

# Less is More: A Minimalistic Model of Opinion Dynamics

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- Minimalism art: more room for imagination
- Minimalistic model: more room for extensions/elaborations.



Chinese traditional ink painting

## French-DeGroot model [1]

$$x_i(t + 1) = \sum_j w_{ij} x_j(t) \quad \text{or} \quad x(t + 1) = Wx(t)$$

Asymptotic consensus under mild connectivity conditions;

## Important milestones (“weighted-averaging family”)

1. Absolutely stubborn agents [2];
2. Friedkin-Johnsen model [3];
3. Bounded-confidence model [4];
4. Altafini model [5];

Extensions of French-DeGroot model generating disagreements.

[1] J. R. P. French, *Psychological Review*, 63(3):181-194, 1956.

[2] D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar, *Mathematics of Operation Research*, 38(1):1–27, 2013.

[3] N. E. Friedkin and E. C. Johnsen, *Journal of Mathematical Sociology*, 15(3-4):193–206, 1990.

[4] R. Hegselmann and U. Krause, *Journal of Artificial Societies and Social Simulation*, 5(3), 2002.

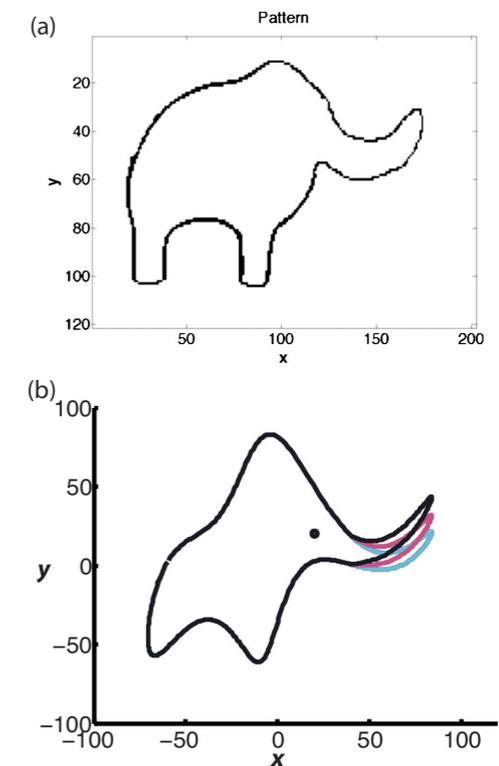
[5] C. Altafini, *IEEE Transactions on Automatic Control*, 58(4):935–946, 2013.

- **Reality: much more complicated than consensus/disagreement;**
  - Public opinion distribution, dispersion of extreme opinions, echo chambers
- **Adding more assumptions and parameters?**
  - Mathematically intractable
  - Curve fitting

*“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.” —John von Neumann*

- **Move forward by looking back:**

**Rethink the micro-foundation of opinion dynamics!**



Fitting an elephant [6]

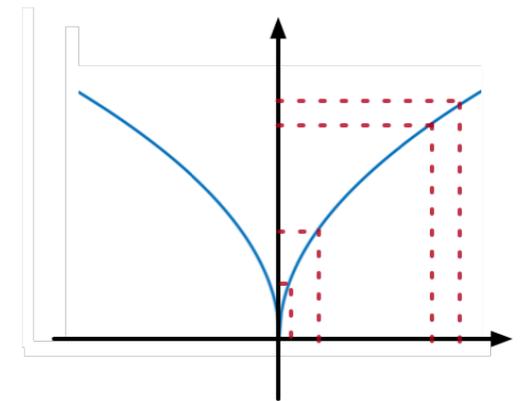
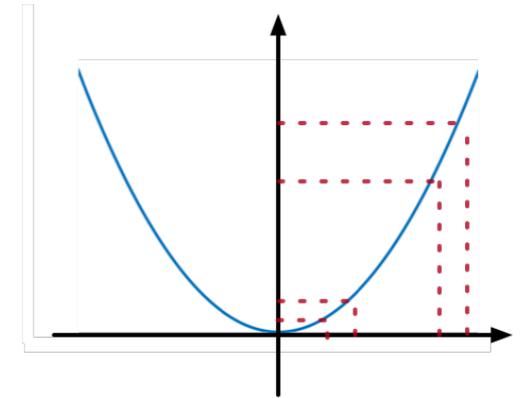
# Weighted-averaging: taken for granted but leads to unrealistic implications

