



**Resilient Control of
Infrastructure
Networks**

DISMA PoliTo



National Research
Council of Italy



Institute of
Electronics
Computer and
Telecommunication
Engineering

AC optimal power flow in the presence of renewable sources and uncertain loads

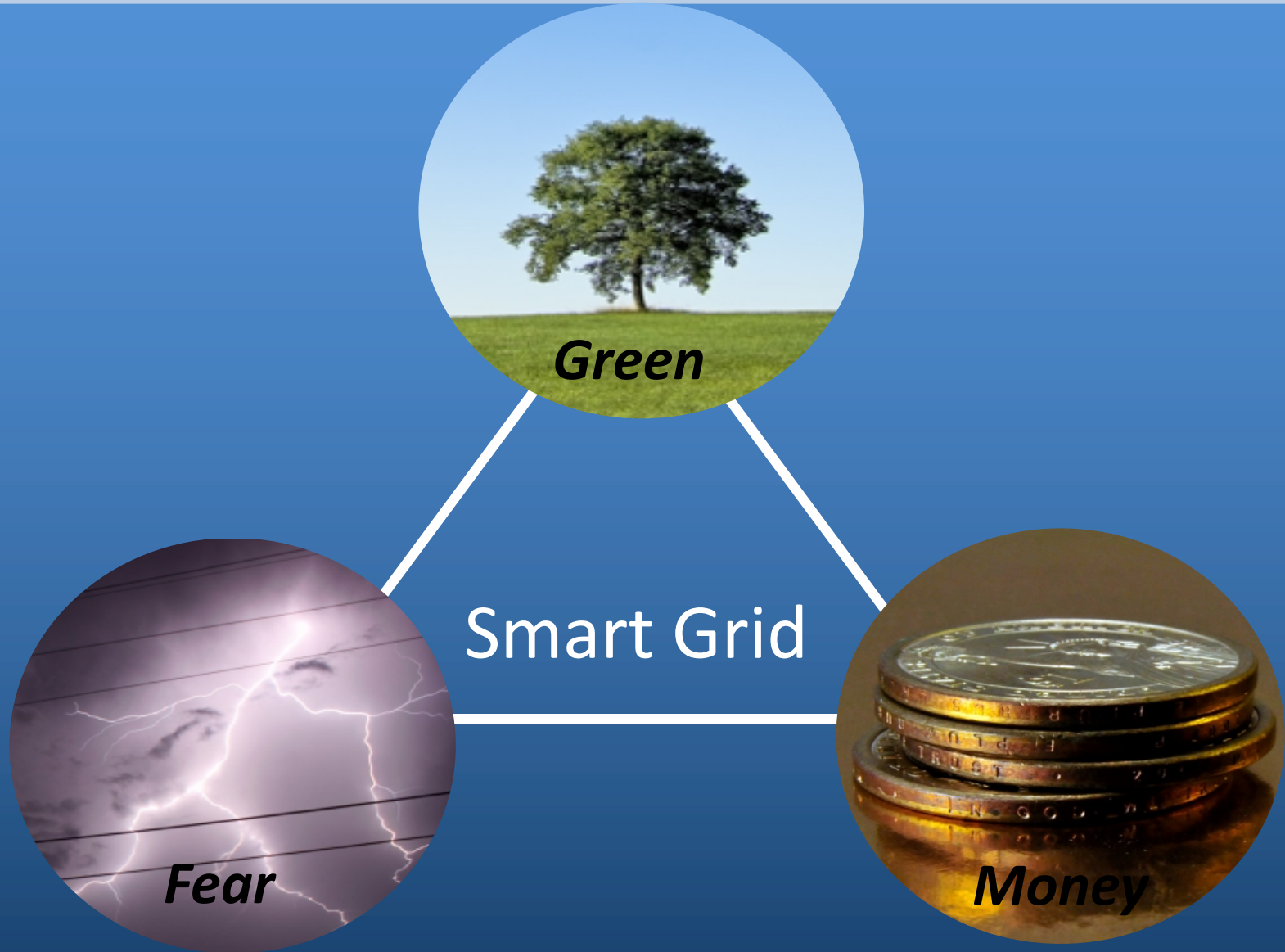
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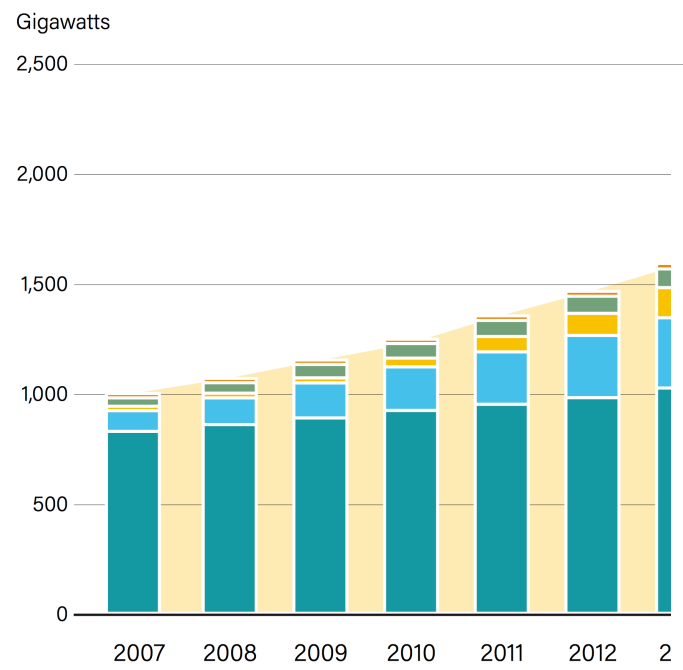
What's Driving Power Grid Design?





Renewable Sources Penetration

FIGURE 5. Global Renewable Power Capacity, 2007-2017



World Total
2,195 Gigawatts

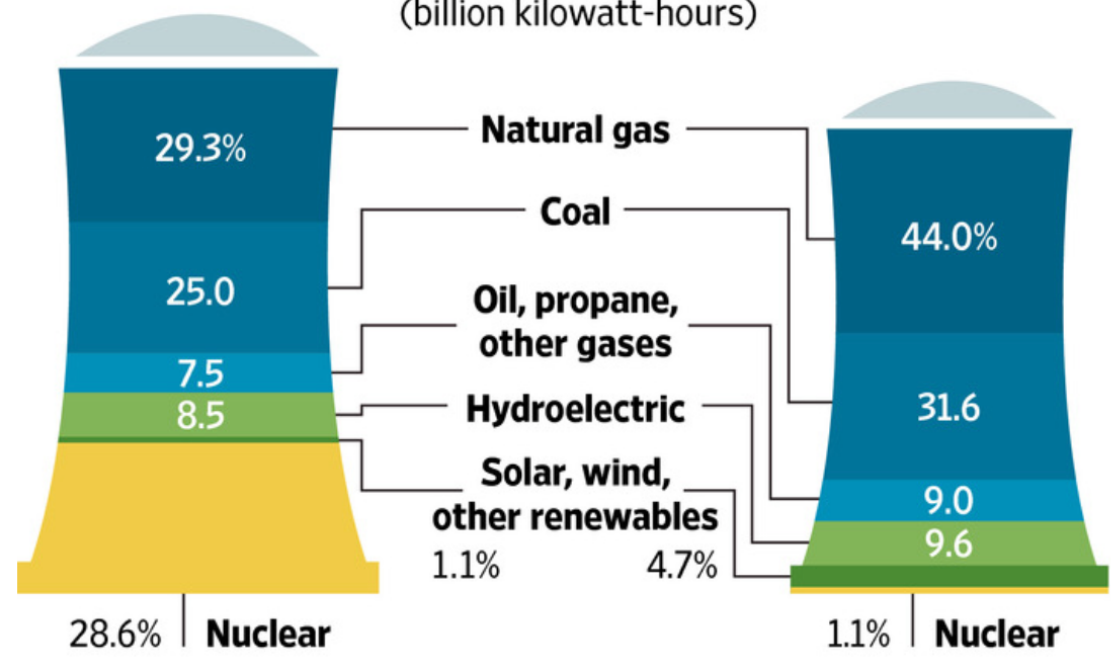
Global renewable power

2010

1,007 << TOTAL POWER GENERATED >> 885

(billion kilowatt-hours)

