Formulas for Data-Driven Control

Stability, Optimality and Robustness

Claudio De Persis

Institute of Engineering and Technology
J.C. Willems Center for Systems and Control

Workshop – Resilient Control of Infrastructure Networks
Department of Mathematical Sciences, Politecnico di Torino,
24-27 September 2019

Joint work with Pietro Tesi (Università di Firenze)

What is this talk about?

...and how it is related to the topic of the workshop **Resilient Control of Infrastructure Networks**?

- Infrastructure networks may have models too difficult to derive from first principles or too complex to work with for design purposes
- Complex networks generate large amount of data

Can we trade off the knowledge of the system's dynamics against experimental data and be able to control the system?

This talk introduces a new approach that enables this transition

Outline

- Closed-loop data-based representations of systems
- Stabilization
- Linear quadratic regulation
- Robustness to noise and disturbances

What is control?

If physics is the science of understanding the physical environment, then **control theory** *may be viewed as* **the science of modifying that environment** *[...] Control theory does not deal directly with physical reality but with* **mathematical models***.*

Rudolf Kalman, Control Theory, *Encyclopædia Britannica*

Mathematical models

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)) \end{aligned} \qquad \begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \quad k = 0, 1, 2, \ldots \end{aligned}$$

Control when the dynamics is unknown

When (A, B) are unknown, one can follow 2 distinct approaches

- **System identification** from data **+ control** of the identified system
  - A. Chiuso and G. Pillonetto. "System identification: A machine learning perspective". Annual Review of Control, Robotics, and Autonomous Systems, 2:281-304, 2019.
  - B. Recht. "A tour of reinforcement learning: The view from continuous control". Annual Review of Control, Robotics, and Autonomous Systems, 2:253-279, 2019.
- **Direct** data-based control design (no identification)
  - M.C. Campi, A. Lecchini, and S.M. Savaresi. "Virtual reference feedback tuning: a direct method for the design of feedback controllers". Automatica, 38(8):1337-1346, 2002.
  - C. Novara, L. Fagiano, and M. Milanese. "Direct feedback control design for nonlinear systems". Automatica, 49(4):849-860, 2013.

Persistence of excitation

In the sequel we propose a new approach, which seeks a data-based representation of the unknown closed-loop dynamics enabled by **persistently exciting** input probing signals

Definition The sequence of input values $u : [0, T-1] \to \mathbb{R}^m$

$$u(0), u(1), \ldots, u(T-1)$$

is persistently exciting (PE) of order L if the Hankel matrix associated to it

$$U_{0,L,T-L+1} = \begin{bmatrix} u(0) & u(1) & \ldots & u(T-L) \\ u(1) & u(2) & \ldots & u(T-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ u(L-1) & u(L) & \ldots & u(T-1) \end{bmatrix}$$

has full rank mL.

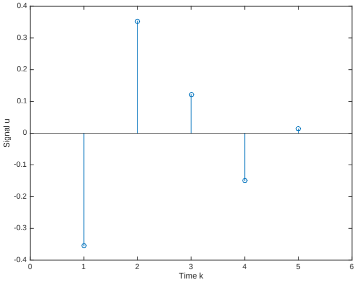
PE requires sufficiently long probing input sequences: $T \geq (m+1)L - 1$

How to generate PE signals

```
aux=zeros(m,T);
aux(:)=0.5;
ud(1:m,1:T)=rand(m,T)-aux;

% Computing the Hankel matrix

for j=1:T-L+1
    for i=1:L
        Ud((i-1)*m+1:(i-1)*m+m,j)=ud(1:m, j+i-1);
    end
end
```



$L = 3, n = 2, m = 1 \Rightarrow T = 5$

$u_{[0,T-1]} = [-0.355\ 0.353\ 0.1221\ -0.149\ 0.0132]$

$$U_{0,L,T-L+1} = \begin{bmatrix} -0.3550 & 0.3530 & 0.1221 \\ 0.3530 & 0.1221 & -0.1490 \\ 0.1221 & -0.1490 & 0.0132 \end{bmatrix}$$

M. Verhaegen and V. Verdult. Filtering and system identification: a least squares approach. Cambridge University Press, 2007.