- Cognitive dissonance caused by disagreement

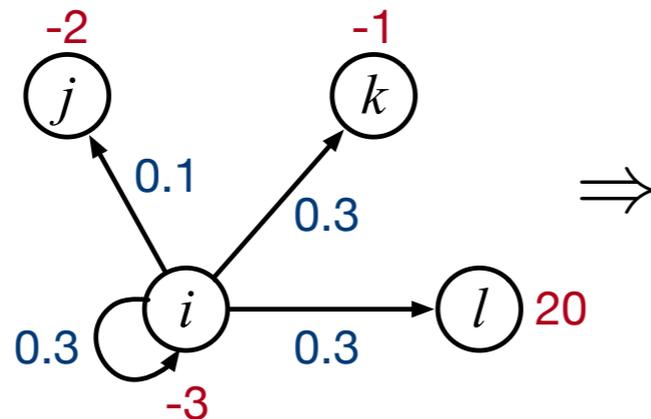
$$u_i(x) = \sum_j w_{ij} |x_j - x_i|^\alpha$$

- $\alpha > 1$ : Distant opinions are more attractive. ( $\alpha = 2$ : DeGroot)
- $\alpha < 1$ : Nearby opinions are more attractive.
- Neutral hypothesis:  $\alpha = 1$ . What will happen?

$$x_i^+ = \operatorname{argmin}_z \sum_j w_{ij} |x_j - z| = \operatorname{Med}_i(x; W)$$



- Weighted-median opinion dynamics**

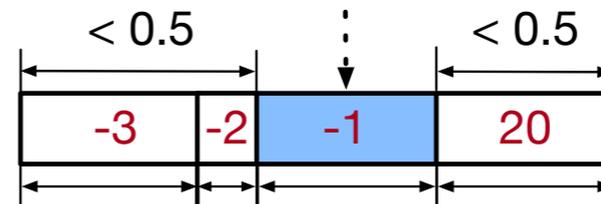


For node  $i$ :

Weighted average: 4.6

Weighted median: -1

Opinions:

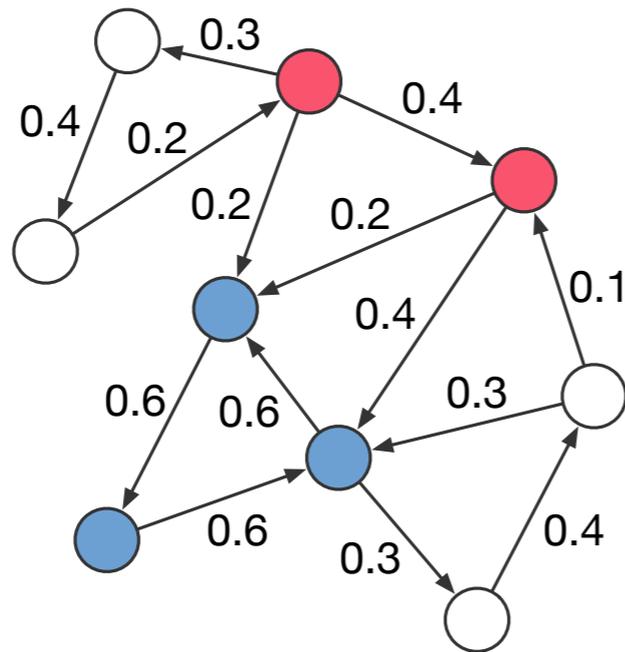


Weights:

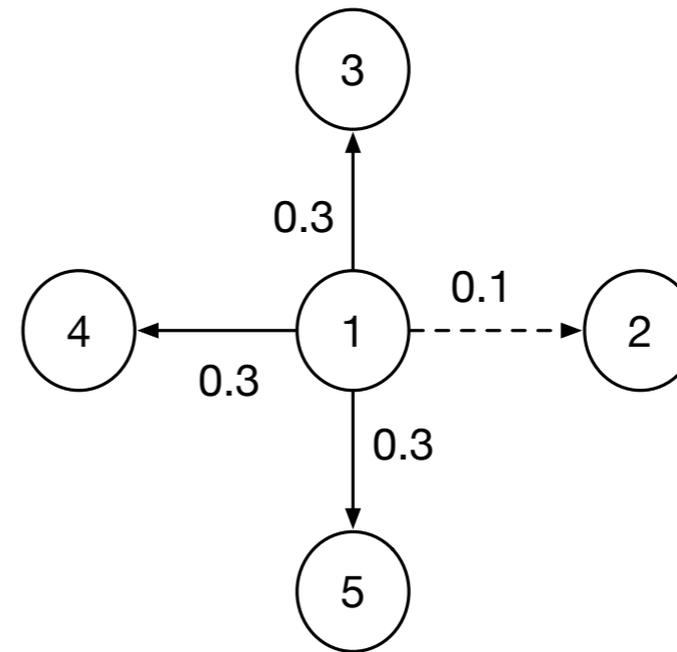
0.3 0.1 0.3 0.3

## Inconspicuous microscopic change $\Rightarrow$ Dramatic macroscopic consequences

1. Broader applicability: ordered multiple-choice issues, e.g., political elections
2. Dependence on more delicate network structures



Cohesive sets:  
echo chambers

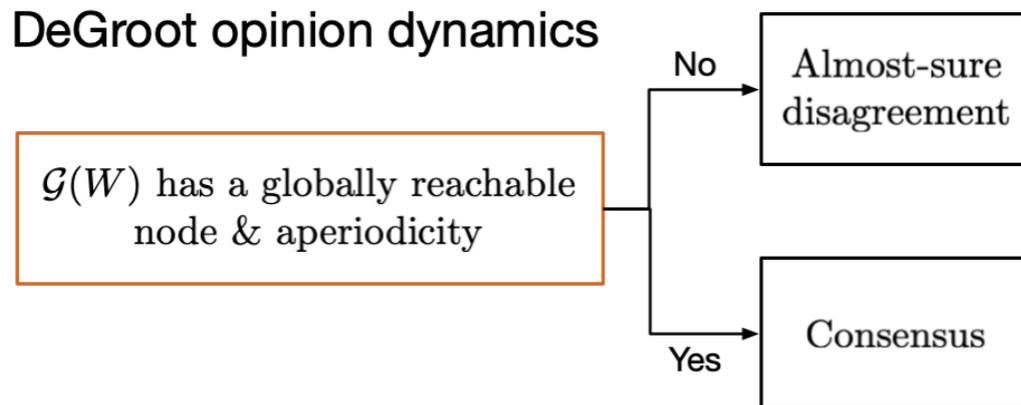


Decisive links:  
Shapley-Schubik index  $> 0$

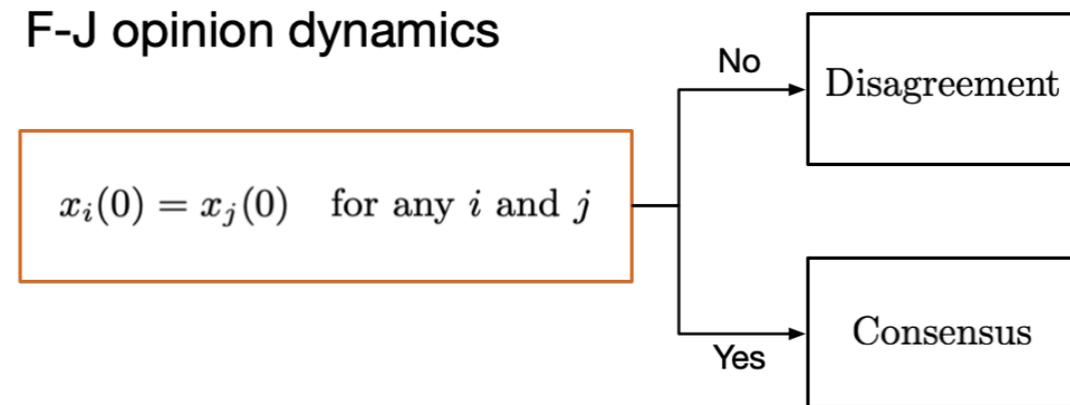
# Inconspicuous microscopic change $\Rightarrow$ Dramatic macroscopic consequences