2015

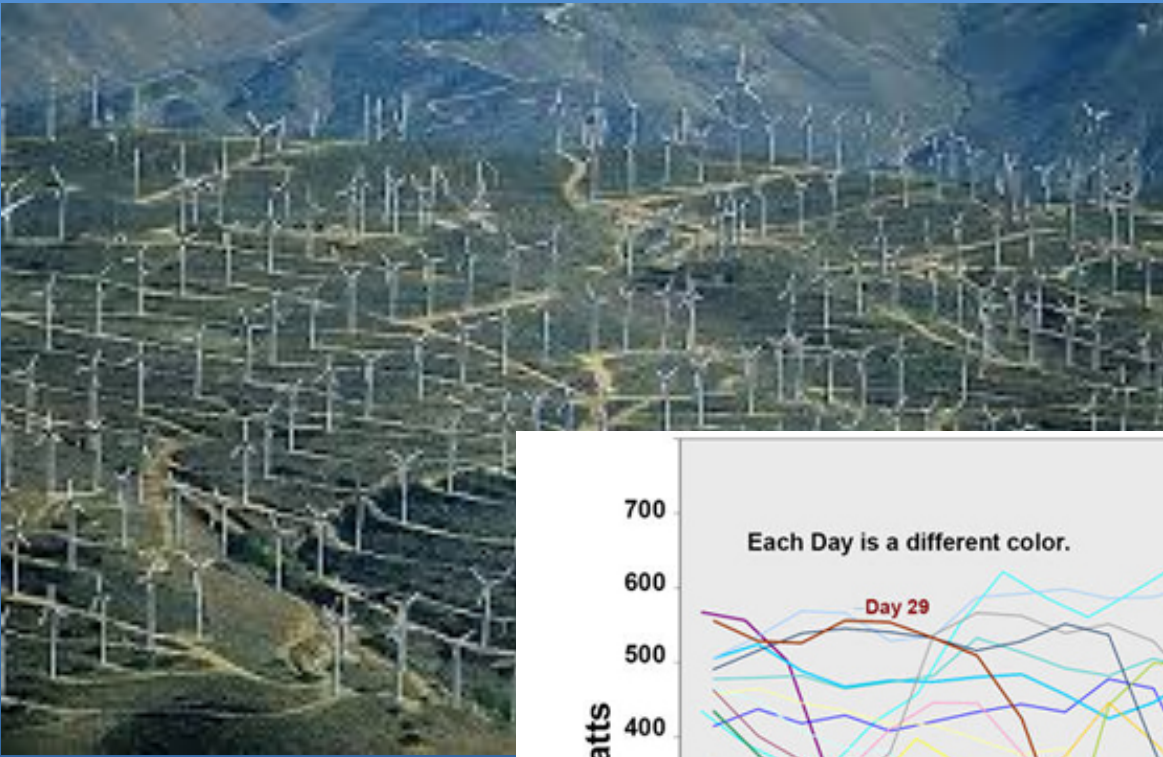


REN21 Renewable Energy Policy Network for the 21st Century

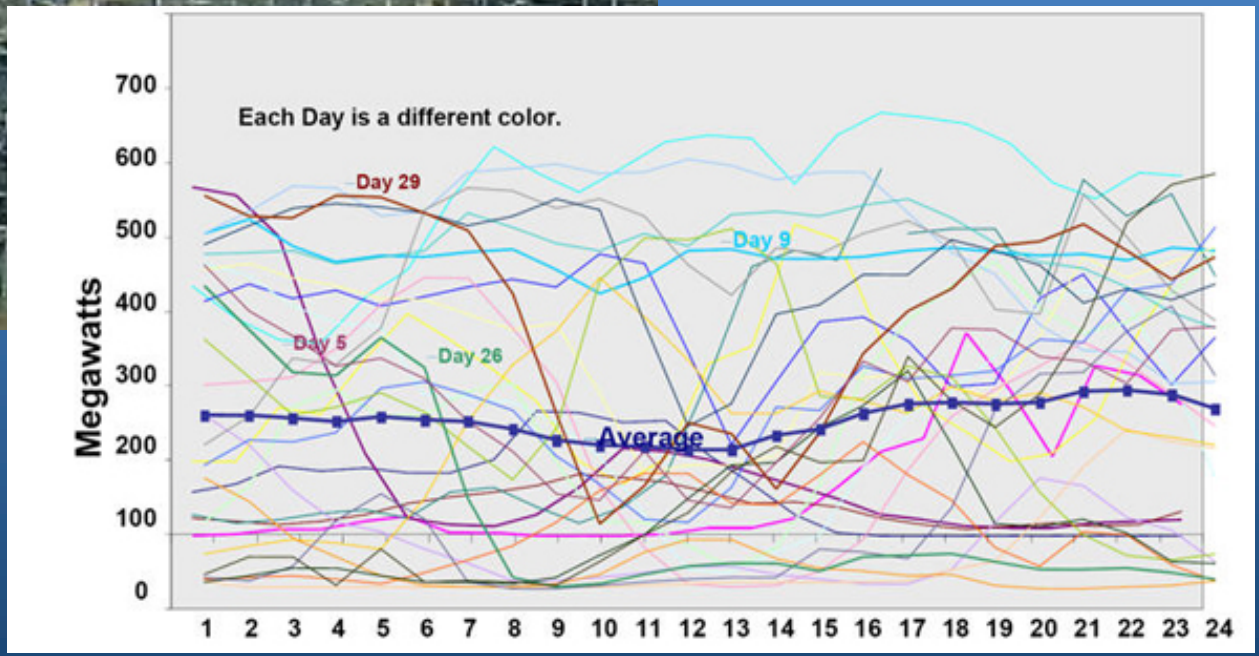
Source: Federation of Electric Power Companies of Japan THE WALL STREET JOURNAL.



High Variability (Wind Farms)



Tehachapi Wind Farm,
Southern CA

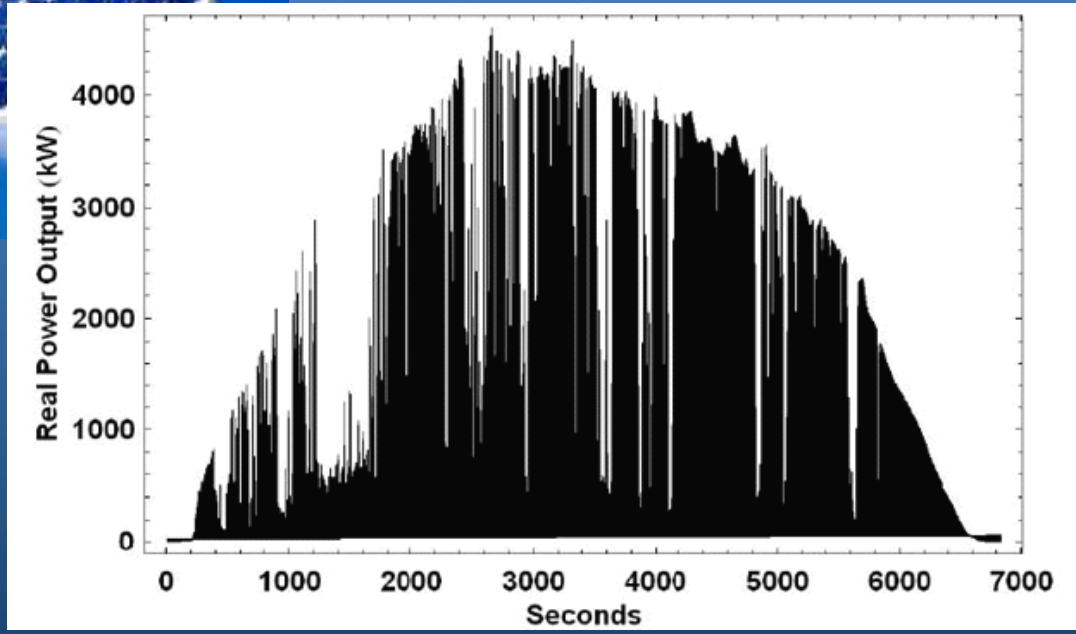


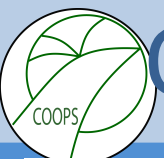


High Variability (Solar PV)



Springville AZ





Optimal Power Flow under Uncertainty

Japan's Shift to Renewable Energy Loses Power

“Most of the growth in renewables has been in solar, **but the industry is suffering.**”

“[...] utilities quickly complained about the **cost of protecting the power grid from imbalances in supply and demand caused by the variability of solar power**”



LNG storage tanks near Tokyo. Low natural-gas prices have blunted Japan's push into renewables. PHOTO: ISSEI KATO/REUTERS

By **MAYUMI NEGISHI**

5 COMMENTS

September 14, 2016

Five years after the disaster at the Fukushima Daiichi nuclear plant, the urgency to go green in Japan has faded.

