Matching and Mechanisms as Potential Approaches for Control of Infrastructure

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Robotics, Intelligent Systems & Control (RISC)

Resilient Control of Infrastructure Networks
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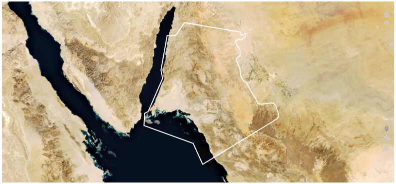
Workshop theme: Infrastructure

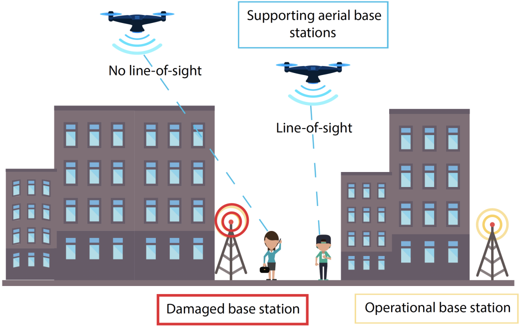


- *Robustness:* Guaranteed level of performance over range of operating conditions.

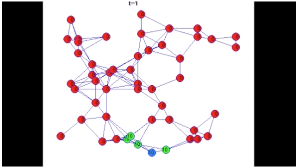
- *Reconfiguration:* Eventual (partial) restoration of performance under disruptions.

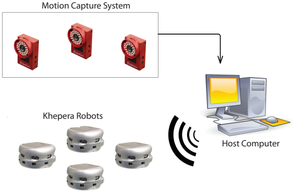
- $X \in$ {Organizing, Healing, Configuring, Optimizing, Adapting, ...}

- *Features:*
  - Dynamic environment
  - Reactive components
  - Distributed architecture



El Hammouti et al., "Learn-As-You-Fly: A Distributed Algorithm for Joint 3D Placement and User Association in Multi-UAVs Networks", preprint.





Features:

- Local information & interaction
- Perpetual reaction

Marden & JSS, "Revisiting log-linear-learning", *Games and Economic Behavior*, 2012.
Lim & JSS, "Robustness of stochastic stability in game theoretic learning", *ACC*, 2013.
Yazicioglu, Egerstedt, & JSS, "Communication-free distributed coverage for networked systems", *IEEE TCNS*, 2017.

Theorem

If agents execute *Distributed Algorithm* then *Desirable Outcome*.

- *Algorithm/Outcome:*
  - Binary Log-Linear-Learning / Perpetual ϵ-Optimal Coverage
  - Gossip algorithm / Consensus
  - Distributed gradient descent / Centralized optimum
  - etc.

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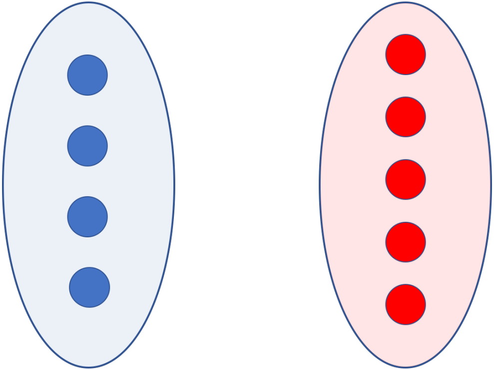
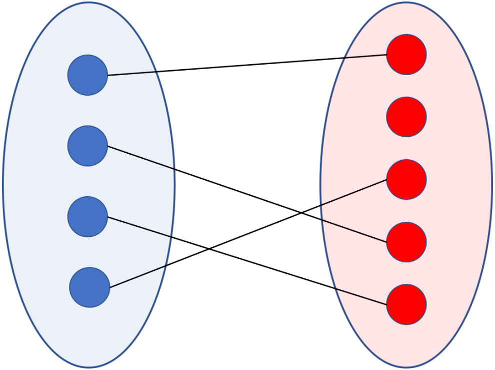
- *Issue:* Programmable components vs Strategic decision makers
- *Outline:*
  - What can go right? (matching)
  - What can go wrong?
  - What to do? (mechanisms)

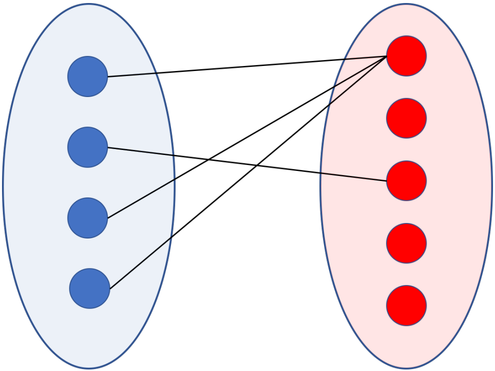


Two Groups

Workers & Firms, Tasks & Processors, Donors & Recipients, Marriage...