1. Broader applicability: ordered multiple-choice issues, e.g., political elections
2. Dependence on more delicate network structures
3. More sophisticated consensus-disagreement phase transitions

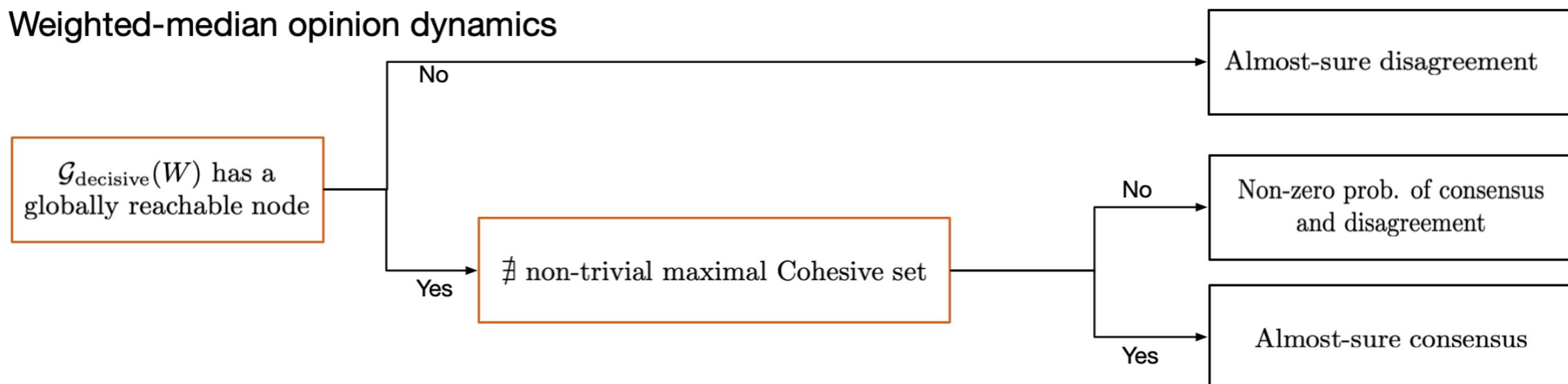
## DeGroot opinion dynamics



## F-J opinion dynamics



## Weighted-median opinion dynamics



## **Inconspicuous microscopic change $\Rightarrow$ Dramatic macroscopic consequences**

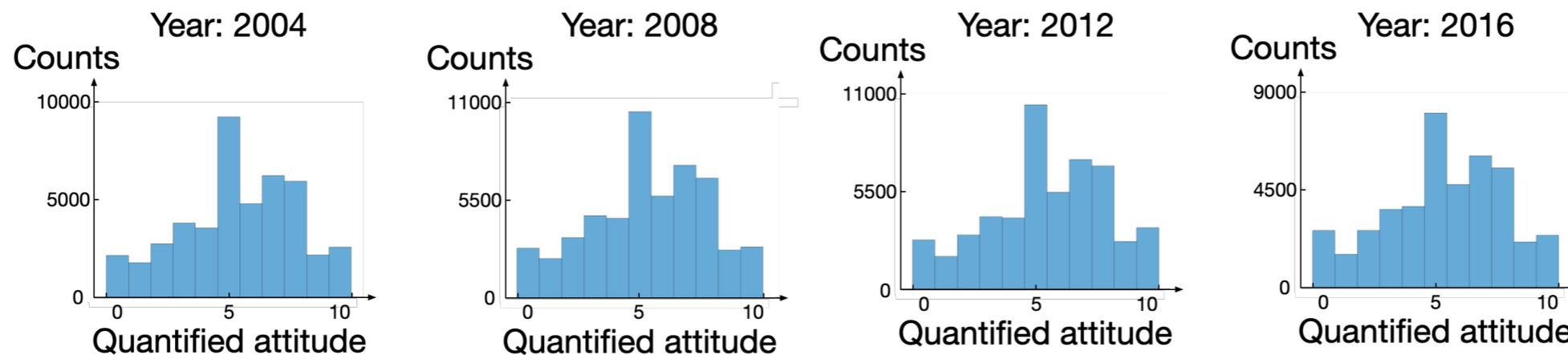
1. Broader applicability: ordered multiple-choice issues, e.g., political elections
2. Dependence on more delicate network structures
3. More sophisticated consensus-disagreement phase transitions