- 1. **EU Will Withstand Effect of Brexit**
- 3. **Sleep Deprivation Is More Harmful Than You Think**
- 4. **Aung San Suu Kyi Set to Meet Obama**
- 5. **Clinton Camp Withheld Pneumonia News – At What Cost?**

- Most Popular Articles
- 1. **iOS 10 Review: You Don't Have to Buy a New iPhone**
 - 2. **Bayer-Monsanto Deal Would Forge New Agricultural Force**
 - 3. **Opinion: The Trump Plan Will Help Working Mothers**



Japan's Shift to Renewable Energy Loses Power

“Most of the growth in renewables has been in solar, **but the industry is suffering.**”

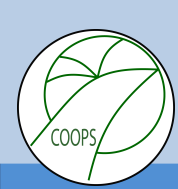
“[...] utilities quickly complained about the **cost of protecting the power grid from imbalances in supply and demand caused by the variability of solar power**”

We need to devise a radically new dispatch philosophy

1. *that minimizes generation costs*
2. *but does not violate generation and transmission constraints for all (most) admissible values of renewable power and variable demand*

1.

The Optimal Power Flow Problem



Optimal Power Flow (OPF)

OPF Goal: Dispatch Generating Units while minimizing Total Cost

OPF is solved routinely (in a **single** time period, e.g.1 hour), to determine

- How much power to generate? Where to generate it?
- Market operation & pricing: functionally combines the power flow with economic dispatch
- Parameter setting (generator injected power)



A Smart Grid

Renewable Gen.



Bus i

Generator



Load



Variable load



Line (i,k)

Bus k

Bus j

Generator nodes $\mathcal{G} \subseteq \mathcal{N}$

$$\mathcal{G} = \{1, \dots, n_G\}$$

Nodes i and j are linked with a complex admittance y_{ij}



Conventional Generators

A generator G_k connected to the k -th bus provides **complex power**

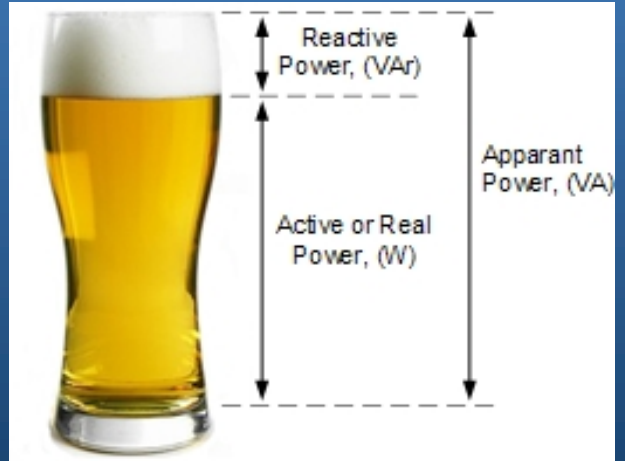


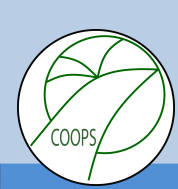
$$P_k^G + iQ_k^G$$

active power (real)
reactive power (imag)

What is reactive power?

Reactive power represents power that oscillates between the sources and the reactive components (inductors, capacitors)



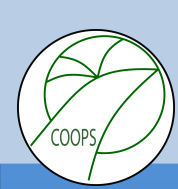


Power Generator Constraints

At any time, the generated power should satisfy given operational constraints for all $k \in \mathcal{G}$

$$P_{k \text{ min}} \leq P_k^G \leq P_{k \text{ max}}$$

$$Q_{k \text{ min}} \leq Q_k^G \leq Q_{k \text{ max}}$$



Voltage Constraints

The complex voltage at bus k is denoted as V_k

At any time, the voltage on the lines should satisfy magnitude and flow constraints

$$V_{k \text{ min}} \leq |V_k| \leq V_{k \text{ max}}$$

$$\forall k \in \mathcal{N}$$

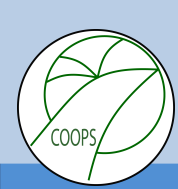
$$|V_l - V_m| \leq \Delta V_{lm}^{\text{max}}$$

$$\forall (l, m) \in \mathcal{L}$$

The second inequality limits the apparent power flow which can path through the line

The complex voltage is expressed in polar coordinates as

- Magnitude $|V_k|$
- Phase θ_k



Generator Bus (PV bus)

In a generator bus, the OPF has to determine the active power of the generator and the bus voltage magnitude

$$P_k^G, |V_k|$$

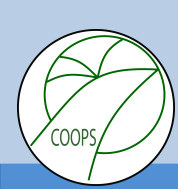


These are called control variables

Also bus voltage phase angle and generator reactive power

$$\theta_k, Q_k^G$$

need to determined by OPF, they are called state variable



Load Bus (PQ bus)

Active and reactive power of the load



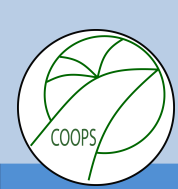
$$P_k^L, Q_k^L$$

are given (their values are known to the network operator)

Feasible bus voltages (magnitude and phase angle)

$$|V_k|, \theta_k$$

are to be determined (state variables)



Balance Equations

A Balance Equations should be satisfied for all $k \in \mathcal{N}$

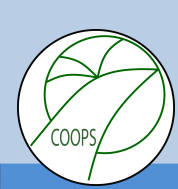
$$P_k^G - P_k^L = \sum_{l \in \mathcal{N}_k} \text{Re} \{ V_k (V_k - V_l)^* y_{kl}^* \}$$

$$Q_k^G - Q_k^L = \sum_{l \in \mathcal{N}_k} \text{Im} \{ V_k (V_k - V_l)^* y_{kl} \}$$

where \mathcal{N}_k is the set of all neighboring nodes of bus k

The solution of these equation is referred to as the power flow problem

Power flow methods find a mathematically but not necessarily physically feasible or optimal solution

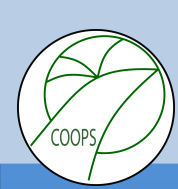


Cost Function

Cost function: Let $c_k(P_k^G)$ be a non-decreasing convex function representing the cost of generation for generator k

The total cost is given by

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$



Economic Dispatch

Used in power exchanges

Supply must meet demand

Takes into account generator limits

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$

$$P_k^{\min} \leq P_k^G \leq P_k^{\max}$$

$$\sum_{k \in \mathcal{G}} P_k^G = P^L$$

Economic Dispatch does not consider any network flow or network constraints!



Optimal Power Flow

Economic dispatch

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$

$$P_{k \min} \leq P_k^G \leq P_{k \max}$$

$$\sum_{k \in \mathcal{G}} P_k^G = P^L$$

Power flow

$$P_k^G - P_k^L = \sum_{l \in \mathcal{N}_k} \text{Re} \{V_k (V_k - V_l)^* y_{kl}^*\}$$

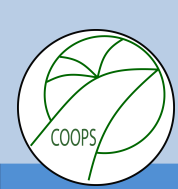
$$Q_k^G - Q_k^L = \sum_{l \in \mathcal{N}_k} \text{Im} \{V_k (V_k - V_l)^* y_{kl}^*\}$$



AC Optimal Power Flow

2.