The Fundamental Lemma

A PE input applied to a controllable system produces data that are sufficiently independent over time

Lemma Let system

$$x(k+1) = Ax(k) + Bu(k)$$

be controllable. Then for any $t \geq 1$

$$u_{[0,T-1]} \text{ PE of order } n+t \quad \Rightarrow \quad \text{rank} \begin{bmatrix} U_{0,t,T-t+1} \\ X_{0,T-t+1} \end{bmatrix} = n + tm$$

$$U_{0,t,T-t+1} = \begin{bmatrix} u(0) & u(1) & \dots & u(T-t) \\ u(1) & u(2) & \dots & u(T-t+1) \\ u(2) & u(3) & \dots & u(T-t+2) \\ \vdots & & & \\ u(t-1) & u(t) & \dots & u(T-1) \end{bmatrix}$$

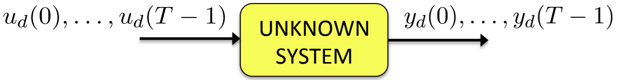
$$X_{0,T-t+1} = \begin{bmatrix} x(0) & x(1) & \dots & x(T-t) \end{bmatrix}$$

J.C. Willems, P. Rapisarda, I. Markovsky, B.L. De Moor. "A note on persistency of excitation." Systems & Control Letters, 54, 4, 325–329, 2005.

Example

Unknown system – linearized predator-prey model $(n = 2,\ m = 1,\ t = 1)$

$$x(k+1) = \begin{bmatrix} 1 & -\dfrac{ad}{b} \\ \dfrac{b\bar{u}}{a} & 1 \end{bmatrix} x(k) + \begin{bmatrix} \dfrac{d}{b} \\ 0 \end{bmatrix} u(k), \quad y(k) = x(k)$$



$$\begin{bmatrix} -0.3550 & 0.3530 & 0.1221 & -0.1490 & 0.0132 \end{bmatrix} \qquad \begin{bmatrix} 0.4027 & 0.3478 & 0.3571 & 0.3216 & 0.2362 \\ 0.4448 & 1.1451 & 1.7499 & 2.3708 & 2.9301 \end{bmatrix}$$

The matrix

$$\begin{bmatrix} U_{0,t,T-t+1} \\ \hline X_{0,T-t+1} \end{bmatrix} = \begin{bmatrix} U_{0,1,5} \\ \hline X_{0,5} \end{bmatrix} = \left[\begin{array}{ccccc} -0.3550 & 0.3530 & 0.1221 & -0.1490 & 0.0132 \\ \hline 0.4027 & 0.3478 & 0.3571 & 0.3216 & 0.2362 \\ 0.4448 & 1.1451 & 1.7499 & 2.3708 & 2.9301 \end{array}\right]$$

has rank $n + tm = 3$

Deep implications for control

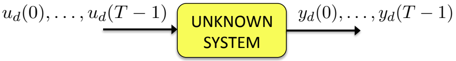
Lemma (i) If $u_{d,[0,T-1]}$ is persistently exciting of order $n+t$, then any t-long input/output trajectory of the system can be expressed as

$$\begin{bmatrix} u_{[0,t-1]} \\ y_{[0,t-1]} \end{bmatrix} = \begin{bmatrix} U_{0,t,T-t+1} \\ Y_{0,t,T-t+1} \end{bmatrix} g$$

where $g \in \mathbb{R}^{T-t+1}$.
(ii) Any linear combination of the columns of the matrix of data, i.e.,

$$\left[\begin{array}{c} U_{0,t,T-t+1} \\ \hline Y_{0,t,T-t+1} \end{array}\right] g,$$

is a t-long input-output trajectory of the system.



$$\left[\begin{array}{c} U_0 \\ \hline Y_0 \end{array}\right] = \left[\begin{array}{ccc} u_d(0) & \ldots & u_d(T-t) \\ \vdots & \ddots & \vdots \\ u_d(t-1) & \ldots & u_d(T-1) \\ \hline y_d(0) & \ldots & y_d(T-t) \\ \vdots & \ddots & \vdots \\ y_d(t-1) & \ldots & y_d(T-1) \end{array}\right]$$

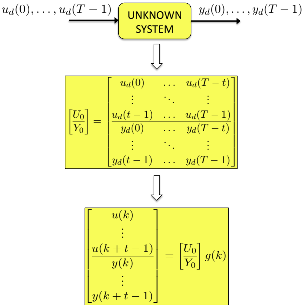
$$\left[\begin{array}{c} u(k) \\ \vdots \\ u(k+t-1) \\ \hline y(k) \\ \vdots \\ y(k+t-1) \end{array}\right] = \left[\begin{array}{c} U_0 \\ \hline Y_0 \end{array}\right] g(k)$$

I. Markovsky and P. Rapisarda. "Data-driven simulation and control". *International Journal of Control*, 81(12), 1946–1959, 2008.