4. Predict various real macroscopic phenomena while the previous models fail to.

### **Models in comparison ( randomized parameters )**

- French-DeGroot model
- French-DeGroot model with absolutely stubborn agents
- Friedkin-Johnsen model
- Networked bounded-confidence model

## Simulation 1: Various types of public opinion distributions

- Empirical data: unimodal, bimodal, multi-modal steady public opinion distributions [7]



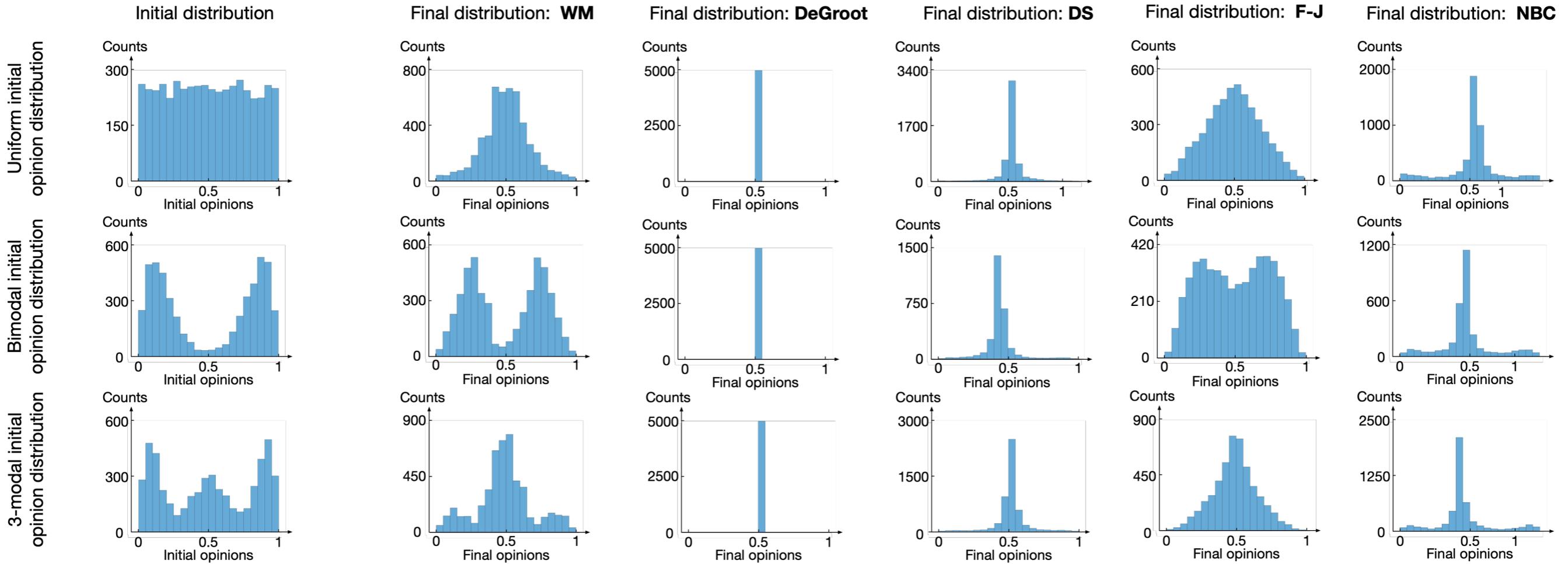
European people's attitudes towards the statement: "Immigrants undermine local culture." Data source: European Social Survey, <http://nesstar.ess.nsd.uib.no/webview/>.