AC-OPF Formulation and Relaxations



AC-OPF

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$

Cost

$$P_k^G - P_k^L = \sum_{l \in \mathcal{N}_k} \operatorname{Re} \{V_k (V_l - V_k)^* y_{kl}^*\}$$
$$Q_k^G - Q_k^L = \sum_{l \in \mathcal{N}_k} \operatorname{Im} \{V_k (V_l - V_k)^* y_{kl}^*\}$$

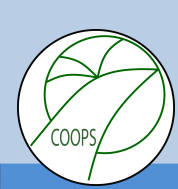
Balance
Constraints

$$P_k^{\min} \leq P_k^G \leq P_k^{\max}$$
$$Q_k^{\min} \leq Q_k^G \leq Q_k^{\max}$$

Power
Constraints

$$V_k^{\min} \leq |V_k| \leq V_k^{\max}$$
$$|V_l - V_m| \leq \Delta V_{lm}^{\max}$$

Voltage
Constraints



Control & State Variable

As already seen, the variables appearing in the OPF formulation are divided into two classes, depending on their role in the optimization problem

Control variables are those used by the network operator to set the operating condition of the network

$$\mathbf{u} \doteq \{P_1^G \cdots P_{n_g}^G, |V_1| \cdots |V_{n_g}|\}$$

State variables are dependent variables that represent the state of a power network

$$\mathbf{x} \doteq \{Q_1^G \cdots Q_{n_g}^G, |V_{n_g+1}|, \dots, |V_n|, \theta_1, \dots, \theta_n\}$$



AC-OPF: Control & State Variables

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$

$$f(\mathbf{u})$$

Cost

$$\begin{aligned}
 P_k^G - P_k^L &= \sum_{l \in \mathcal{N}_k} \text{Re} \{V_k(V_l - V_k)^* y_{kl}^*\} \\
 Q_k^G - Q_k^L &= \sum_{l \in \mathcal{N}_k} \text{Im} \{V_k(V_l - V_k)^* y_{kl}^*\}
 \end{aligned}$$

$$g(\mathbf{u}, \mathbf{x}) = 0$$

Balance Constraints

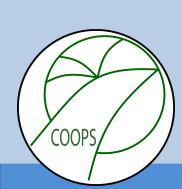
$$\begin{aligned}
 P_{k \min} &\leq P_k^G \leq P_{k \max} \\
 Q_{k \min} &\leq Q_k^G \leq Q_{k \max}
 \end{aligned}$$

$$f(\mathbf{u}, \mathbf{x}) \leq 0$$

Power Constraints

$$\begin{aligned}
 V_{k \min} &\leq |V_k| \leq V_{k \max} \\
 |V_l - V_m| &\leq \Delta V_{lm}^{\max}
 \end{aligned}$$

Voltage Constraints



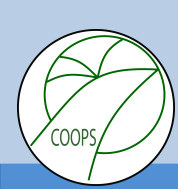
AC-OPF: Control & State Variables

$\min_{\mathbf{u}} f(\mathbf{u})$ s.t.: there exist \mathbf{x} such that

$$g(\mathbf{x}, \mathbf{u}) = 0$$

$$h(\mathbf{x}, \mathbf{u}) \leq 0$$

Given the demand, we optimally design the generators (injected power & voltage magnitude at PV nodes) so that there exists a network configuration (reactive power and voltage phase, complex voltage at PQ nodes) satisfying the operational constraints



AC-OPF: Comments

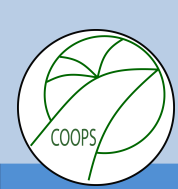
The AC-OPF is **nonconvex**, due to the quadratic balance equations

Several commercial solvers are available (e.g. MATPOWER)

No guarantee that we find the global optimum

Solution strategies: relax/approximate the problem

- DC OPF
- Convex relaxations



A Convex Relaxation of AC-OPF

Define the vector of complex bus voltages

$$\mathbf{V} \doteq [V_1, \dots, V_n]^T$$

The OPF quadratic constraints can be reformulated as linear ones by introducing a new variable

$$\mathbf{W} = \mathbf{V}\mathbf{V}^*$$

This variable should satisfy the following two constraints

$$\mathbf{W} \succeq 0$$

$$\text{rank}\{\mathbf{W}\} = 1$$

[Low] S.H. Low, *IEEE Trans. on Control of Network Systems*, 2014

[Madani] R. Madani, S. Sojoudi, J. Lavaei, *IEEE Trans. on Power Systems*, 2015



CR-AC-OPF

$$\min \sum_{k \in \mathcal{G}} c_k(P_k^G)$$

Cost

$$\mathbf{W} \succeq 0 \quad \text{rank}\{\mathbf{W}\} = 1$$

Rank

$$P_k^G - P_k^L = \sum_{l \in \mathcal{N}_k} \text{Re} \{ (W_{kk} - W_{kl})^* y_{kl}^* \}$$

$$Q_k^G - Q_k^L = \sum_{l \in \mathcal{N}_k} \text{Im} \{ (W_{kk} - W_{kl})^* y_{kl}^* \}$$

Balance
Constraints

$$P_{k \min} \leq P_k^G \leq P_{k \max}$$

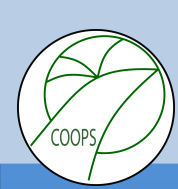
$$Q_{k \min} \leq Q_k^G \leq Q_{k \max}$$

Power
Constraints

$$(V_{k \min})^2 \leq W_{kk} \leq (V_{k \max})^2, \quad \forall k \in \mathcal{N}$$

$$W_{ll} + W_{mm} - W_{lm} - W_{ml} \leq (\Delta V_{lm}^{\max})^2$$

Voltage
Constraints



CR-AC-OPF

By removing the rank constraint, the ensuing problem becomes convex and can be efficiently solved by suitable SDP solvers

The relaxation is exact whenever we find a rank one solution

It turned to be exact for various IEEE test problems

3.

Taming Uncertainty in AC-OPF

Towards resilient smart grids



Wind Characterization

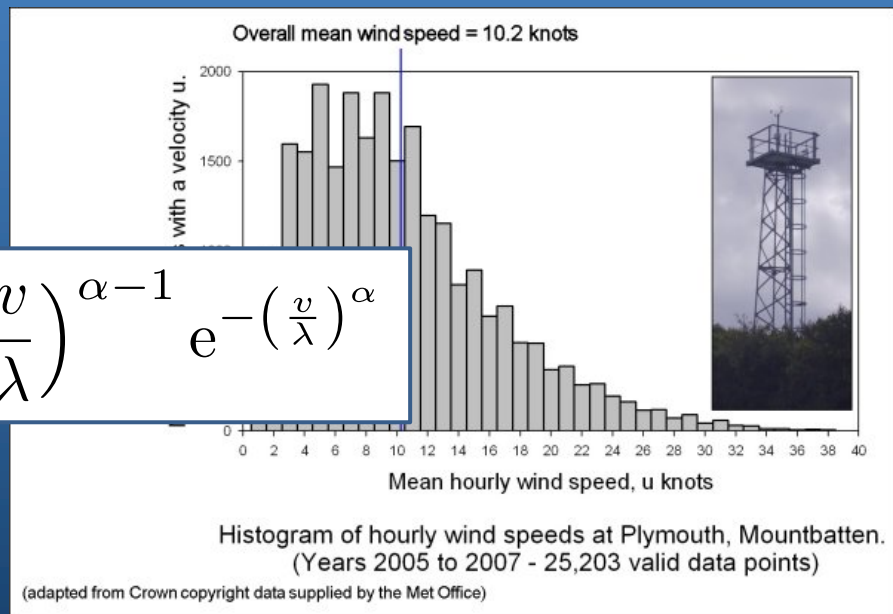
Electricity generated by a wind farm depends on the wind speed v_W and location of installation

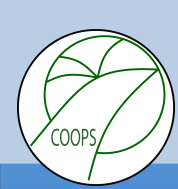
$$P_R = \frac{\rho}{2} c_p A_r (v_W)^3$$



Weibull distribution to describe wind speed variations

$$f_v(v) = \frac{\alpha}{\lambda} \left(\frac{v}{\lambda}\right)^{\alpha-1} e^{-\left(\frac{v}{\lambda}\right)^\alpha}$$