DeePC - Data enabled Predictive Control

$$\max_{g(k),w(k)} \quad \sum_{\ell=0}^{t-1} \left(\|y(k+\ell)\|_Q^2 + \|u(k+\ell)\|_R^2\right) + \|w(k)\|^2$$

$$\text{s.t.} \quad \begin{bmatrix} u_{[k,k+t-1]} \\ y_{[k,k+t-1]} \end{bmatrix} = \begin{bmatrix} U_{0,t,T-t+1} \\ Y_{0,t,T-t+1} \end{bmatrix} g(k) + w(k)$$



De Pintelier Café September 2018

J. Coulson, J. Lygeros, F. Dörfler. "Data-Enabled Predictive Control: In the Shallows of the DeePC." *European Control Conference*, 2019.

Direct data-based control design: stability

Closed-loop data-based representation

Now we introduce a closed-loop data-based representation that enables the design of controllers without the intermediate step of estimating the model.

Why should we care about this different solution?

Because

- Sometimes system identification is difficult:
  - complex dynamics
  - noisy data (without statistics)
  - finite number of samples
- We skip one step (save computation)
- It is intellectually stimulating
- It is a systematic approach
- It leads to clean analytic formulas

Closed-loop data-based representation

Arrange the closed loop system as

$$A + BK = \begin{bmatrix} B & A \end{bmatrix} \begin{bmatrix} K \\ I \end{bmatrix}$$

By the Fundamental Lemma and Rouché-Capelli Theorem

$$\begin{bmatrix} K \\ I \end{bmatrix} = \begin{bmatrix} U_{0,1,T} \\ X_{0,T} \end{bmatrix} G_K, \quad \text{for some } T \times n \text{ matrix } G_K$$

Hence

$$A + BK = \begin{bmatrix} B & A \end{bmatrix} \begin{bmatrix} U_{0,1,T} \\ X_{0,T} \end{bmatrix} G_K = X_{1,T} G_K$$

having set

$$\begin{aligned} AX_{0,T} + BU_{0,1,T} = &\ A \begin{bmatrix} x(0) & x(1) & \dots & x(T-1) \end{bmatrix} + B \begin{bmatrix} u(0) & u(1) & \dots & u(T-1) \end{bmatrix} \\ = &\ \begin{bmatrix} x(1) & x(2) & \dots & x(T) \end{bmatrix} \\ =: &\ X_{1,T} \end{aligned}$$

Data-based parametrization of the closed-loop

Theorem System

$$x(k+1) = Ax(k) + Bu(k)$$

in closed-loop with a state feedback $u = Kx$ has the following equivalent representation

$$x(k+1) = X_{1,T} G_K x(k)$$

where G_K is a $T \times n$ matrix satisfying

$$\begin{bmatrix} K \\ I_n \end{bmatrix} = \begin{bmatrix} U_{0,1,T} \\ X_{0,T} \end{bmatrix} G_K$$

Shift design from K to G_K – then $K = U_{0,1,T} G_K$

C. De Persis, P. Tesi. "Formulas for data-driven control: stability, optimality, robustness". arXiv:1903.06842 , 15 March 2019.