- Open problem in social science: what models generate various steady distributions? [8]
- Simulation setup: scale-free or small-world networks, different initial opinion distributions

[7] A. Downs, *Journal of Political Economy*, 65(2):135-150, 1957.

[8] N. E. Friedkin, *IEEE Control Systems*, 35(3):40-51, 2015.

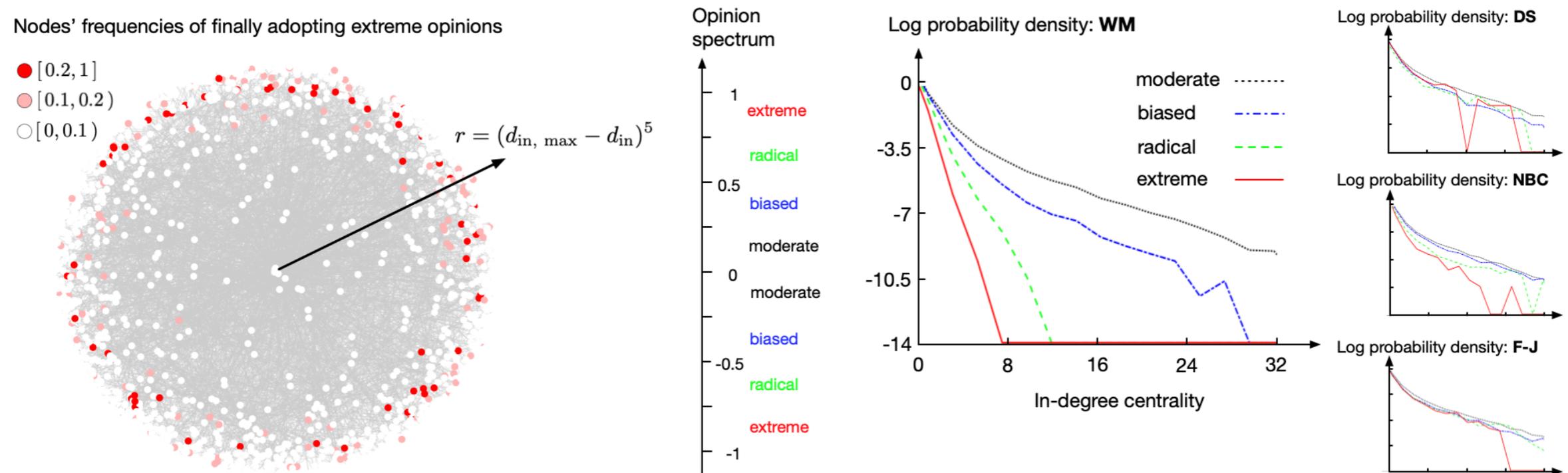
# Simulation 1: Various types of public opinion distributions



Acronyms: **WM** = the weighted-median model; **DS** = the DeGroot model with absolutely stubborn agents; **F-J** = the Friedkin-Johnsen model; **NBC** = the networked bounded-confidence model.

