Chaos in Electronics and Telecommunications

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Chapter 1

Introduction

The world is non-linear and chaotic. Even though engineers are always trying to approximate and linearize everything often times non-linearity has to be directly dealt with. At times non-linear properties of systems are can even be exploited for certain applications. In this report we will analyze a particular consequence of non-linearity and describe possible applications, especially in telecommunications.

Some applications of non-linear systems in electronics and telecommunications can be found in amplifiers, multipliers, oscillators, among others. Chaos theory has also seen a fair amount of applications in other fields. Most have serve to better model complex systems such as biological systems, economical models, and networks with many components such as the wide world web or even just facebook!

An interesting characteristic of some non-linear systems is chaos. Such will be the focus of this report. A brief introduction to chaos theory will be given. Thus giving a qualitative definition as to what exactly we mean by chaos. Afterwards some applications of chaos theory will be analyzed in telecommunications but also applications in other scientific fields will be mentioned. Finally a simple physical example of chaos in electronics will be discussed.
Chapter 2

What is Chaos?

2.1 Qualitative Description

What exactly do we mean by? When people hear the word chaos, even in a non-formal way, usually they already have an overall idea of what it represents. Yet, when trying to explain it, the answers are usually vague at the very least. Yes, chaos can be disorder, confusing, complex, unpredictable, but it is actually more than just that. A whole new branch of mathematics has arisen which tries to model and explain precisely chaos, but the scope of this report is just to briefly introduce chaos and then focus on possible applications. To understand what we mean by applications for chaos we must first though understand the basics of what we mean for chaos, hence in this introductory part we focus on doing so.

It might be easier to grasp the meaning of chaos by briefly analyzing so called chaotic systems. As any physical system, they can be modeled with differential equations, yet for there to be chaos this system must be non-linear. With this in mind we can now give an initial qualitative definition of chaos.

A chaotic system is a system of differential equations where any two solutions at a given time are exponentially different for two very similar initial conditions[5].

Therefore one can easily think of such a system as one where the output after some time will be unpredictable, even if the initial conditions are known and given. One could almost think that it a random behavior such as in for example Gaussian white noise. Nevertheless a chaotic system is fully described by a set of deterministic differential equation therefore we can call this deterministic chaos.

Another qualitative definition given by Strogatz in the same book gives us a few more definition. This one starts to get into more specifics and into interesting informations for applications since it is slightly less mathematical.

“Chaos is aperiodic long-term behavior in a deterministic system that exhibits
sensitive dependence on initial conditions”[3]

This latter definition by prof. Steven Strogatz we can remark three main points. A chaotic “signal”, or lets just say variable for the moment, will be aperiodic. One can imagine on having a periodic signal with an infinite period, thus basically never repeating a regular pattern. This in turn this signal again can almost be imagined to be similar to noise. The other two last main points of this definition is as before, chaotic systems are fully deterministic and very sensible to it’s initial set of conditions.

So why exactly would we ever want to use chaos in real life applications? Well, the simple answer would be: “Why not?”. It is a new fascinating scientific and mathematicla branch so researchers, especially engineers, will always try to find uses to new theories.

2.2 Mathematical Basis

Before going any deeper into the fascinating world of chaos we will first introduce a dew basic concepts and approaches for a better analysys of chaos. We will focus on examples, first simple linear examples then a non-linear system leading to a more complex (yet basic) example of a system which presents chaotic behavior. Most of the material usedfor this section of the report was obtained from the notes by Prof.Stievano and Prof.Biey [1].

To introduce the conepts in this chapter a few examples will be presented. They are all electrical systems were one or more features can be highlighted so as to better understand later theory.

2.2.1 First Order Linear Systems

Starting from the simplest possible system, a first order linear system. Consider a typical RC circuit with a switch which connects the generator to the RC cell at time \( t = 0 \)s such as the one in figure 2.1.

