A model for multilane traffic flow on simple networks

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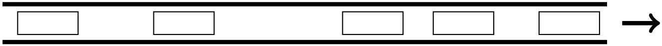
LWR macroscopic traffic flow model

$$\begin{cases} \partial_t \rho + \partial_x \rho\, v(\rho) = 0 & (t, x) \in [0, T] \times \mathbb{R} \\ \rho(0, x) = \rho_o(x) & x \in \mathbb{R} \end{cases}$$



$\rho(t, x)$ density of vehicles $\in [0, R]$

$v(\rho)$ speed law (density dependent)

[Lighthill-Whitham, 1955; Richards, 1956]

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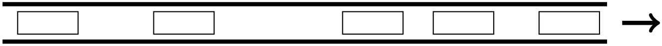

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$v \geq 0, \; v' < 0, \; v(R) = 0$

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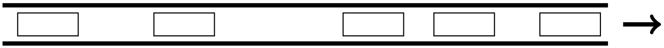
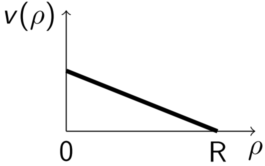


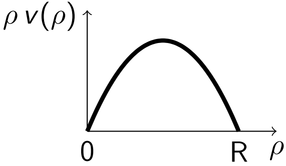
$\rho(t,x)$ density of vehicles $\in [0,R]$

$v(\rho)$ speed law (density dependent)

$v \geq 0,\ v' < 0,\ v(R) = 0$

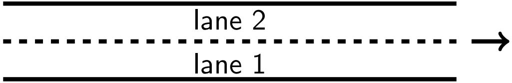
$$v(\rho) = V\left(1 - \frac{\rho}{R}\right)$$





[Lighthill-Whitham, 1955; Richards, 1956]

A multilane model: two lanes



$$\begin{cases} \partial_t \rho_1 + \partial_x \rho_1 \, v_1(\rho_1) = -S(\rho_1, \rho_2) & (t,x) \in [0, T] \times \mathbb{R} \\ \partial_t \rho_2 + \partial_x \rho_2 \, v_2(\rho_2) = S(\rho_1, \rho_2) & (t,x) \in [0, T] \times \mathbb{R} \\ \rho_1(0,x) = \rho_{o,1}(x) & x \in \mathbb{R} \\ \rho_2(0,x) = \rho_{o,2}(x) & x \in \mathbb{R} \end{cases}$$

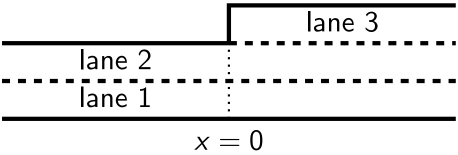
$$S(\rho_1, \rho_2) = \left(v_2(\rho_2) - v_1(\rho_1)\right) \begin{cases} \rho_1 & v_2(\rho_2) \geq v_1(\rho_1) \\ \rho_2 & v_2(\rho_2) < v_1(\rho_1) \end{cases}$$

[Holden-Risebro, 2019]

Extending the multilane model

- The number of lanes can change
- The speed laws can change

Example 1: 1-to-1 junction, from 2 to 3 lanes



[Goatin, Rossi, to appear on SIAM J. Appl. Math.]

Extending the multilane model

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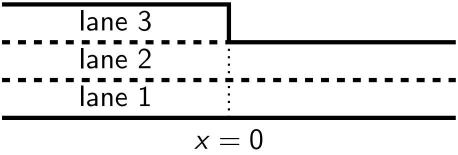
Example 2: 1-to-1 junction, from 3 to 2 lanes



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Extending the multilane model

- The number of lanes can change
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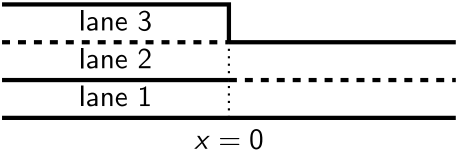
Example 3: 2-to-1 junction, from 1+2 to 2 lanes



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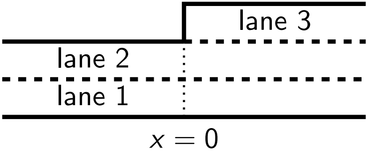
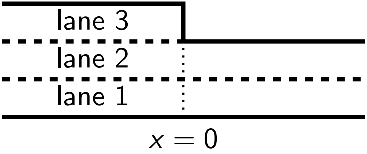
The model