One-to-One Matching



Many-to-One Matching

- *Two groups:*
  - $\mathcal{K} = \{1, 2, ..., K\}$
  - $\mathcal{L} = \{1, 2, ..., L\}$

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- *Orderings:* $\succeq_k$ & $\succeq_\ell$
    - $\ell \succeq_k \ell' \iff$ Agent k prefers ℓ over ℓ'
    - $k \succeq_\ell k' \iff$ Agent ℓ prefers k over k'

- *Firms & Workers:*
  - $\overline{S}_{k\ell}$: Maximum salary firm k is willing to pay to worker ℓ
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- *Utility:*
  - Suppose $\{k, \ell\}$ are matched at salary, s:

$$\underline{S}_{k\ell} \leq s \leq \overline{S}_{k\ell}$$

  - Firm: $\overline{S}_{k\ell} - s$
  - Worker: $s - \underline{S}_{k\ell}$

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$$\left(\overline{S}_{k\ell} - s\right) + \left(s - \underline{S}_{k\ell}\right) = \overline{S}_{k\ell} - \underline{S}_{k\ell}$$

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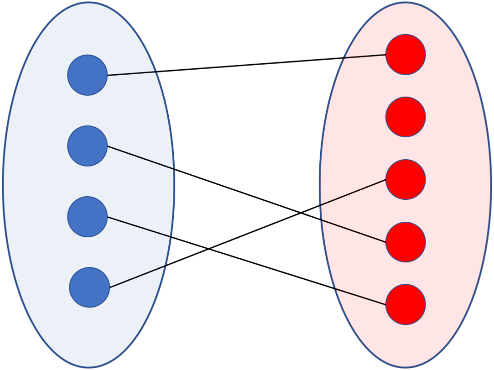
- *Generalization:* Agreement sets *(coming soon...)*



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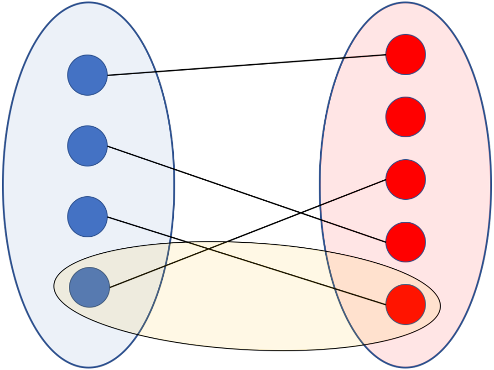
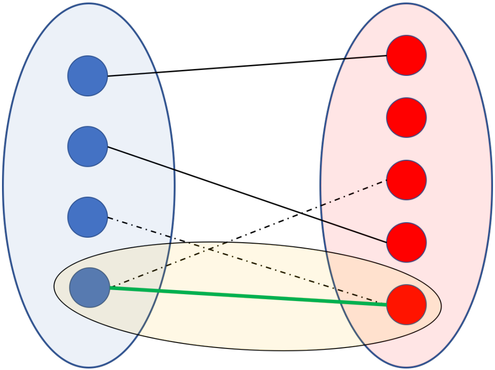
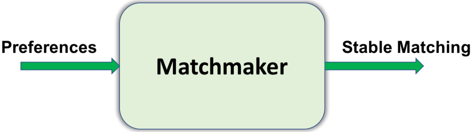


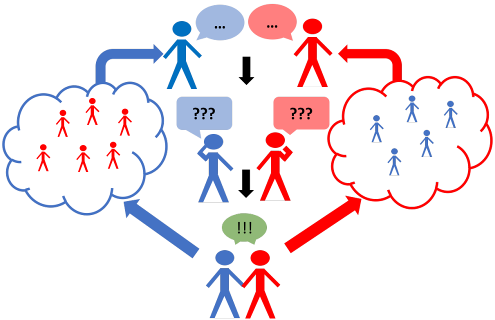
Reevaluation



Unstable?