![Figure 2.1. Schematic of the simple RC circuit](image-url)
The current flowing through the circuit is equal to the flow of charges which in turn depends on the capacitor. Therefore

\[ i(t) = \frac{dq(t)}{dt} = C \frac{dv_c(t)}{t} = \frac{V_o - v_c(t)}{R} \] (2.1)

From now onwards the dot notation will be used for time derivatives for simplicity. Which can be re-written as:

\[ \dot{v}_c(t) = -\frac{v_c(t)}{RC} + \frac{V_o}{RC} \] (2.2)

And thus we have found a differential equation which fully describes the simple system. But instead of the usual approach of analytically solving the system, by direct integration or by using other method such as Laplace transform, we focus on a more graphical approach. Plotting \( \dot{v}_c \) in function of \( v_c \) one can obtain an interesting graph such as the left most one in figure 2.2. If we think of the time derivative as a rate of change, i.e. as a velocity of the quantity under analysis, it is safe to say that such \( v_c \) is at a stall or at a fixed point when the velocity is equal to zero which in this case is when \( v_c = V_o \). Moreover when the velocity, or rate of change, is positive then \( v_c \) will move towards the right, towards increasing values; whereas for negative velocities it will move back towards decreasing values.

![Phase Space Trajectory](image1)

![Time Domain](image2)

Figure 2.2. A graphical representation of the different methods

Notice in the first graph of figure 2.2 the two arrows pointing in different directions. They represent the concept of positive and negative velocities explained before. This type of graph is usually called the phase space curve. Now, getting back on track, to get to a final solution to the ordinary differential equation problem graphically consider that, for any given initial value of \( v_c \) this value will always move towards the so called fixed point. Moreover it’s derivative has a negative slope which
means that the shape of the $v_c(t)$ has to be concave down. In such a way the final solution can be graphically approximated and should result in something similar to what can be found using other methods as in the second graph of figure 2.2.

A further note on this example, the fixed point in this case is said to be a stable fixed point since for any initial value of the $v_c$ the system will be attracted towards it. This is also way it is also sometimes called an attractor since in more complex systems it may not be a point but more than that.

2.2.2 Second Order Linear systems

Consider a simple LC circuit. Described a set of linear ordinary differential equations shown in equation 2.3.

We consider the ideal case for obvious reasons. This again is a second order and most importantly linear system therefore we will not expect to meet chaos.

$$\frac{dv_c(t)}{dt} = \frac{1}{C}i_c(t)$$

$$\frac{di_c(t)}{dt} = -\frac{1}{L}v_c(t)$$

Figure 2.3. A schematics of the circuit

Instead of solving viewing the typical solution in time one could plot the variables of the system versus each other and obtain the phase portrait which is very useful when analyzing nonlinear systems.
Nothing really special in this simple circuit, but if we add some more complexity and non-linearity then bifurcations\(^1\) (such as period doubling) can occur leading to more interesting behavior. See how in this second order example the attractor is a circle which determines the frequency. For any set of initial conditions (different from the null one of course) the circuit will behave towards such a circle condition since it will always oscillate.

### 2.2.3 Non-Linear Systems

Usually when confronting non linearity one is used to setting it straight by hammer using some sort of linearization such as Taylor’s series. Here since we try to use a more graphical approach to things we don’t really care about the exact quantitative analysis of the particular system. What is of most interest in this case is the overall behavior given a set of initial conditions.

For example, consider the famous Van Der Pol’s oscillator. It is a well known non-linear second degree differential system. It’s phase corresponding phase portrait is shown in figure 2.6. We can immediately see how it is similar to the previous phase Portrait analysed, for the LC circuit. It is closed shape and this is due to the fact that this is an oscillating system.

Nonetheless the phase portrait is somewhat skewed and shaped awkwardly. This is due to the effects of non-linearity.

### 2.2.4 Third Order Systems

As the reader might know or might have understood at this point is that to have an oscillating system, it must also be a second order system.

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\(^1\)The study of bifurcations is out of the scope of this mini-project.
Figure 2.6. Phase Portrait of Van Der Pol’s Oscillator
Chapter 3

Chaos in Electronics

One can see chaos in real life in everyday life, our society, the thousands or complex networks that surrounds us and the everyday fact that all the small conditions at one moment can completely change the outcome in our day. But we also want to analyse chaos more closely and in a more familiar context. That was probably what Prof. Chua was thinking when he thought of his famous circuit ”Chua’s Circuit”.

