see Eq.(13) of Ref.[11] and Eq.(1) below). The distribution functions expected by the Tsallis statistics vanish at some tenths of $k_B T$, depending on the value of q. The depleted Maxwellian distribution represents a good approximation (see also Ref.[16] for a general discussion on non-Maxwellian distributions).

The correction to the SSM predictions can be factorized following the Clayton approach independently of the initial SSM values and can be calculated as a function of the parameter δ. The corrections of the four SSM fluxes are reported below:
$f_{red}^{B} = \Phi_B^{SSM}/\Phi_B = 1 + 92581.75\,\delta^{1.92}$,
$f_{red}^{CNO} = \Phi_{CNO}^{SSM}/\Phi_{CNO} = 1 + 22.28\,\delta^{0.43}$,
$f_{red}^{Be} = \Phi_{Be}^{SSM}/\Phi_{Be} = 1 + 851.71\,\delta^{1.287}$,
$f_{incr}^{p} = \Phi_p/\Phi_p^{SSM} = 1 + 0.89\,\delta^{0.62}$.

For the sake of utility we have reported above the reduction factor $f_{\rm red}$ as a function of δ ($\Phi_{B,CNO,Be} = \Phi_{B,CNO,Be}^{SSM}/f_{red}$). The factor of enhancement $f_{\rm incr}$ concerns the proton flux which changes very smoothly as a function of δ, compared to the corresponding reduction of Φ_B, Φ_{CNO} and Φ_{Be} ($\Phi_p = \Phi_p^{SSM}\, f_{incr}$).

We can observe that the reduction needed to explain the value of the Boron flux implies δ(Boron)=0.003, for the CNO flux $\delta(CNO) > 0.0025$ but smaller than 0.03, for the Be flux $\delta(Be) > 0.02$, but smaller that 0.06. The proton flux depends very smoothly on δ and within the errors we can accept any values of δ between 0 and 0.01 because also a value a little larger than 60 (f.i. say 64) still satisfies the constraints.
We know that the flux Φ_{Be} is proportional to [17]: $\Phi_{Be} \propto N(e^-)\,N(^7Be) < e^-\ \ ^7Be >$, and that $\Phi_B \propto N(e^-)\,N(^7Be) < p\ \ ^7Be >$.
Therefore Φ_{Be} depends on the electron distribution while Φ_B on the proton distribution. Also Φ_{CNO} depends on the proton distribution because all nuclei involved in this cycle react with protons. Considering the Kamiokande experiment correct, it is just its result on Φ_B that fixes the value of δ_B, δ_{CNO} and δ_p.

As a consequence $\delta_p = \delta_B = \delta_{CNO}$ and we may select the value 0.003, indicated above. Very different is the value of the depletion parameter to be used for Φ_{Be}. It can be fixed

to $\delta_{Be} = 0.024 \div 0.06$ (the upper value holds if the Be flux nearly vanishes). Without loosing generality we select 0.024.

The shape of the distribution of the ^3He and ^4He and the parameter δ_{He} do not enter directly and explicitly in the derivation of the fluxes. Rather these quantities are contained in the percentage of the reacting nuclei, responsible of the neutrino emission, through the density $N(^7Be)$.
Since the presence of a resonance in Helium channels seems o be excluded [18], our results are independent of it. An improvement of this phenomenological analysis can be accomplished using the relation between δ and n

$$\delta(n+1) = \frac{1}{2}\frac{k_B T N}{\mathcal{P}} \quad , \tag{1}$$

derived in Ref.[11] ($k_B T$ is the temperature, $\mathcal{P}$ the pressure and N the number of particles per unit volume $N = \mathcal{N}_{AV}\rho/\mu$, ρ is the density and μ the molecular weight).
In order to simplify our discussion, we do not consider the dependence of $\mathcal{P}$ on δ and the very small variation, within the Tsallis statistics, of the perfect gas law [19]. Therefore for a system of particles of average molecular weight μ, we have:

$$\delta = \frac{1}{2}\frac{1}{n+1} \quad , \tag{2}$$

(for instance: $n = 3.5$ $\delta = 0.111$, $n = 5$ $\delta = 0.0083$).
The quantity N can be written as $N = \sum_i N_i$ where i refers to H, ^3He and ^4He, Z (nuclei heavier than helium) and e^-. The parameter δ is therefore the sum of the single parameters δ_i, given by

$$\delta_i = \frac{1}{2}\frac{1}{n+1}\frac{N_i}{N} \quad . \tag{3}$$

We can also write δ as the sum of the three depletion parameters of the partial proton distributions responsible of the reactions allowing the emission of B neutrinos, $CNO-\nu$'s and $p-\nu$'s plus the depletion parameter of the electron distribution responsible for the emission of the $Be-\nu$'s.

Assuming that the Kamiokande experiment is correct we can fix the depletion or Tsallis parameter δ_B and consequently δ_{CNO} and δ_p with the same value of δ_B. The three fluxes Φ_B, Φ_{CNO} and Φ_p depend on the distribution function of protons which is non-Maxwellian with a smoothly depleted tail. Contrarily to the other fluxes, the magnitude of Φ_p increase slightly with δ. Berillium flux behaves very differently since it depends on the electron distribution which is much more depleted than the ions distribution. This means that electrons are submitted to more intense interactions than ions, probably due to the strong electromagnetic fields in which they are immersed in addition to the long-range gravitational potential. We recall that the corrected fusion nuclear rates can be found in Refs. [11,14]; the electron capture on ^7Be rate is the SSM one [20] corrected by the factor $(1 - 2\delta + 12\delta^2)$.

We calculate the dependence of the four neutrino fluxes upon the central temperature T_c when all the SSM calculations are accomplished at the central SSM temperature T_c^{SSM} ($T_c = T_c^{SSM}(1 + 3.12\delta)$). Recently both Bahcall and Ulmer [17] and Calabresu et al.[21]

have analyzed this dependence. The "experimental" values of δ we have selected from the measured fluxes imply that the central temperature T_c must be 1% greater than T_c^{SSM} when we evaluate the p, B, CNO fluxes and 7% greater than T_c^{SSM} for Be flux. The first lower temperature must correspond to the ionic plasma temperature, the second greater to the electronic plasma temperature. The meaning of these relations is very different from the one coming from the relations proposed by Bahcall et al. [17] and Calabresu et al. [21]. According to these authors, the central temperature must be lower than the SSM value to reproduce the behavior of the ratio. In our approach instead the central temperature must increase slightly with δ and the fluxes (except Φ_p) decrease because of the depleted Maxwellian tail and of the consequent rearrangement of the particle distribution.

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