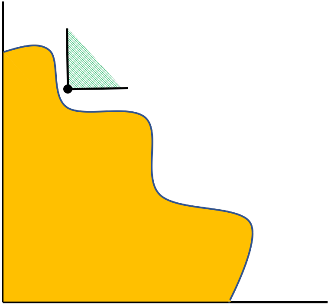
Centralized



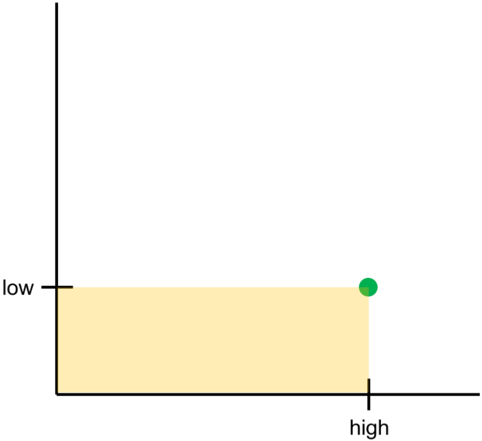
Peer-to-peer

- Binary agreement functions:

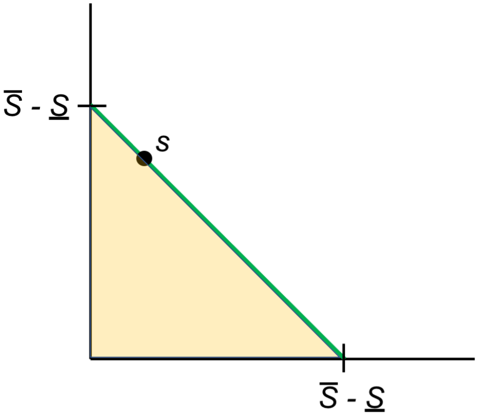
$$\mathcal{A}_{k\ell} : \mathbb{R}_+ \times \mathbb{R}_+ \to \{0, 1\}$$



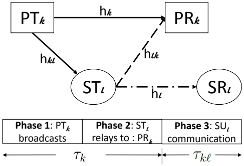
- Assumptions:
  - Monotonicity: $\mathcal{A}_{k\ell}(a, b) = 0 \Rightarrow \mathcal{A}_{k\ell}(a', b') = 0$ if $a' \geq a$ and $b' \geq b$
  - Uniform bound, γ: $\mathcal{A}_{k\ell}(a, b) = 0$ if $a \geq \gamma$ or $b \geq \gamma$



Ranking



Transferable Utility



- *Setup:*
  - Primary users own spectrum
  - Secondary users willing to offer compensation for access
  - Compensation in form of services
- *Tradeoff:*
  - P2S: Will you transmit my data at requested power?
  - S2P: Will you yield access to channel for requested time?
  - All depends on channel conditions & available power

- *Setup:*
  - Stages $t = 0, 1, 2, \ldots$
  - Agents maintain aspiration levels: $a_k(t)$ & $b_\ell(t)$
  - Agents seek to improve by at least ϵ
  - Unsatisfied single agents reduce by δ

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- *Main loop:* At stage t
  - Activate a pair $\{k, \ell\}$
  - If $\mathcal{A}_{k\ell}(a_k(t) + \epsilon, b_k(t) + \epsilon) = 1$:
    - Match with positive probability
    - Update aspirations
  - Otherwise: $\{k, \ell\}$ reduce aspirations by δ *if single*

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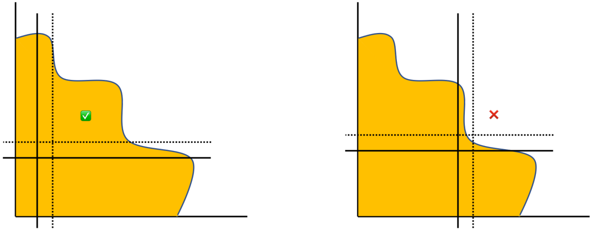

BLind Matching Algorithm (BLMA)