Direct data-based stabilization

Problem Find G_K such that the closed-loop system

$$x(k+1) = X_{1,T} G_K x(k)$$

is asymptotically stable

A necessary and sufficient condition is given by the Lyapunov inequality

$$\begin{aligned} P &\succ 0 \\ X_{1,T} G_K \cdot P \cdot G_K^\top X_{1,T}^\top - P &\prec 0 \end{aligned}$$

with

$$\begin{bmatrix} K \\ I_n \end{bmatrix} = \begin{bmatrix} U_{0,1,T} \\ X_{0,T} \end{bmatrix} G_K$$

Direct data-based stabilization

- The variable transformation

$$Y = G_K P$$

and Schur's complement reduces it to the data-based LMI

$$\begin{bmatrix} X_{0,T} Y & Y^\top X_{1,T}^\top \\ X_{1,T} Y & X_{0,T} Y \end{bmatrix} \succ 0$$

with

$$\begin{bmatrix} K \\ P \end{bmatrix} = \begin{bmatrix} U_{0,1,T} G_K \\ X_{0,T} Y \end{bmatrix}$$

- The solution to the LMI returns Y. The control gain is obtained via

$$\begin{array}{rl} K = & U_{0,1,T} G_K \\ Y = & G_K P \\ P = & X_{0,T} Y \end{array} \quad \Rightarrow \quad K = U_{0,1,T} Y (X_{0,T} Y)^{-1}$$

Direct data-based stabilization

Theorem Any matrix Y satisfying

$$\begin{bmatrix} X_{0,T}\,Y & X_{1,T}\,Y \\ Y^\top X_{1,T}^\top & X_{0,T}\,Y \end{bmatrix} \succ 0$$

is such that

$$K = U_{0,1,T}\,Y(X_{0,T}\,Y)^{-1}$$

is a stabilizing state-feedback gain for system

$$x(k+1) = Ax(k) + Bu(k)$$

Converse result if K is a stabilizing state-feedback gain for the system, then it can be written as $K = U_{0,1,T}\,Y(X_{0,T}\,Y)^{-1}$

C. De Persis, P. Tesi. "Formulas for data-driven control: stability, optimality, robustness". arXiv:1903.06842, 15 March 2019.

Example

Data-based stabilization of the linearized predator-prey model

State response to PE input from experiment

$$X_{0,5} = \begin{bmatrix} 0.4027 & 0.3478 & 0.3571 & 0.3216 & 0.2362 \\ 0.4448 & 1.1451 & 1.7499 & 2.3708 & 2.9301 \end{bmatrix}$$

$$X_{1,5} = \begin{bmatrix} 0.3478 & 0.3571 & 0.3216 & 0.2362 & 0.1541 \\ 1.1451 & 1.7499 & 2.3708 & 2.9301 & 3.3409 \end{bmatrix}$$

Solve for Y

```
cvx_begin sdp
variable Y(T,n)
[X0T*Y X1T*Y; Y'*X1T' X0T*Y]>=eye(2*n);
cvx_end
```

which returns

$$Y = \begin{bmatrix} 27.4724 & -20.8515 \\ -25.5235 & -8.8555 \\ -1.6399 & -2.0356 \\ 5.3938 & 3.6399 \\ 0.1696 & 18.8019 \end{bmatrix}$$

S. Boyd. "Solving semidefinite programs using cvx," http://stanford.edu/class/ee363/notes/lmi-cvx.pdf

Example

Feedback gain

$$K = U_{0,1,5} Y (X_{0,5} Y)^{-1} = \begin{bmatrix} -8.2995 & -1.2512 \end{bmatrix}$$

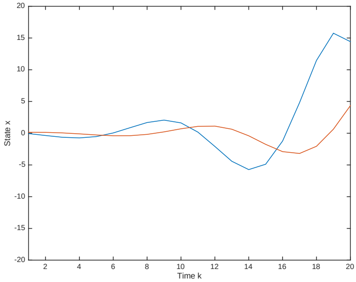


Figure: Unforced solution $u(k) = 0$

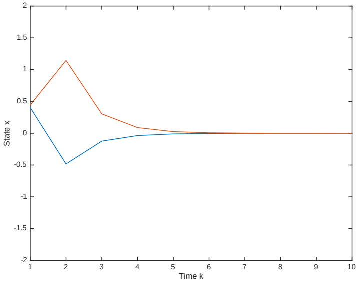


Figure: Solution under data-based feedback $u(k) = Kx(k)$

Spectral radius data-based controlled system $\rho(A + BK) = 0.5666$

Discussion I

- Simple solution: data-dependent Lyapunov stability theory
- The data-based problem is solvable via efficient numerical algorithms (cvx)
- It only requires a finite number of data collected in one-shot low sample-complexity experiments ($T \geq (m+1)(n+1)-1$)
- A dynamic output feedback control can be designed from data obtained with a PE input of order $(2n+1)$
- There is no attempt to estimate A, B from data. Data are only used to represent the gain K.