We have been analysing circuits with a greater complexity. As previously suggested, to obtain a chaotic systems we need at least a third order differential system. Actually the three rules for having a chaotic electrical circuit are the following:\[3:\]

- Nonlinear elements.
- Locally active resistors.
- At least Three energy-storage elements.

The term of locally activity in the electrical world simply put means we need a negative and active resistance. This active element is basically what gives the required energy to the systems. Now let’s try to see how this applies to the simplest of examples.

3.1 Chua’s Circuit

It is the simplest electrical circuit which presents chaotic behavior. Can be described with the third degree differential equation system in equation 3.1. The two main features are the fact that it a third order system and that it also nonlinear. The non linearity and the local activity is given by a single non linear element. This element is often calle Chua’s diode but can be seen as a non-linear resistance with an I-V
characteristic shown in figure 3.2

\[
\frac{dv_1(t)}{dt} = \frac{1}{C_1}[(v_2 - v_1)G - f(v_1)]
\]

\[
\frac{dv_2(t)}{dt} = \frac{1}{C_2}[(v_1 - v_2)G + i_L]
\]

\[
\frac{di_L(t)}{dt} = \frac{-1}{L}[v_2 + R_o i_L]
\]

(3.1)

Figure 3.1. Chua’s Circuit

As one can appreciate in the plot in figure 3.3 the signals of the circuit in the time domain give very little information. The signal might seem periodic at a first glance but actually if one were to take a closer look there is no regular pattern. This is why we can talk of an aperiodic signal.

It is a non-linear oscillator which with a set of specific parameters shows chaos. The circuit is nevertheless an oscillator, hence under the right conditions (very often actually) it will behave in a very similar way to the Van der Pol’s oscillator. Yet for some specific parameters the phase portrait of the system will results in something like in figure 3.4 Where one can finally appreciate the fascination of chaos. In this case the attractor is this strange butterfly shaped figure. Actually this is precisely why chaotic attractors are called strange attractors. Moreover this was also the initial reason for the name of the butterfly effect which is simply a lay’s man definition of chaos if one thinks about it.

The strange attractor as in the simple examples from before basically attracts any solution starting from a given set of initial conditions. We can already see how for two infinitesimally similar initial condition the output at a certain time
Figure 3.2. I-V characteristics of the non-linear element

Figure 3.3. iL(t)

will be completely different. The shape of the attractor changes depending on the conditions and the even so the points of the system are changing quickly on the attractor.
Figure 3.4. Phase Portrait of Chua’s Circuit
Chapter 4
Synchronization

4.1 Synchronization with Chua’s Oscillators

Simple example on how it could be done using Chua’s Circuit. It has been done with more complex circuits. In this case the coupling is done directly but it can also be done wireless by propagating the coupling signal.

![Figure 4.1. Synchronization with Chua’s Circuit](image)

$i_L$ of the master with respect to the slave’s
Figure 4.2. Not Synchronized

Figure 4.3. Synchronized
Chapter 5

Application in Telecommunications

5.1 Motivations

Why should we want to introduce chaos into the practical world of engineering? Chaos has been a focus of many studies of many research groups and scientists for many years now but it is only until relatively recent that some interesting applications have started to be conceived. Especially in the field of telecommunications.

In general we can argue with the following list of reasons[4].

- **Low-power** implementations: circuits can be very simple.

- **Noiselike** appearance: Chaotic signals seem random therefore the are difficult to detect but actually they are deterministic.

- **Broadband spectra**: Just like noise they have very large bandwidth.

- **Nonlinearity**: Better efficiency and encoding.

- **Self-Synchronization**: Two or more chaotic circuits can synchronous with each other leading to many useful applications.