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  - Stages $t = 0, 1, 2, \ldots$
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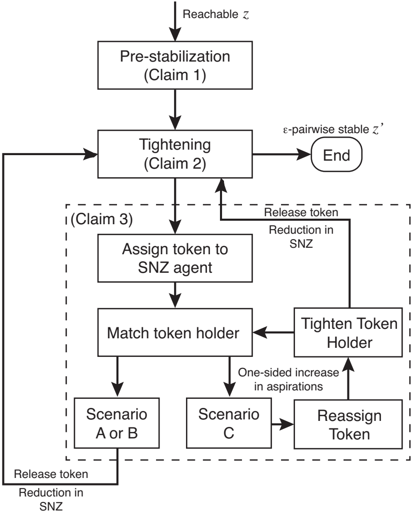
Theorem

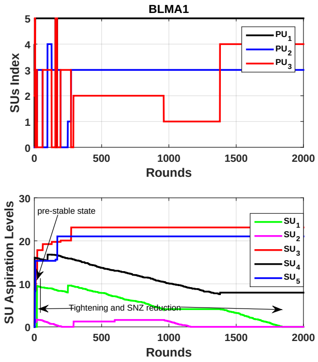
If agents execute* *BLMA*, then iterations converge to an *ϵ-stable matching*.

*for $\delta < \epsilon$

Nax & Pradelski, "Evolutionary dynamics and equitable core selection in assignment games", *Int Journal of Game Theory*, 2014.
Hamza & JSS, "BLMA: A blind matching algorithm with application to cognitive radio networks", *IEEE Journal on Selected Areas in Communications*, 2017.

- SNZ: **S**ingle with **N**on-**Z**ero aspirations
- Pre-stable: No profitable matches but SNZ non-empty
- Tight: SNZ's are δ-reduction away from profitable match
- Assume k in SNZ
    - Scenario A: ℓ in SNZ
    - Scenario B: ℓ in SZ
    - Scenario C: ℓ taken
- Pathway: Reduction in SNZ





- Implementation: Power/Time proposals
- Small number of match updates
- Long epochs of tightening

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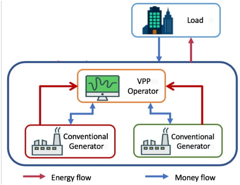
If agents execute *Distributed Algorithm* then *Desirable Outcome*.

Issue: What if agents do not faithfully execute prescribed algorithm?



Economic dispatch:

- Multiple generators to meet fixed demand: $\sum_{i=1}^{N} p_i = p_L$
- Constraints: $\underline{p}_i \leq p_i \leq \overline{p}_i$
- Cost of generation (convex): $F_i(p_i)$
- Minimum cost generation:

$$\min_{p} \sum_{i} F_i(p_i)$$

Distinguished setup:

- Generators propose payment schedules: $B_i(\cdot)$
- VPP solves minimum payout:

$$\min_{p} \sum_{i} B_i(p_i)$$

- Induces a *game* among generators.

- *Assumption:* Linear payment schedule

$$B_i(p_i) = b_i \cdot p_i$$

- *Algorithm:*
  - Initialization: $b_i(0)$ & $p_i(0)$
  - Iterations: $t = 0, 1, 2, \ldots$
    - VPP responds with proposed allocations $\big(b(\tau \leq t), p(\tau \leq t)\big) \mapsto p(t+1)$
    - Agents update $\big(b_i(\tau \leq t), p_i(\tau \leq t)\big) \mapsto b_i(t+1)$
    - Repeat for fixed number of steps, T.
  - Output: Final allocation $p(T)$

- *References:*
  - Cherukuri & Cortes (2016), "Decentralized Nash equilibrium learning by strategic generators for economic dispatch".
  - Macana & Pota (2017), "Optimal energy management system for strategic prosumer microgrids: An average bidding algorithm for prosumer aggregators".

- *"VPP responds with proposed allocations* $\big(b(\tau \leq t), p(\tau \leq t)\big) \mapsto p(t+1)$*:"*

$$p_i^+ = p_i - \gamma\Big(b_i - \frac{1}{N}\sum_{i=1}^{N} b_i\Big) \tag{M\&P}$$

Reduce allocation if price more expensive than average

- *"VPP responds with proposed allocations* $\big(b(\tau \le t), p(\tau \le t)\big) \mapsto p(t+1)$*:"*

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  Reduce allocation if price more expensive than average

- *"Agents update* $\big(b_i(\tau \le t), p_i(\tau \le t)\big) \mapsto b_i(t+1)$*:"*

$$b_i^+ = b_i + \gamma\big(p_i - \pi_i^*(b_i)\big)$$
$$\pi_i^*(b_i) = \arg\max_{q_i} b_i q_i - F_i(q_i)$$