Generator Variability

A renewable energy generator connected to bus k provides an uncertain complex power

$$P_k^R(\delta_k^R) + Q_k^R(\delta_k^R)i = P_k^{R,0} + Q_k^{R,0}i + \delta_k^R$$

The complex uncertainty

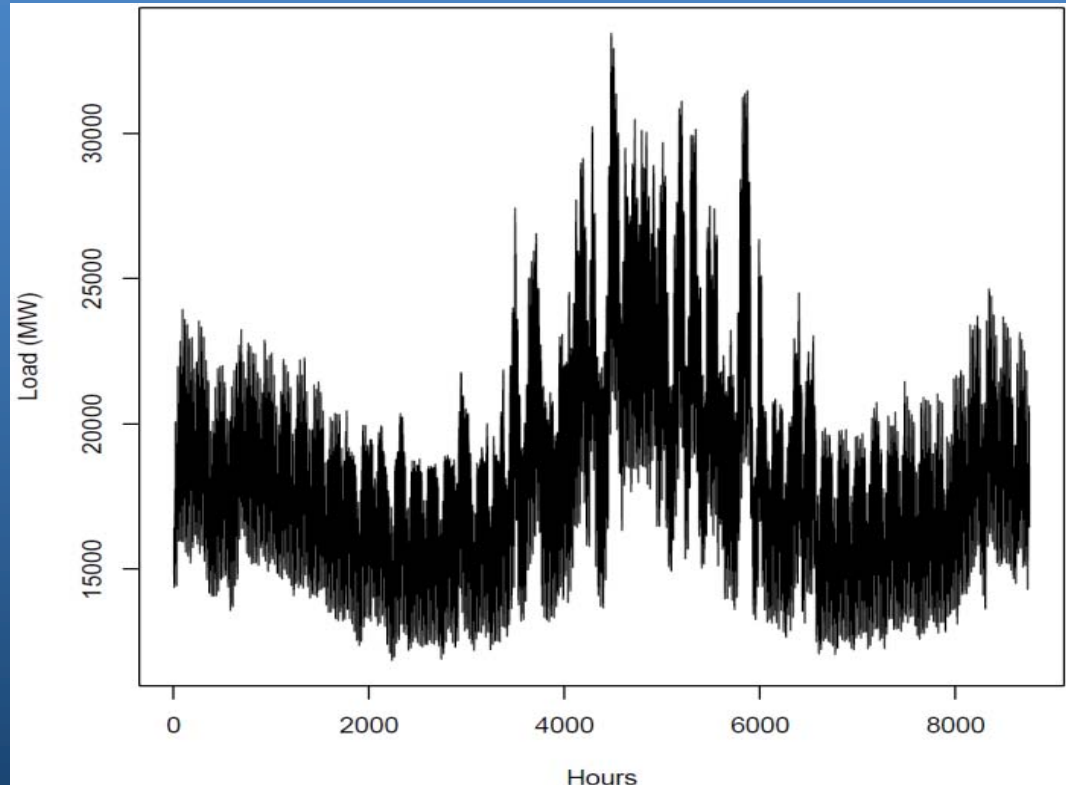
$$\delta_k^R \in \Delta_k^R \subset \mathbb{C}$$

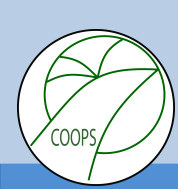
represents the power generation fluctuations which mainly depend on the environmental conditions (wind/sun)



Load Variability

Similarly, loads exhibit high variability
New York Independent System Operator
(NYISO) hourly load for the year 2010, in MW





Variable Loads

The complex load connected to bus k is represented as

$$P_k^L(\delta_k^L) + Q_k^L(\delta_k^L)i = P_k^{L,0} + Q_k^{L,0}i + \delta_k^L$$

where $P_k^{L,0}, Q_k^{L,0}$ represent expected active and reactive load and

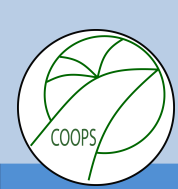
$$\delta_k^L \in \Delta_k^L \subset \mathbb{C}$$

is the complex fluctuation in the demand at bus k

All the uncertainty is captured in the vector

$$\delta \doteq [\delta_1^L \cdots \delta_n^L \delta_1^R \cdots \delta_n^R]^T,$$

which lies in $\Delta \doteq \Delta_1^L \times \cdots \times \Delta_n^L \times \Delta_1^R \times \cdots \times \Delta_n^R$.



A resilient design

The difference between real-time and predicted demand is distributed among all generators

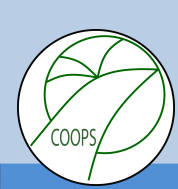
More precisely, a deployment vector is introduced

$$\alpha \doteq [\alpha_1, \dots, \alpha_{n_g}]^T, \quad \sum_{k \in \mathcal{G}} \alpha_k = 1, \quad \alpha_k \geq 0$$

Real-time active power of generator k is designed as

$$\bar{P}_k^G = P_k^G + \alpha_k \left(\sum_{j \in \mathcal{N}} \text{Re}\{\delta_j^L\} - \sum_{k \in \mathcal{R}} \text{Re}\{\delta_k^R\} \right)$$

α is an additional control variable



Robust Resilient AC-OPF

Resilient formulation

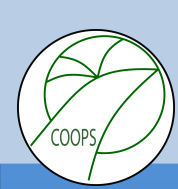
$\min_{\mathbf{u}} f(\mathbf{u})$ subject to

for any $\boldsymbol{\delta} \in \Delta$ there exists $\mathbf{x} = \mathbf{x}(\boldsymbol{\delta})$ such that

$$g(\mathbf{u}, \mathbf{x}, \boldsymbol{\delta}) = 0$$

$$h(\mathbf{u}, \mathbf{x}, \boldsymbol{\delta}) \geq 0$$

How to solve this? Apply [Low]'s relaxation



Towards a Convex Relaxation

Assume w.l.o.g. that the first n_g buses are generator buses and the remaining ones are load buses, we have

$$\mathbf{W} = \mathbf{V}\mathbf{V}^* = \begin{bmatrix} |V_1|^2 & V_1V_2^* & \dots & V_1V_{n_g}^* & \dots & V_1V_n^* \\ & |V_2|^2 & V_2V_3^* & \dots & \dots & V_2V_n^* \\ & & \ddots & & & \\ & & & |V_{n_g}|^2 & & \\ & & & & \ddots & \\ & & & & & |V_n|^2 \end{bmatrix}.$$

$$\mathbf{W} = \mathbf{W}^u + \mathbf{W}^x$$

Convexified Resilient AC-OPF

$$\text{minimize}_{\mathbf{P}^G, \alpha, \mathbf{W}^u} \sum_{k \in \mathcal{G}} f_k(P_k^G)$$

subject to for all $\delta \in \Delta$ there exist $\mathbf{Q}^G = \mathbf{Q}^G(\delta)$, $\mathbf{W}^x = \mathbf{W}^x(\delta)$

$$\min_{\mathbf{u}} f(\mathbf{u}) \text{ subject to}$$

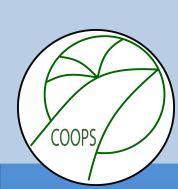
for any $\delta \in \Delta$ there exists $\mathbf{x} = \mathbf{x}(\delta)$ such that

$$\left. \begin{array}{l} \tilde{g}(\mathbf{u}, \mathbf{x}, \delta) = 0 \\ \tilde{h}(\mathbf{u}, \mathbf{x}, \delta) \geq 0 \end{array} \right\} \text{convex in } \mathbf{u}, \mathbf{x} \text{ for given } \delta$$

$$\mathbf{u} = \{\mathbf{P}^G, \mathbf{W}^u, \alpha\} \quad \mathbf{x} = \{\mathbf{Q}^G, \mathbf{W}^x\}$$

$$(V_{k \min})^2 \leq W_{kk} \leq (V_{k \max})^2, \quad \forall k \in \mathcal{N}$$

$$W_{ll} + W_{mm} - W_{lm} - W_{ml} \leq (\Delta V_{lm}^{\max})^2, \quad \forall (l, m) \in \mathcal{L}$$