So there are quite a few reasons for using it. The two main reasons are firstly the fact that chaotic electrical signals have a very broad spectrum therefore one could think of using them in spread spectrum techniques. Secondly, and probably most importantly, some chaotic systems have the ability to completely synchronize with one or more other similar systems. This is a unique characteristic which has led to fascinating ideas [4].
5.2 Chaos in Encryption

As demonstrated by Cuomo and company in [2] a circuit demonstrating chaotic behavior can be exploited in some very interesting applications. In their case they based their circuit on the well known Lorenz system. It is a third order set of differential equations given by

\[
\begin{align*}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= rx - y - xz \\
\dot{z} &= xy - bz
\end{align*}
\] (5.1)

But what is of real interest here is not the system per se but how this system was used. As mentioned several times before, the key feature of some chaotic systems is that they are able to synchronize. An in depth discussion on synchronization is out of the scope of this report so we just limit to saying that for some given systems this behavior is very robust even under great disturbances such as in this case.

- Since chaotic signals are random-like they are perfect to encrypt data.
- The receiver has to be Synchronized with the transmitted in order to extract the information from the chaotic signals.
- Several different implementations have been made, Cuomo did it with a Lorentz-based chaotic circuit.

Possible implementation with Chua’s circuits.

![Figure 5.1. Block Diagram of a possible configuration](image-url)

As a first simple example we can think of using a chaotic circuit both at the transmitter’s and at the receiver’s side. Then one of the signals of the transmitter
is used to mask the transmitted signal. This can be done by simply adding the two signals. The receiver circuit is then a nearly identical chaotic circuit as the one used in the transmitter. This means that it can easily be synchronized. To do this operation, the signals which masks the signal of interest is sent to the receiver and is then incorporated into it’s system so that synchronisation is possible. After the receiver circuit is synchronized to the transmitter the it’s own chaotic signal can be used to subtract from the received signal to extract the signal of interest. This signal can be several orders of magnitude smaller than the chaotic signal therefore almost invisible at first. Any unwanted listener who catches the signal will just see something very similar to noise, the chaotic signal in essence.

Figures 5.2 and 5.3 show the most significant results from Cuomo’s research. In the former there are two plots: the transmitted signal before the masking and the extraced signal from the chaos. They are very similar, actually it was an audio signal used and the result was quite decent according to the researchers. On the other hand the latter shows the spectrum analysis where we can see a difference of more than 20 dB between the chaotic signal and the information signal.

Figure 5.2. Original and unmasked audio signal
5.3 Chaos in Modulation

In this section we present the use of chaos for modulation in telecommunications systems in literature [4]. The focus will be on spread spectrum (SS) techniques since that is where chaos can be exploited the best. In the example presented this was done using a PSK (Phase Shift Key) digital technique. We will not go into detail of the PSK and SS techniques since they are completely out of the scope of this report.

In typical communication systems signals are transmitted over a bandwidth more or less equivalent to their own bandwidth and usually this gives decent results. In some cases it might be convenient to transmit over a much wider bandwidth since maybe the signal has be hidden from unwanted receivers or more channels are desired. In digital application this is obtained usually with a Direct Sequence Spreading which lead to a technique called DS-SS. This can then be applied to one of the more typical digital modulation techniques such as PSK.

In a more conventional system the DS-SS technique can be implemented using a pseudo-noise sequence of bits (PN sequence) which is then used to modulate the sequence of bits containing the information [4]. This PN sequence is merely an extremely long sequence of bits which can be viewed as random since the sequence is so long. In [4] literature a few implementations of this have been proposed but using a chaotic sequence instead of the PN sequence. The block diagram of such a system is shown in figure 5.4.
5 – Application in Telecommunications

Figure 5.4. Block diagram of the structure
Chapter 6

Conclusions

The theory of chaos has sure gone a long way since it first came into being as side-product of the study of nonlinear dynamic systems. Sure, there is still a lot of work to be done since for many it still remains an esoteric field. The real life applications of chaos are still few but interesting, nevertheless books such as [4] shows us that the possibilities are many. For what concerns electronics they have mainly been used to study chaos in a more direct way by actually doing measurements. Whereas for communication systems the use of chaotic signals has given the possibilities of using another approach to the typical PN signals for encryption and spread spectrum techniques.

The book by Steven Strogatz[5] gives examples of several other applications other from electronics and telecommunications whereas [4] would be an optimal place to study many more applications in telecommunications, most of which were too complex for the purpose of this report. In the end, we can say that in many cases the use of chaos could be avoided since more conventional techniques exist already. Yet innovation, in all ways, comes from attempting new unconventional approaches.
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