Discussion II

- Variations of the arguments used to prove the main result show that the result holds even using data not obtained from PE data. Recall our main result

$$\begin{bmatrix} X_{0,T}Y & X_{1,T}Y \\ Y^\top X_{1,T}^\top & X_{0,T}Y \end{bmatrix} \succ 0 \quad K = U_{0,1,T}Y(X_{0,T}Y)^{-1} \quad P = X_{0,T}Y$$

By Schur's complement, the LMI is rewritten as

$$X_{1,T}Y(X_{0,T}Y)^{-1} \cdot P \cdot (X_{0,T}Y)^{-1}(X_{1,T}Y)^\top - P \succ 0$$

which shows that

$$X_{1,T}Y(X_{0,T}Y)^{-1}$$

is stable. Then

$$\begin{aligned} X_{1,T}Y(X_{0,T}Y)^{-1} = & \begin{bmatrix} B & A \end{bmatrix} \begin{bmatrix} U_{0,1,T} \\ X_{0,T} \end{bmatrix} Y(X_{0,T}Y)^{-1} \\ = & \begin{bmatrix} B & A \end{bmatrix} \begin{bmatrix} K \\ I_n \end{bmatrix} \\ = & A + BK \end{aligned}$$

shows that $K = U_{0,1,T}Y(X_{0,T}Y)^{-1}$ is stabilizing

H. van Waarde, J. Eising, H. Trentelman, K. Camlibel. "Data informativity: a new perspective on data-driven analysis and control". arXiv:1908.00468, 1 August 2019.

Optimality

Optimality - Linear Quadratic Regulation

LQR problem Design $u(0), u(1), u(2), \ldots$ that minimizes

$$J_\infty(x_0, u) := \sum_{k=0}^{\infty} (x(k) Q x(k) + u(k)^\top R u(k)), \quad Q \succeq 0, R \succ 0$$

for the system $x(k+1) = Ax(k) + Bu(k)$, $x(0) = x_0$

The solution is given by

$$u_\star := K_\star x, \quad K_\star := -(R + B^\top P B)^{-1} B^\top P A$$

with P the stabilizing solution of the DARE

$$A^\top P A - P - A^\top P B (R + B^\top P B)^{-1} B^\top P A + Q = 0$$

Data-based solution to LQR

A reformulation of LQR as an optimization problem

$$\min_{K,P,X}\ \mathrm{trace}\,(QP) + \mathrm{trace}\,(X)$$
$$\text{subject to}$$
$$\begin{cases} (A+BK)P(A+BK)^\top - P + I_n \preceq 0 \\ P \succeq I_n \\ X - R^{1/2}KPK^\top R^{1/2} \succeq 0 \end{cases}$$

E. Feron, V. Balakrishnan, S. Boyd, L. El Ghaoui, "Numerical methods for H_2 related problems," in 1992 American Control Conference, pp. 2921–2922.

Data-based solution to LQR

A similar transformation as before leads to the semidefinite program

$$
\begin{array}{l}
\min_{Y,X} \ \mathrm{trace}\,(QX_{0,T}Y) + \mathrm{trace}\,(X) \\
\text{subject to} \\
\left\{
\begin{array}{l}
\begin{bmatrix} X & R^{1/2}U_{0,1,T}Y \\ Y^\top U_{0,1,T}^\top R^{1/2} & X_0 Y \end{bmatrix} \succeq 0 \\[2ex]
\begin{bmatrix} X_{0,T}Y - I_n & X_{1,T}Y \\ Y^\top X_{1,T}^\top & X_{0,T}Y \end{bmatrix} \succeq 0
\end{array}
\right.
\end{array}
$$

The resulting optimal gain matrix is given by

$$K_\star = U_{0,1,T}Y(X_{0,T}Y)^{-1}$$

which coincides with the DARE-based solution

$$K_\star = -(R + B^\top PB)^{-1}B^\top PA$$

C. De Persis, P. Tesi. “Formulas for data-driven control: stability, optimality, robustness”. arXiv:1903.06842, 15 March 2019.