CR-AC-OPF

This clearly improves upon robust formulations based on DC power flow

To deal with the «there exists», previous works introduce a finite (linear) parameterization of $\mathbf{W}(\boldsymbol{\delta})$

$$\mathbf{W}(\boldsymbol{\delta}) = \mathbf{A} + \sum_k \mathbf{B}_k \delta_k,$$

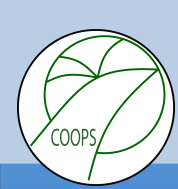
CR-AC-OPF formulation is less conservative, since no *specific dependence* is imposed

[ETH-DC] M. Vrakopoulou *et al.*, IEEE Trans. on Power Systems, 2013

[Bienstock-DC] D. Bienstock *et al.*, SIAM Review, 56(3):461–495, 2014

[Modarresi] M. S. Modarresi *et al.*, IEEE Trans. on Power Systems, 2018

[ETH-AC] M. Vrakopoulou *et al.*, IEEE POWERTECH, 2013



Convexified Resilient AC-OPF

CR-AC-OPF can be proven to be exact for several problem structures

THEOREM (EXACTNESS OF CR-AC-OPF)

Consider a lossless weakly-cyclic network with cycles of size three, and assume $Q_{k \min} = -\infty$ for every $k \in \mathcal{G}$. Then, the convex relaxation CR-AC-OPF is exact.

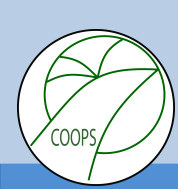
However this is in general an hard semi-infinite optimization problem

Need computable approximations

[FD-AC] M. Chamambaz, FD, C. Lagoa, IEEE Trans. Control of Network Systems, 2016

4.

Sampled-based Solutions

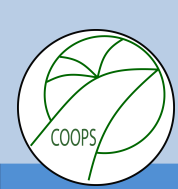


Sample-Based Methods

Based on the availability of a prescribed number of random uncertainty samples

Solve the «sample based» problem

Provides guarantees on the goodness of the solution



Sample-Based Solution

$\min_{\mathbf{u}} f(\mathbf{u})$ subject to

for any $\delta \in \Delta$ there exists $\mathbf{x} = \mathbf{x}(\delta)$ such that

$$\left. \begin{array}{l} g(\mathbf{u}, \mathbf{x}, \delta) = 0 \\ h(\mathbf{u}, \mathbf{x}, \delta) \geq 0 \end{array} \right\} \text{convex in } \mathbf{u}, \mathbf{x} \text{ for given } \delta$$

We generate N samples of the uncertainty

$$\delta^{(1)}, \dots, \delta^{(N)}$$

For every sample, we define a different certificate

$$\mathbf{x}_1, \dots, \mathbf{x}_N$$



Scenario with Certificates

$\min_{\mathbf{u}} f(\mathbf{u})$ subject to

for any $\delta \in \Delta$ there exists $\mathbf{x} = \mathbf{x}(\delta)$ such that

$$g(\mathbf{u}, \mathbf{x}, \delta) = 0$$

$$h(\mathbf{u}, \mathbf{x}, \delta) \geq 0$$



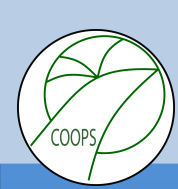
$\min_{\mathbf{u}, \mathbf{x}_1, \dots, \mathbf{x}_N} f(\mathbf{u})$ subject to

SwC

$$g(\mathbf{u}, \mathbf{x}_i, \delta^{(i)}) = 0$$

$$h(\mathbf{u}, \mathbf{x}_i, \delta^{(i)}) \geq 0$$

$i = 1, \dots, N$



Sample Complexity Result

Define the probability of violation of design

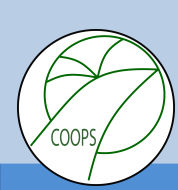
$\text{Viol}(\mathbf{u}) =$

$$\Pr \{ \boldsymbol{\delta} \mid \nexists \mathbf{x} \text{ satisfying } g(\mathbf{u}, \mathbf{x}, \boldsymbol{\delta}) = 0, h(\mathbf{u}, \mathbf{x}, \boldsymbol{\delta}) \geq 0 \} .$$

THEOREM (GUARANTEES OF SWC)

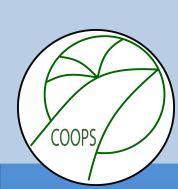
If **number of samples high enough**

then **things will most probably go well**



SWC-AC-OPF

$$\begin{aligned}
 & \text{minimize} && \gamma \\
 & \mathbf{P}^G, \mathbf{W}^u, \alpha, \mathbf{Q}^{G,[1]}, \dots, \mathbf{Q}^{G,[N]}, \mathbf{W}^{\times,[1]}, \dots, \mathbf{W}^{\times,[N]} \\
 & \text{subject to: for } I = 1, \dots, N \\
 & \mathbf{W}^{[I]} = \mathbf{W}^u + \mathbf{W}^{\times,[i]}, \quad \mathbf{W}^{[i]} \succeq 0, \quad \alpha_k \geq 0, \quad \forall k \in \mathcal{G} \\
 & L_{lm}^{[I]} = |(W_{ll}^{[I]} - W_{lm}^{[I]})^* y_{lm}^*| + |(W_{mm}^{[I]} - W_{ml}^{[I]})^* y_{lm}^*| \\
 & \sum_{k \in \mathcal{G}} f_k(P_k^G) + \gamma_b \sum_{k \in \mathcal{G}} Q_k^{G,[I]} + \gamma_\ell \sum_{(l,m) \in \mathcal{L}^{\text{prob}}} L_{lm}^{[I]} \leq \gamma \\
 & P_k^G + \alpha_k \mathbf{s}^T \text{Re}\{\boldsymbol{\delta}^{(i)}\} + P_k^R(\boldsymbol{\delta}^{(i)}) - P_k^L(\boldsymbol{\delta}^{(i)}) = \\
 & \quad \sum_{l \in \mathcal{N}_k} \text{Re}\left\{ (W_{kk}^{[I]} - W_{kl}^{[i]})^* y_{kl}^* \right\}, \quad \forall k \in \mathcal{N} \\
 & Q_k^{G,[i]} + Q_k^R(\boldsymbol{\delta}^{(i)}) - Q_k^L(\boldsymbol{\delta}^{(i)}) = \\
 & \quad \sum_{l \in \mathcal{N}_k} \text{Im}\left\{ (W_{kk}^{[I]} - W_{kl}^{[i]})^* y_{kl}^* \right\}, \quad \forall k \in \mathcal{N} \\
 & P_k^{\min} \leq P_k^G + \alpha_k \mathbf{s}^T \text{Re}\{\boldsymbol{\delta}^{(i)}\} \leq P_k^{\max}, \quad \forall k \in \mathcal{G} \\
 & Q_k^{\min} \leq Q_k^{G,[i]} \leq Q_k^{\max}, \quad \forall k \in \mathcal{G} \\
 & (V_k^{\min})^2 \leq W_{kk}^{[I]} \leq (V_k^{\max})^2, \quad \forall k \in \mathcal{N} \\
 & W_{ll}^{[I]} + W_{mm}^{[I]} - W_{lm}^{[I]} - W_{ml}^{[I]} \leq (\Delta V_{lm}^{\max})^2, \quad \forall (l,m) \in \mathcal{L}
 \end{aligned}$$



Handling N-1 Security Constraints

The N-1 security constrained OPF easily fits in this framework in N-1 SC-OPF framework, only the outages of a single component are taken into account

$$\mathcal{I}^{\text{out}} = \{0, 1, \dots, N_{\text{out}}\}$$

In a real network some buses may have a larger probability of incurring into an outage, i.e. due to geographical location, or because they employ older technologies