  Increase price if allocation is greater than ideal

Cherukuri & Cortes (2016), "Decentralized Nash equilibrium learning by strategic generators for economic dispatch".
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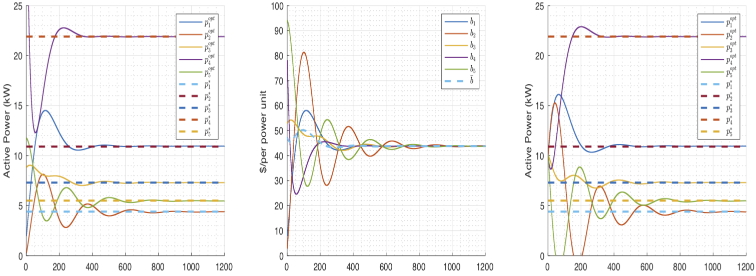


(b) Optimal power quantities $r_i^{opt}(k)$ (c) Bids $b_i(k)$ (a) PA power references $r_i(k)$

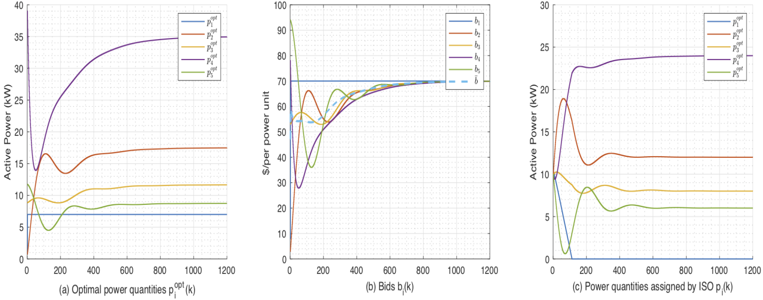
Outcome:

- Bids equalize
- Allocations converge to optimal economic dispatch

Theorem

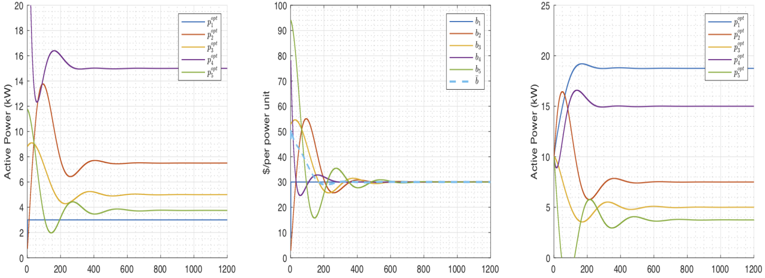
If agents execute *Distributed Algorithm* then *Desirable Outcome*.

† courtesy Eder Baron Prada



Outcome:

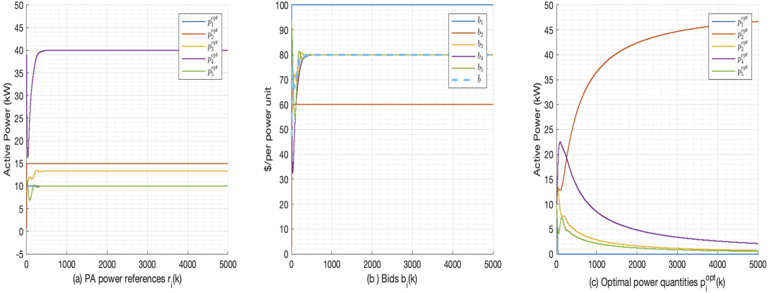
- Cheating agent maintains high price
- Sabotages VPP and self



(a) Optimal power quantities $p_i^{opt}(k)$ (b) Bids $b_i(k)$ (c) Power quantities assigned by ISO $p_i(k)$

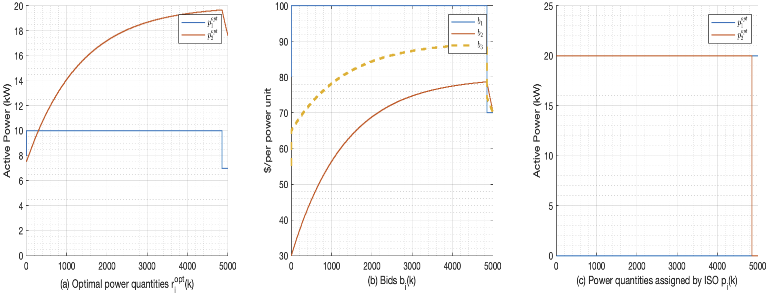
Outcome:

- Cheating agent maintains low price
- Sabotages other agents and self



Outcome:

- Colluding pair maintain high & low price
- Low price takes all



Outcome:

- Cheating agent undercuts others
- Sabotages VPP and others

Note: Uses VPP winner-take-all rule from C&C

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- *Mechanism design:*

$$\text{Private info} \stackrel{\mathcal{D}}{\Longrightarrow} \text{Social decision}$$

vs

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  - Mechanism $\mathcal{M}$ induces a game in reporting strategies.
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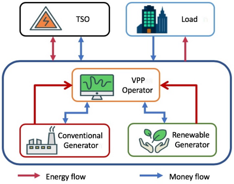
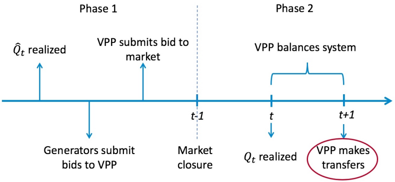
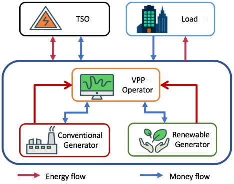
- *Literature:* Demand response, crowdsourcing, EV charging, air traffic control, ...

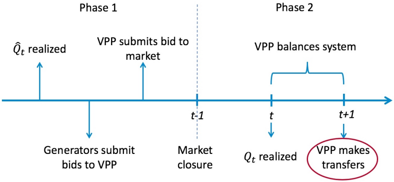
Mechanism design (static): Two-stage markets





- *Actors:* renewable generator, thermal generator, VPP





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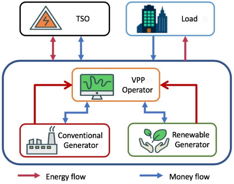
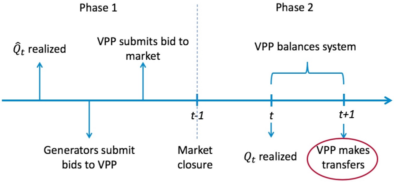
- *Decisions:*
  - Phase 1: Planning
    - RG: Energy bid to VPP
    - TG: Energy bid to VPP, up/down thermal balancing fee/rebate
    - VPP: Energy bid to ISO





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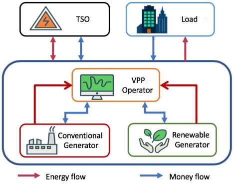
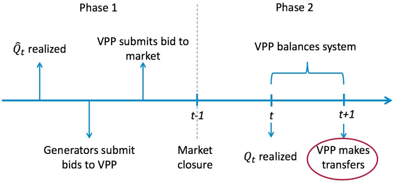
  - Phase 2: Real-time
    - VPP: Balancing between ISO vs RG





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  - Phase 2: Real-time
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- *Uncertainty (Phase 2):* Load, Renewable Energy, Balancing Prices

- *Renewable:* payment for bid + reward/penalty for imbalance

$$\max_{B^{\mathsf{R}}} \lambda^{\mathsf{Ph1}} B^{\mathsf{R}} + \mathbb{E}_{P^{\mathsf{R}}} \left[\mathcal{M}(P^{\mathsf{R}} - B^{\mathsf{R}}) \right]$$

- *Thermal:* payment for bid − production cost + balancing fee/rebate

$$\max_{B^{\mathsf{T}}, \lambda^{\mathsf{T}+}, \lambda^{\mathsf{T}-}} \lambda^{\mathsf{Ph1}} B^{\mathsf{T}} + \mathbb{E}_{\tilde{B}^{\mathsf{T}}} \left[-\pi(B^{\mathsf{T}} + \tilde{B}^{\mathsf{T}}) + \mathcal{V}(\tilde{B}^{\mathsf{T}}) \right]$$