Discussion

- The data-based problem is solvable via efficient numerical algorithms (cvx)

```
cvx_begin sdp
    variable Q(T,n)
    variable X(m,m) symmetric
    minimize ( trace(Q*X0*Y) +trace(X) )
    [X, sqrtm(R)*U0*Y; Y'*U0'*sqrtm(R)', X0*Y] >= 0
    [X0*Y-eye(n), X1*Y; Y'*X1', X0*Y] >= 0
cvx_end

K = U0*Y*(inv(X0*Y));
```

- It only requires data collected in one-shot low sample-complexity experiments
- Solution is exactly computed via a single SDP and not approximated via sequential iterations as in, e.g., Q-learning applied to LQR

Q-learning and LQR

Algorithm 1 The Q-learning algorithm applied to the LQR problem

```
Guess initial stabilizing gain K_0
Set initial time k = 0
for i = 0 to ∞ do
    for j = 1 to N do
        Apply u(k) = K_i x(k) + e(k), e(k) PE "exploration signal"
        Estimate K_i(j) using RLS and I/O measurements
        k = k + 1
    end for
    Set K_{i+1} = K_i(N)
end for
```

There exists an estimation interval N such that the algorithm generates a sequence $\{K_i : i = 0, 1, 2, \ldots\}$ such that $\lim_{i \to \infty} \|K_i - K_\star\| = 0$

S.J. Bradtke, B.E. Ydstie and A.G. Barto. Adaptive linear quadratic control using policy iteration. Proceedings of the 1994 American Control Conference, 3475–3479, 1994.
J.C.H. Watkins and P. Dayan. Q-learning. Machine learning, 8(3-4):279–292, 1992.

Robustness

Noisy measurements

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
\zeta(k) &= x(k) + w(k) \qquad k = 0, 1, 2, \ldots
\end{aligned}
$$

where w is an unknown measurement noise

Experiment

- Consider a PE input $u_{[0,T-1]}$ of order $n+t$ with $t=1$
- Apply it to the system and collect the **measured** (hence, **noisy**) state response in the $n \times T$ matrix

$$
Z_{0,T} = X_{0,T} + W_{0,T}
$$

where

$$
\begin{aligned}
X_{0,T} &= \begin{bmatrix} x(0) & x(1) & \ldots & x(T-1) \end{bmatrix} \\
W_{0,T} &= \begin{bmatrix} w(0) & w(1) & \ldots & w(T-1) \end{bmatrix}
\end{aligned}
$$

Data-based representation with noisy measurements

As before, by the Fundamental Lemma

$$\begin{bmatrix} K \\ I \end{bmatrix} = \begin{bmatrix} U_{0,1,T} \\ Z_{0,T} \end{bmatrix} G_K, \quad \text{for some } G_K$$

Hence

$$A + BK = \begin{bmatrix} B & A \end{bmatrix} \begin{bmatrix} U_{0,1,T} \\ Z_{0,T} \end{bmatrix} G_K = (\underbrace{Z_{1,T}}_{\textit{known}} + \underbrace{R_{0,T}}_{\textit{uncertainty}})G_K$$

having set

$$\begin{aligned} AZ_{0,T} + BU_{0,1,T} &= AX_{0,T} + AW_{0,T} + BU_{0,1,T} \\ &= X_{1,T} + AW_{0,T} \\ &= Z_{1,T} - W_{1,T} + AW_{0,T} \\ &=: Z_{1,T} + R_{0,T} \end{aligned}$$

Robust stabilization with noisy measurements

Theorem Let

$$R_{0,T} R_{0,T}^\top \preceq \gamma Z_{1,T} Z_{1,T}^\top$$

"noise-to-signal ratio"

for some $\gamma > 0$.

Any matrix Y and scalar $\alpha > 0$ satisfying $\gamma < \alpha^2/(4 + 2\alpha)$ and

$$\begin{bmatrix} Z_{0,T}\,Y - \alpha Z_{1,T} Z_{1,T}^\top & Z_{1,T}\,Y \\ Y^\top Z_{1,T}^\top & Z_{0,T}\,Y \end{bmatrix} \succeq 0 \quad \begin{bmatrix} I_T & Y \\ Y^\top & Z_{0,T}\,Y \end{bmatrix} \succeq 0$$

is such that

$$K = U_{0,1,T}\,Y (X_{0,T}\,Y)^{-1}$$

is a stabilizing state-feedback gain for system $x(k+1) = Ax(k) + Bu(k)$.