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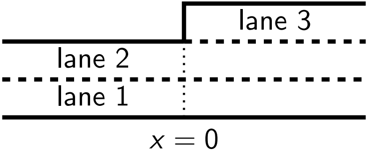
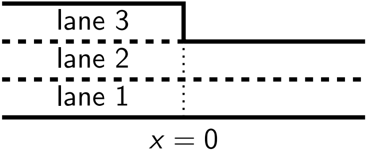
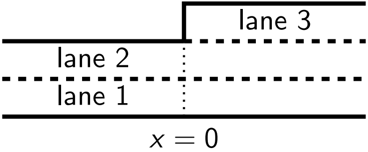
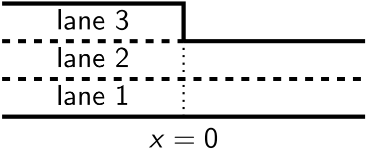




- Same number of lanes M on the left and on the right of $x = 0$

The model





- Same number of lanes M on the left and on the right of $x = 0$
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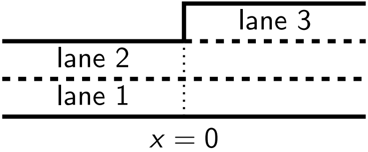
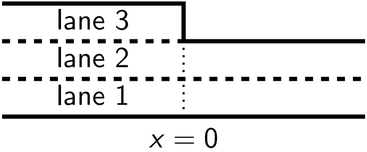
The model





- Same number of lanes M on the left and on the right of $x = 0$
  - No lane change from *active* to fictive lane
  - Adjust the initial data: $\rho_{o,j} : \mathbb{R} \to [0,1]$ for $j = 1, \ldots, M$ with

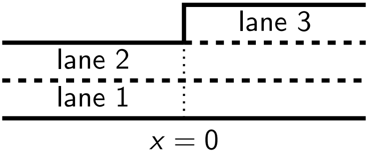
$$\begin{aligned} \rho_{o,j}(x) &= 0 && \text{for } x \in \,]-\infty, 0[\text{ and } j \notin \mathcal{M}_\ell, \\ \rho_{o,j}(x) &= 1 && \text{for } x \in \,]0, +\infty[\text{ and } j \notin \mathcal{M}_r. \end{aligned}$$

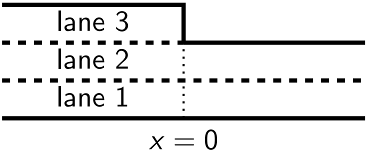
The model

For $j = 1, \ldots, M$, each equation of the system reads

$$\partial_t \rho_j + \partial_x \rho_j \, v_j(x, \rho_j) = S_{j-1}(x, \rho_{j-1}, \rho_j) - S_j(x, \rho_j, \rho_{j+1})$$

The model





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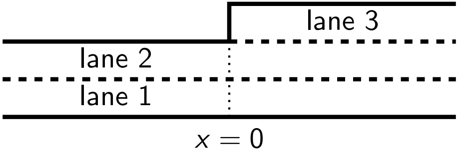
with

$$\begin{aligned} v_j(x,u) &= H(x)\, v_{r,j}(u) + (1 - H(x)) v_{\ell,j}(u) && \text{for } j = 1, \ldots, M, \\ S_j(x,u,w) &= H(x)\, S_{r,j}(u,w) + (1 - H(x)) S_{\ell,j}(u,w) && \text{for } j = 1, \ldots, M-1, \\ S_0 = S_M &= 0 \end{aligned}$$

1-to-1 junction: from two to three lanes

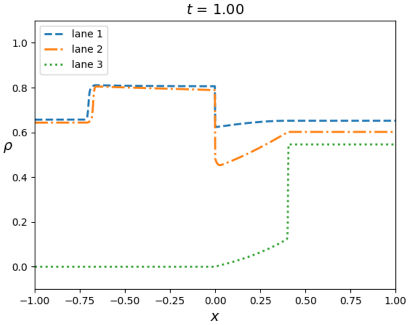


$$\begin{cases} \partial_t \rho_1 + \partial_x \rho_1 \, v_1(x, \rho_1) = -S_1(x, \rho_1, \rho_2) \\ \partial_t \rho_2 + \partial_x \rho_2 \, v_2(x, \rho_2) = S_1(x, \rho_1, \rho_2) - S_{r,2}(\rho_2, \rho_3) \\ \partial_t \rho_3 + \partial_x \rho_3 \, v_3(x, \rho_3) = S_{r,2}(\rho_2, \rho_3) \end{cases}$$

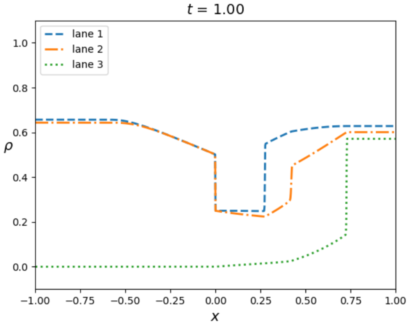
We impose $\rho_{o,3}(x) = 0$ for $x < 0$ and $S_{\ell,2}(u, w) = 0$.