We associate to each element of \mathcal{I}^{out} a different “outage probabilities”

$$p_i^{\text{out}} \in [0, 1]$$

obtaining a **probabilistic SC-OPF**

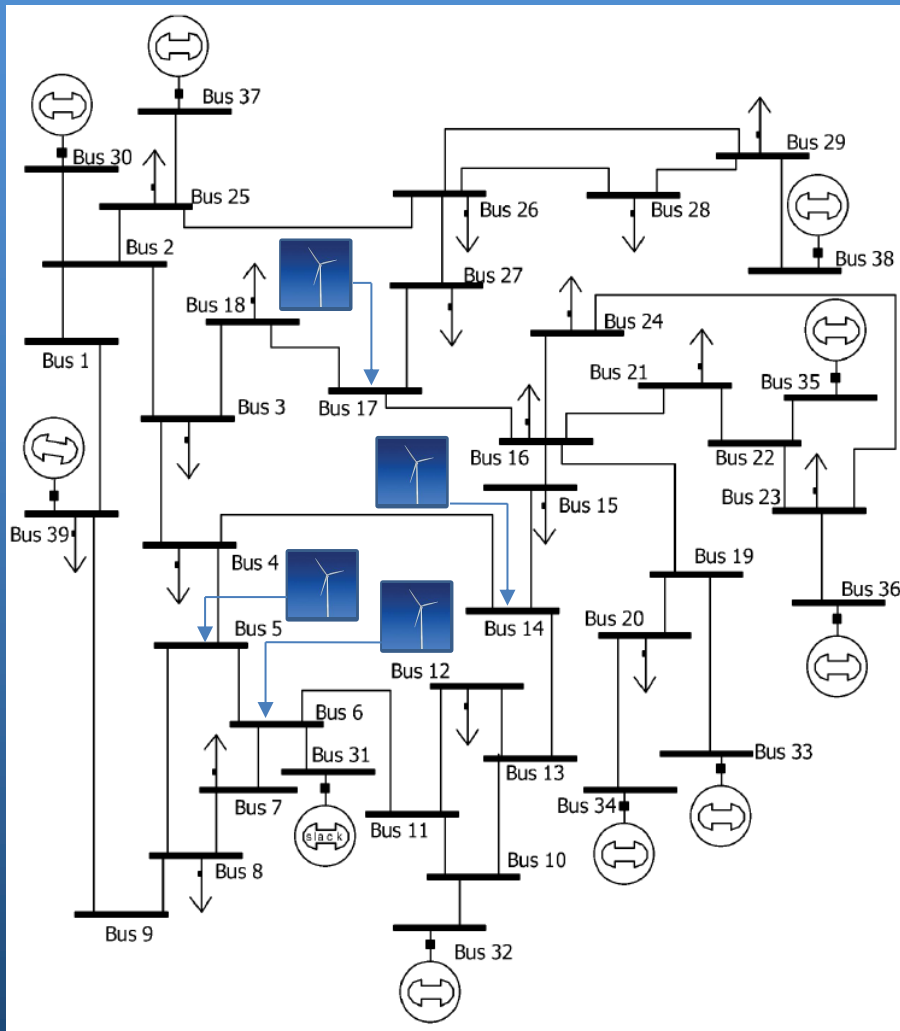
5.

Numerical Results

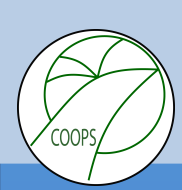


IEEE 39-bus

- 21 loads with a total demand of 6254 MW
- 46 transmission lines
- 11 tap changing transformers
- Penetration level 30%



- 10 order IV synchronous generators with max capacitance of 8404 MW
- TG and AVR control devices
- Individual cost parameters for every generator



IEEE 39-bus – Comparison with nominal design

We consider a 24 hour demand pattern and solve the OPF for each hour

The problem has 31 design variables

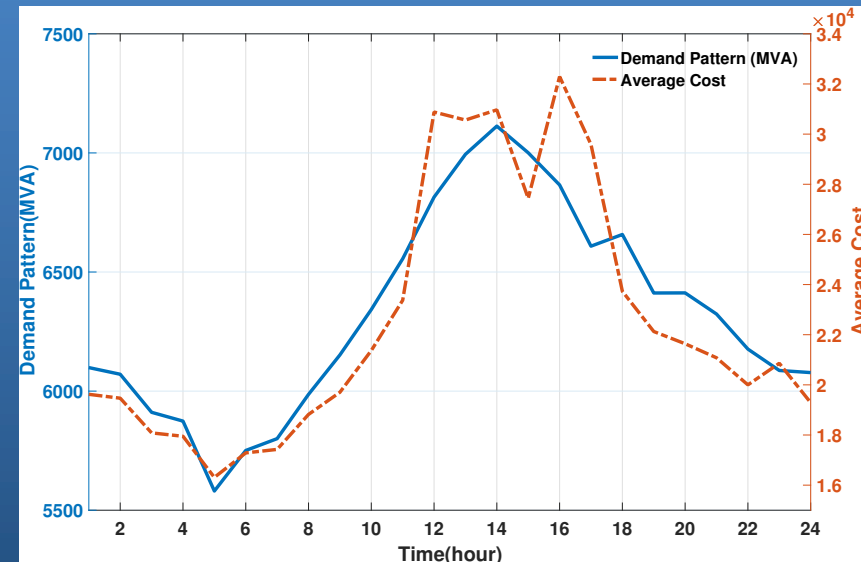
Setting $\epsilon = .02$ and $\delta = 1 \times 10^{-15}$ we get $N = 5, 105$

We run SWC-AC-OPF obtaining

$$\mathbf{u} = \{\mathbf{P}^G, \mathbf{W}^u, \alpha\}$$

We then checked the solution with an a-posteriori Monte Carlo with 10,000 samples: For each sample we solved the non-linear AC-OPF

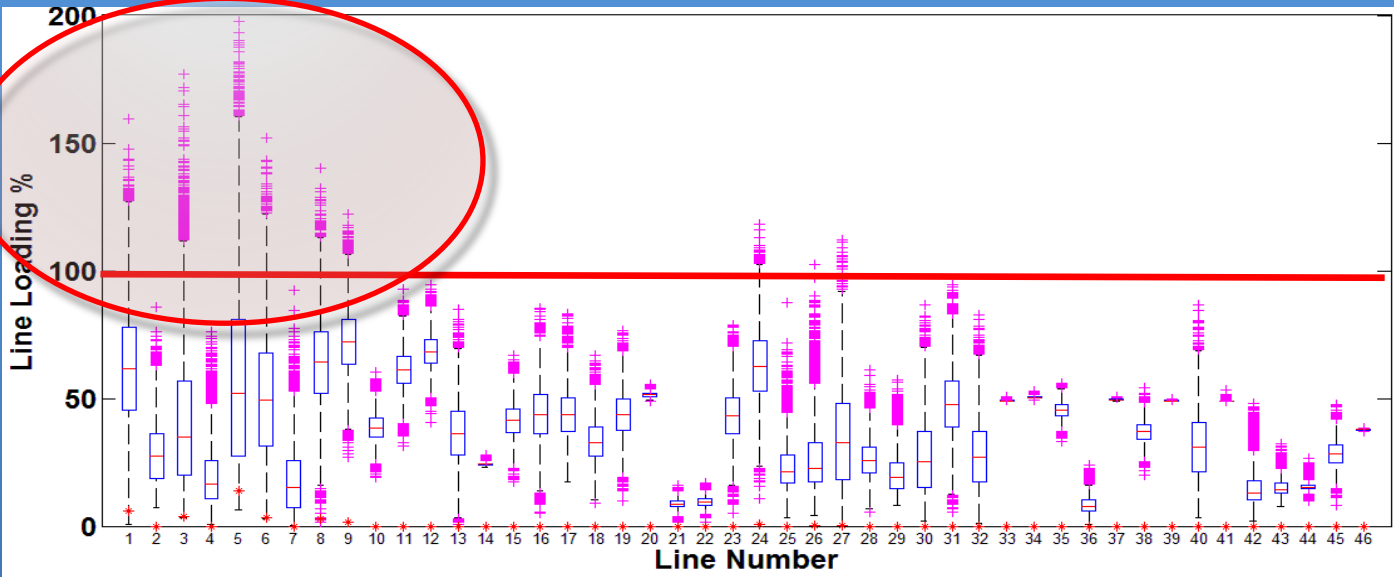
Approach	Empirical Violation
SwC	0.0014
Nominal	0.5374