- In practice, search for the feasible solution maximizing α
- Same results applicable to process disturbances

$$x(k+1) = Ax(k) + Bu(k) + d(k)$$

where $d(k)$ can model, e.g., neglected nonlinearities $\Rightarrow$ stabilization in the first approximation of nonlinear systems

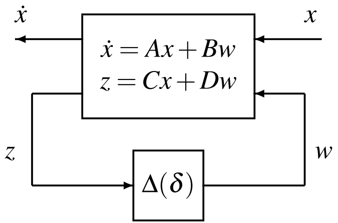
C. De Persis, P. Tesi. "Formulas for data-driven control: stability, optimality, robustness". arXiv:1903.06842, 15 March 2019.

Data-based representation and robust control

An LFT representation

$$x^+ = (Z_{1,T} + R_{0,T})G_K\, x$$
$$\Updownarrow$$
$$\begin{bmatrix} x^+ \\ z \end{bmatrix} = \begin{bmatrix} Z_{1,T}G_K & I \\ G_K & \mathbb{0} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}$$
$$w = R_{0,T}\, z$$

Linear fractional representation



Concrete full-block S-procedure [Scherer-Weiland – Th. 6.8] If there exists a symmetric multiplier M such that

$$\begin{bmatrix} R_{0,T} \\ I \end{bmatrix}^\top \overbrace{\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix}}^{M} \begin{bmatrix} R_{0,T} \\ I \end{bmatrix} \succeq 0$$

and

$$\begin{bmatrix} \star & \star & \star & \star \\ \star & \star & \star & \star \end{bmatrix} \begin{bmatrix} \mathbb{0} & P & \mathbb{0} & \mathbb{0} \\ P & \mathbb{0} & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & Q & S \\ \mathbb{0} & \mathbb{0} & S^\top & Q \end{bmatrix} \begin{bmatrix} I & \mathbb{0} \\ Z_{1,T}G_K & I \\ \mathbb{0} & I \\ G_K & \mathbb{0} \end{bmatrix} \prec 0$$

then a robust stabilizer can be designed.

J. Berberich, A. Romer, C.W. Scherer, F. Allgöwer. "Robust data-driven state-feedback design". arXiv:1909.04314, 10 Sep 2019.

Conclusions

A systematic method for the direct design of data-driven control policies for linear systems

- Stabilization, LQR, output feedback, MIMO systems
- One-shot experiment of duration $n+1$ (or $2n+1$ – output feedback)
- Robustness to noise
- Stabilization in first approximation of nonlinear systems

Formulas for Data-driven Control: Stabilization, Optimality and Robustness

C. De Persis and P. Tesi

***Abstract*—In a paper by Willems and coauthors it was shown that persistently exciting data can be used to represent the input-output behavior of a linear system. Based on this fundamental result, we derive a parametrization of linear feedback systems that paves the way to solve important control problems using**

control theory [6], iterative feedback tuning [7], and virtual reference feedback tuning [8]. This topic is now attracting more and more researchers, with problems ranging from PID-like control [9] to model reference control and output tracking

arXiv:1802.08457 – 15 March 2019

Outlook

- LMIs and SDPs are ubiquitous in control – this approach can be used to deal with problems using data to replace models. Start from, e.g., Scherer-Weiland's textbook (LFT, IQC, LPV) and expand.
- Complex dynamics Nonlinear dynamics uplifted to higher dimension systems using observable functions give a more accurate representation (Claude-Fliess-Isidori's immersion, Carleman linearization, polyflow approximation, Koopman operator...). Thus using more data enables non-regional data-based control of nonlinear systems.
- Fundamental lemma for nonlinear systems Input-output relation based on truncated Fliess's fundamental formula (nonlinear realization theory - Isidori, Chapter 3). This I/O relation can be organized in the form of a Hankel matrix whose entries depend on experimental data. Major difference: Lie rank vs. Hankel rank.
- Large-scale systems Experiments in open-loop for unstable large-scale systems is unfeasible. Either experiments in closed-loop + dither or design local experiments for global results.

Thank you!