1-to-1 junction: from two to three lanes

$$\rho_{o,1}(x) = 0.7, \qquad \rho_{o,2}(x) = 0.6, \qquad \rho_{o,3}(x) = 0.5\chi_{[0,+\infty[}(x).$$

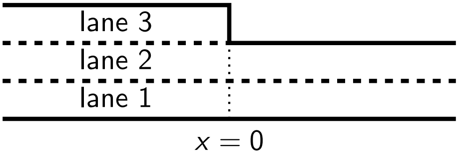


$V_\ell = 1.5, \ V_r = 1$



$V_\ell = 1.5, \ V_r = 2$

1-to-1 junction: from three to two lanes



$$\begin{cases} \partial_t \rho_1 + \partial_x \rho_1 \, v_1(x,\rho_1) = -S_1(x,\rho_1,\rho_2) \\ \partial_t \rho_2 + \partial_x \rho_2 \, v_2(x,\rho_2) = S_1(x,\rho_1,\rho_2) - S_{\ell,2}(\rho_2,\rho_3) \\ \partial_t \rho_3 + \partial_x \rho_3 \, v_3(x,\rho_3) = S_{\ell,2}(\rho_2,\rho_3) \end{cases}$$

We impose $\rho_{o,3}(x) = 1$ for $x > 0$ and $S_{r,2}(u,w) = 0$.

1-to-1 junction: from three to two lanes

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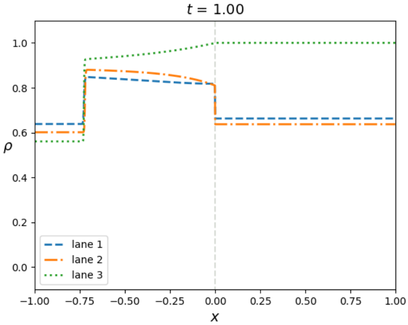

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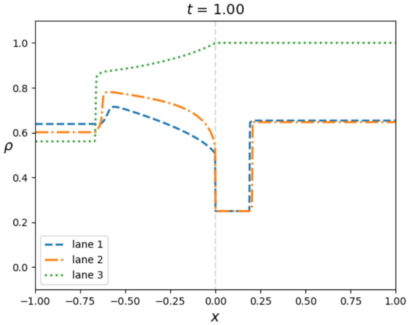

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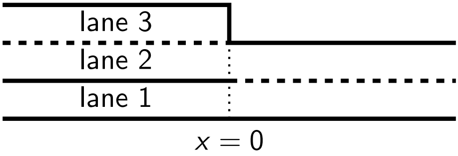
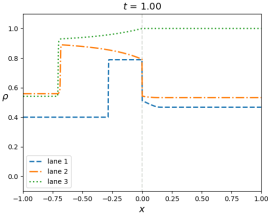


$$\begin{cases} \partial_t \rho_1 + \partial_x \rho_1 \, v_1(x, \rho_1) = -S_{r,1}(\rho_1, \rho_2) \\ \partial_t \rho_2 + \partial_x \rho_2 \, v_2(x, \rho_2) = S_{r,1}(\rho_1, \rho_2) - S_{\ell,2}(\rho_2, \rho_3) \\ \partial_t \rho_3 + \partial_x \rho_3 \, v_3(x, \rho_3) = S_{\ell,2}(\rho_2, \rho_3) \end{cases}$$

We impose $\rho_{o,3}(x) = 1$ for $x > 0$, $S_{r,2}(u, w) = 0$ and $S_{\ell,1}(u, w) = 0$.

2-to-1 junction: from one+two to two lanes

$$\rho_{o,1}(x) = 0.4, \quad \rho_{o,2}(x) = 0.6, \quad \rho_{o,3}(x) = 0.5\,\chi_{]-\infty,0]}(x) + 1\,\chi_{]0,+\infty[}(x).$$



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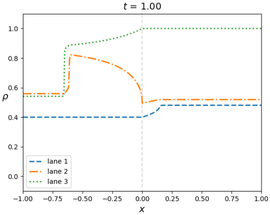

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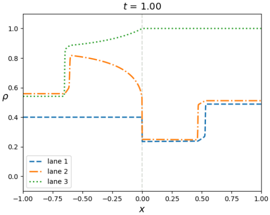

$V_\ell = 1.5, \; V_r = 2$

Main Publications

- H. Holden, N.H. Risebro,
  Models for Dense Multilane Vehicular Traffic,
  SIAM J. Math. Anal., 51(5), 3694-3713, 2019.

- P. Goatin, E. Rossi,
  A multilane macroscopic traffic flow model for simple networks,
  to appear on SIAM J. Appl. Math.