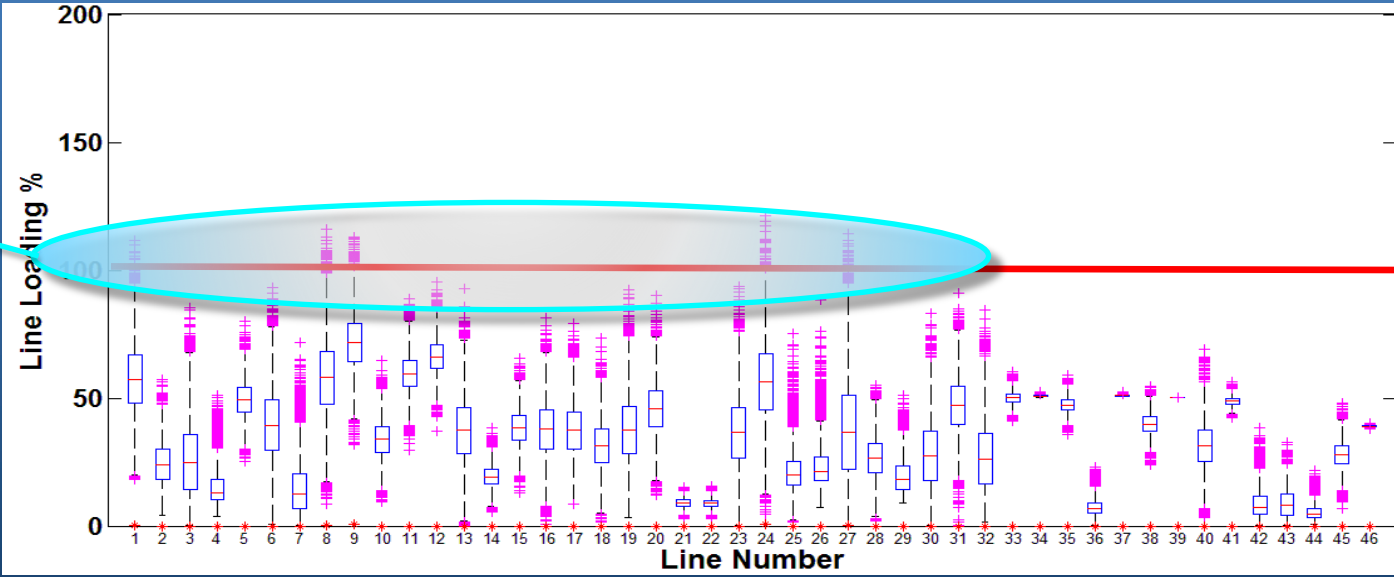


IEEE 39-bus – Comparison with nominal design

High violation



Very low violation

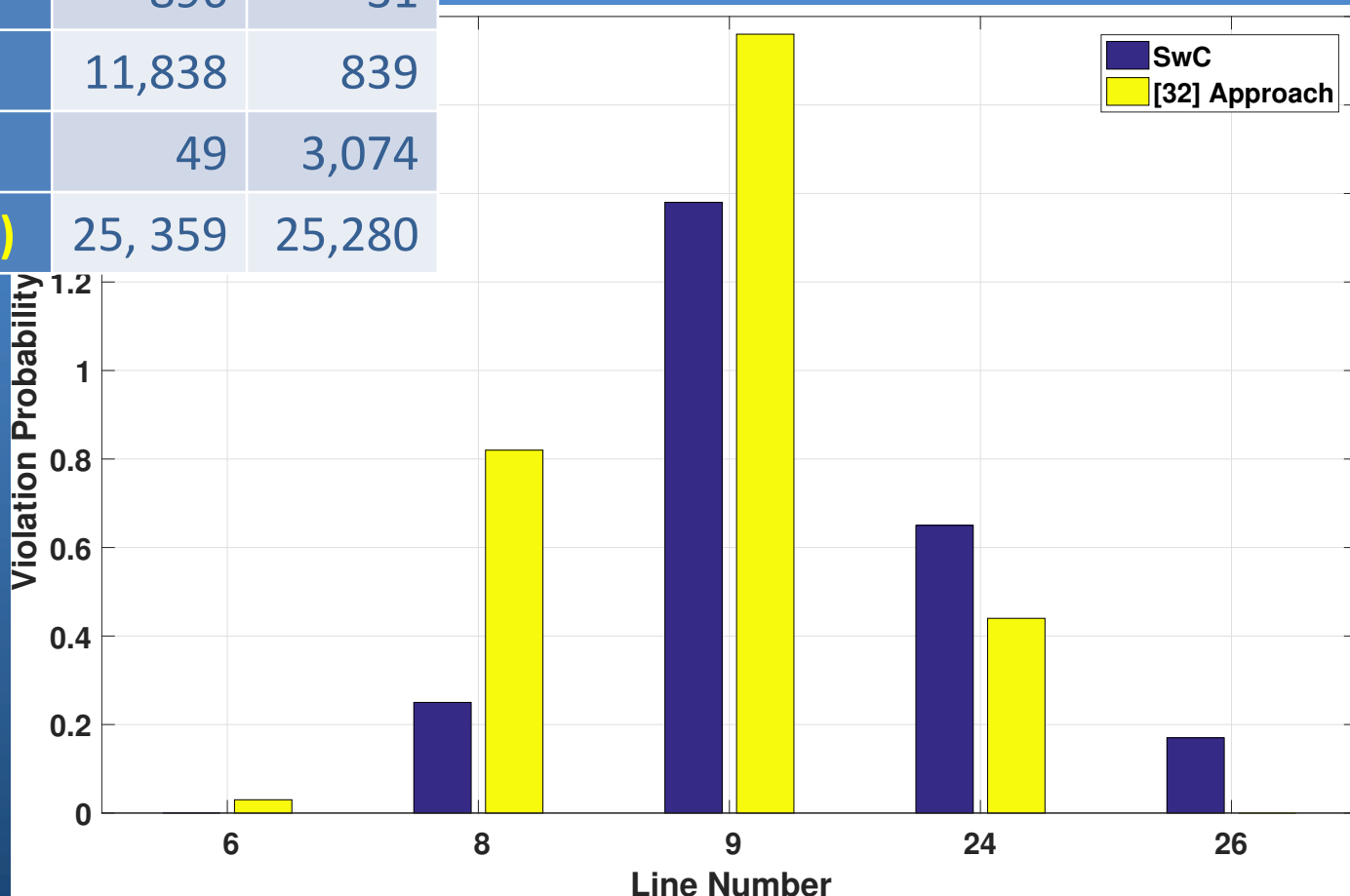


Results: Robust OPF

IEEE 39-bus – Comparison with [ETH]

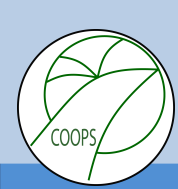


	[ETH]	SWC
Design Variables	896	31
N (samples)	11,838	839
Time (min)	49	3,074
Generation cost (\$)	25,359	25,280



6.

**Conclusions and Future
Directions**



Conclusions

The combination of Convex Relaxations and SwC provides a new useful tool

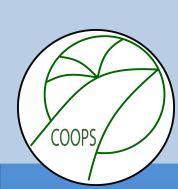
The approach is mildly scalable...

New opportunities using new technologies

*"**Distributed** renewable energy systems are emerging as the **least expensive** and fastest option for providing energy [...] Enabling technologies can help to accommodate higher shares of VRE by contributing to **more flexible** and **integrated** system*

[Global status report, 2018]





Future Directions

New paradigm: Distributed solutions

(local computation + coordination)

- to reduce complexity
- to reduce transmissions and delays
- to improve resilience
- to guarantee privacy



Distributed scenario with certificates optimization

- each node has
 - its own design variables and its own **uncertainty**
- certificates need to be shared between adjacent nodes



Constantino
Lagoa



Mohammadreza
Chamanbaz



Mahmoud
Ashour



Chiara
Ravazzi



Luca
Zaccarian



Simone
Formentin



THANK YOR FOR YOUR ATTENTION