- *VPP (cost):* payment for bids − payment from load + balancing cost

$$\mathsf{Ph1} : \min_{B^{\mathsf{AG}}} \lambda^{\mathsf{Ph1}} (B^{\mathsf{AG}} + B^{\mathsf{T}} + B^{\mathsf{R}}) + \mathbb{E}_{Q} \left[J_{\mathsf{Ph2}} \right]$$

$$\mathsf{Ph2} : \min_{\tilde{B}^{\mathsf{T}}, \tilde{B}^{\mathsf{AG}}} \lambda^{\mathsf{Ph2}} \tilde{B}^{\mathsf{AG}} + \mathcal{V}(\tilde{B}^{\mathsf{T}})$$

- *Balancing market:*
  - Phase 1 Market Price: λ^{Ph1}
  - Balancing Market Prices: λ^{Up} (buying) & λ^{Down} (selling)

$$\begin{array}{lll} \text{Load} > \text{Supply:} & \lambda^{\text{Up}} \geq \lambda^{\text{Ph1}} & \lambda^{\text{Down}} = \lambda^{\text{Ph1}} \\ \text{Load} < \text{Supply:} & \lambda^{\text{Up}} = \lambda^{\text{Ph1}} & \lambda^{\text{Down}} \leq \lambda^{\text{Ph1}} \end{array}$$

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- *Mechanism:* Penalize renewable forecast

$$\mathcal{M}(P^{\text{R}} - B^{\text{R}}) = \begin{cases} \lambda^{\text{Down}}(P^{\text{R}} - B^{\text{R}}) & \text{if } P^{\text{R}} - B^{\text{R}} > 0 \\ -\lambda^{\text{Up}}(B^{\text{R}} - P^{\text{R}}) & \text{if } B^{\text{R}} - P^{\text{R}} > 0 \end{cases}$$

Incentivizes accurate $B^{\text{R}} \approx P^{\text{R}}$

Kulmukhanova, Al-Awami, El-Amin, and JSS, "Mechanism design for virtual power plant with independent distributed generators", *IFAC Workshop on Smart Grid and Renewable Energy Systems (CGRES2019)*, June 2019.

$$\text{Private info} \quad \overset{\mathcal{D}}{\Longrightarrow} \quad \text{Social decision}$$

vs

$$\text{Private info} \quad \overset{\mathcal{S}}{\Longrightarrow} \quad \text{Reports} \quad \overset{\mathcal{M}}{\Longrightarrow} \quad \text{Social decision \& Incentives}$$

- Private info is revealed *sequentially*
- Agents do not know own types in advance
- Decisions based on sequential reports

- Two agents: $\{1, 2\}$
- Agent's need for resource is low or high: $\theta_i \in \{L, H\}$

Illustration: Sequential resource allocation

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$$v_i(a, L) = 0, \quad v_i(-i, H) = 0$$

$$v_1(1, H) = c_1, \quad v_2(2, H) = c_2$$

$$0 < c_1 < c_2$$

i.e., Agent 2 has priority

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- Uncoupled state transitions:

$$\mathbf{Pr}\left[\theta_i^+ = H \;\middle|\; a = i, \theta_i = L/H\right] = p_i$$

$$\mathbf{Pr}\left[\theta_i^+ = H \;\middle|\; a = -i, \theta_i = L\right] = p_i'$$

$$\mathbf{Pr}\left[\theta_i^+ = H \;\middle|\; a = -i, \theta_i = H\right] = 1$$

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- Coupled state transitions according to 4×4 matrices over set

$$\{(L, L), (H, L), (L, H), (H, H)\}$$

- *Agent dynamics:*

$$\theta_i^+ \sim f_i(\theta, a)$$

  - θ_i is evolving private information
  - a is action of central planner
  - **Special case:** Uncoupled dynamics

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$$\sum_{t=0}^{\infty} \delta^t v_i(a(t), \theta_i(t))$$

- *Planner objective:*

$$a = \pi(\theta)$$

e.g., **efficiency optimizer**

$$\sum_{t=0}^{\infty} \delta^t \left(\sum_{i=1}^{n} v_i(a(t), \theta_i(t)) \right)$$

- *Mechanism:* Implement $\pi(\cdot)$ and introduce transfers

$$\sum_{t=0}^{\infty} \delta^t \Big(v_i(\pi(r_1(t), ..., r_n(t)), \theta_i(t)) - q_i(r_1(t), ..., r_n(t)) \Big)$$

  - *Agent reports:* $r = (r_1, r_2, ..., r_n)$
  - *Agent strategies* map histories to reports

$$\sigma_i : (\theta_i^0, a^0, ..., \theta_i(t-1), a^{t-1}, \theta_i(t)) \mapsto r_i(t)$$

- Mechanism induces *stochastic game* among agents
- *Issue:* Is truthful reporting, $r_i(t) = \theta_i(t)$, a Nash equilibrium?