## Simulation 2: Social marginalization and opinion radicalization

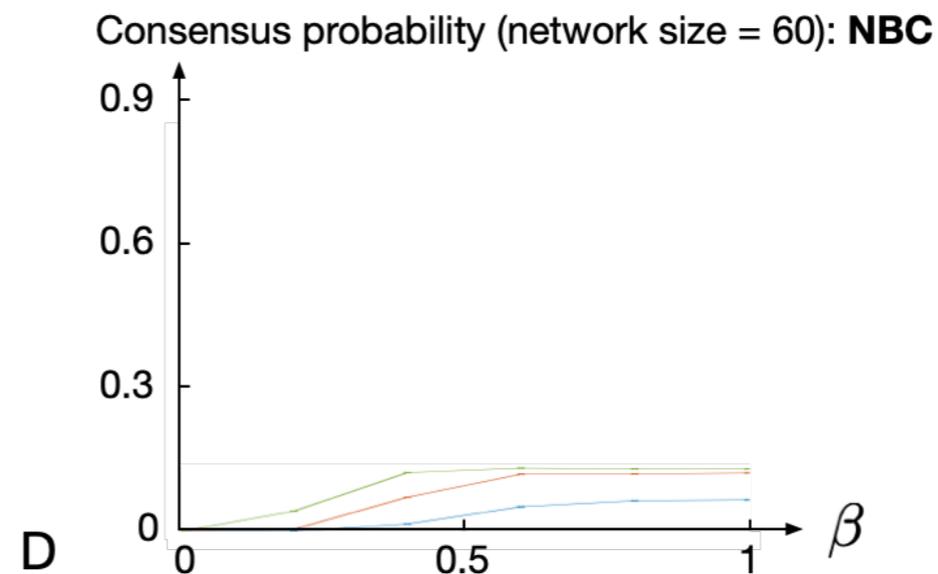
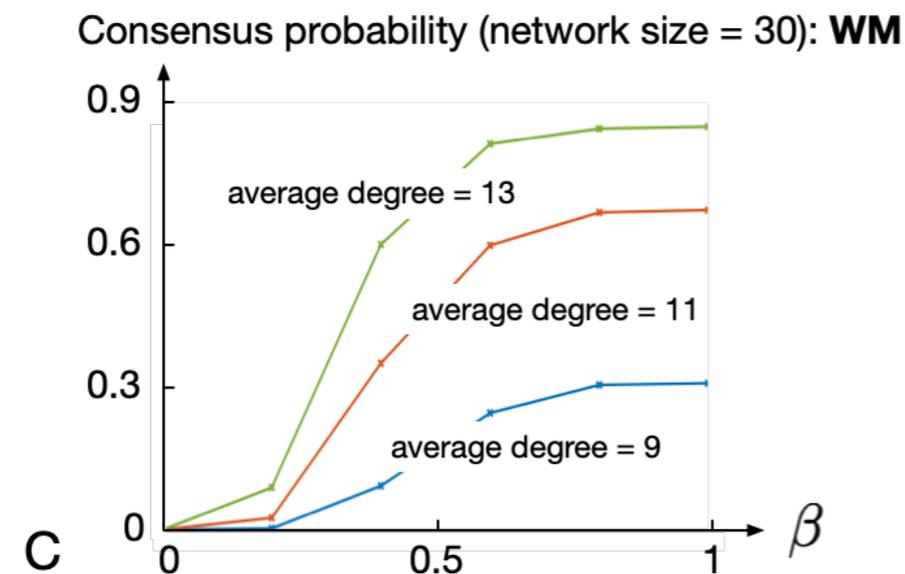
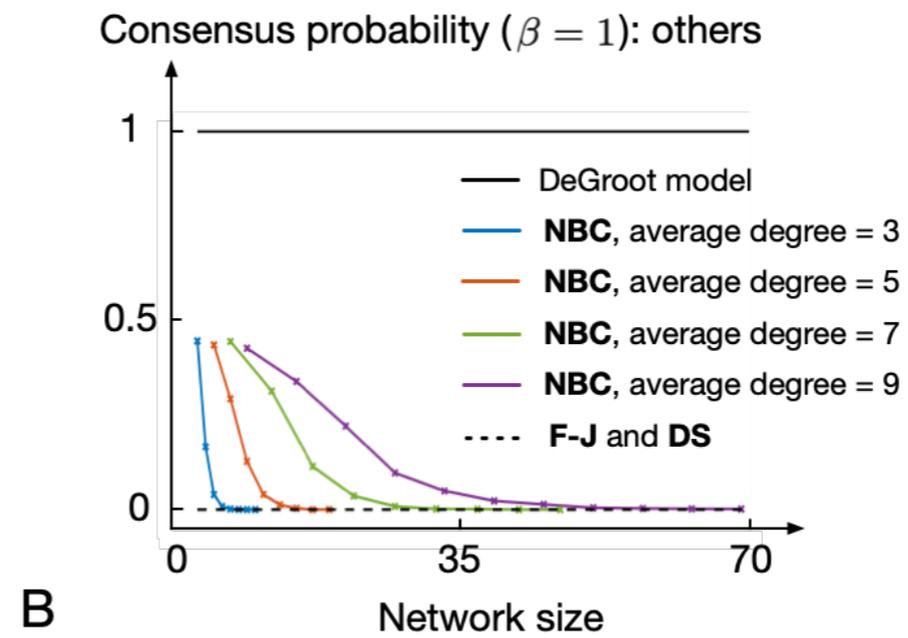
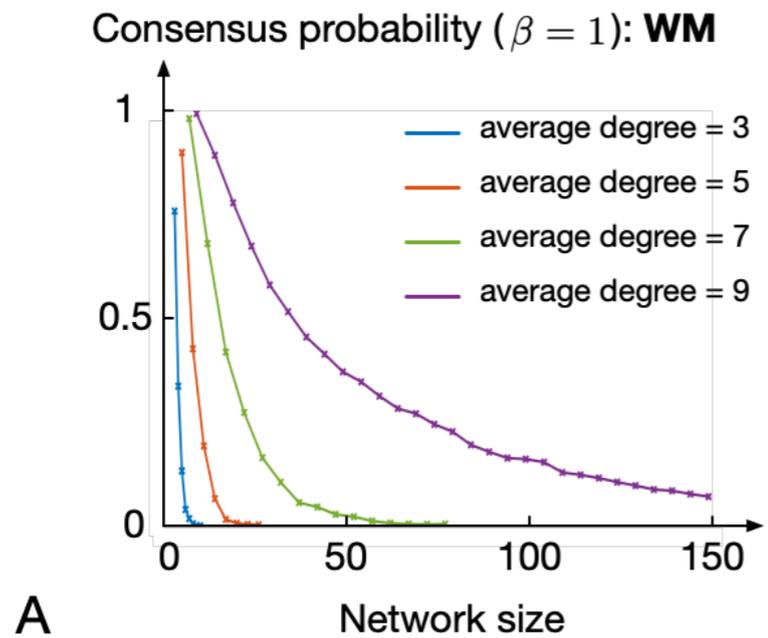
- Empirical evidence of such correlation [8]
- Simulation set-up: scale-free network, 4 categories of final opinions



**Weighted-median model: Peripheral areas are more vulnerable to extreme opinions.**

### Simulation 3: Lower consensus likelihoods in larger or more clustered groups.

- Obvious everyday experience, but not predicted by previous models;
- Simulation set-up: small-world networks (average degree & clustering coefficient)



## Summary: Weighted-median Opinion Dynamics

1. As simple as the classic French-Degroot model
2. Broader applicability
3. Richer and more robust dynamical behavior
4. More realistic prediction



Occam's razor

## Future research directions:

1. Incorporating the compromise behavior (updates with inertia, working paper);
2. Measuring of social power (Shapley-Schubik influence networks, working paper);
3. Conditions for consensus when  $\alpha > 1$ ,  $\alpha = 1$ , or  $\alpha < 1$ ;
4. Networks with heterogenous individuals;
5. Other extensions to DeGroot model (time-varying graphs, negative weights, etc);

**Thank you!**