- *Presumptions at Nash equilibrium:*
  - Each agent solves POMDP
  - Optimal solution depends on environment & opponent dynamics
  - Beliefs over beliefs...
- *Anecdotal bookkeeping:* LTI systems
  - n^{th} order plant + m^{th} order opponent
  - $(n+m)^{\mathrm{th}}$ order best response
  - $(2n+m)^{\mathrm{th}}$ order best response
  - etc
- *Reformulation:* "Empirical Evidence Equilibrium"

Dudebout & JSS (2012), "Empirical evidence equilibria in stochastic games".

Theorem (LP conditions): If there exist

$$J_i : \Theta \to \mathbb{R} \quad \& \quad q_i : \Theta \to \mathbb{R}$$

such that for all i & $r, \theta \in \Theta$

$$J_i(\theta) = v_i(\pi(\theta), \theta_i) - q_i(\theta) + \delta \sum_{\theta^+ \in \Theta} \mathbf{Pr}\left[\theta^+ \mid \theta, \pi(\theta)\right] J_i(\theta^+)$$

$$J_i(\theta) \geq v_i(\pi(r_i, \theta_{-i}), \theta_i) - q_i(r_i, \theta_{-i}) + \delta \sum_{\theta^+ \in \Theta} \mathbf{Pr}\left[\theta^+ \mid \theta, \pi(r_i, \theta_{-i})\right] J_i(\theta^+)$$

then optimal response to truthful reporting is truthful reporting.

Kotsalis & JSS (2013), "Dynamic mechanism design in correlated environments".

- Agent i has partial observations:

$$\theta_i \text{ vs } \{\theta_1, ..., \theta_n\}$$

- Influences social decision through $\pi(r_i, \theta_{-i})$
- Construct stage reward so that $\pi(\theta_i, \theta_{-i})$ is optimal
- Consequence: $r_i = \theta_i$ is optimal

- Setup (independent notation):
  - Finite state space $i = 1, 2, ..., n$
  - Finite action space $u \in \mathcal{C}(i)$
  - Transition probabilities $p_{ij}(u)$
  - Stage reward $g(i, u)$
- Bellman LP:

$$J^* = \arg\min_J \mathbf{1}^{\mathrm{T}} J$$

$$\text{s/t} \quad J(i) \geq g(i, u) + \delta \sum_j p_{ij}(u) J(j), \quad \forall i, u \in \mathcal{C}(i)$$

- Current setting exploits known u_{opt} for i

- Optimal efficient policy: Favor Agent 2

$$\pi(L,L) = 1 \text{ or } 2, \quad \pi(H,L) = 1, \quad \pi(L,H) = 2, \quad \pi(H,H) = 2$$

- Agent 2 can monopolize by misreporting

- Payment rule:

$$q_1(\cdot,\cdot) = 0$$
$$q_2(L,L) < 0, \quad q_2(L,H) < 0, \quad q_2(H,L) < 0$$
$$q_2(H,H) > 0$$

- Ex ante payment from agent 2 $= 0$

- Prior work (Bergemann & Valimaki) has explicit construction for
  - *uncoupled* dynamics
  - $\pi(\cdot)$ optimizes *efficiency*
- Current (non-explicit) formulation is more general
- Both approaches "reverse engineer" stage rewards
- LP formulation allows constraints on free variables (optimal values and transfers)
- LP formulation allows *robust optimization* over coefficients (transition probabilities)

Bergemann & Valimaki (2010), "The dynamic pivot mechanism", *Econometrica*.

Theorem

If agents execute *Distributed Algorithm* then *Desirable Outcome*.

- *Recap*
  - Distributed matching & pairwise stability
  - Mechanism design: Static, Iterative, Dynamic
  - Dynamic mechanism addressed truthfulness (not obedience)
- *Mechanism design critique*
  - Presumes agents can (and will) solve dynamic optimization
  - Presumes knowledge of full system dynamics available to all
  - Tuned for behavior only at specific Nash equilibrium
  - Neglects model mismatch
  - *Alternative approach: Robust control/optimization?*
  - *Adaptive mechanisms for adaptive agents?*