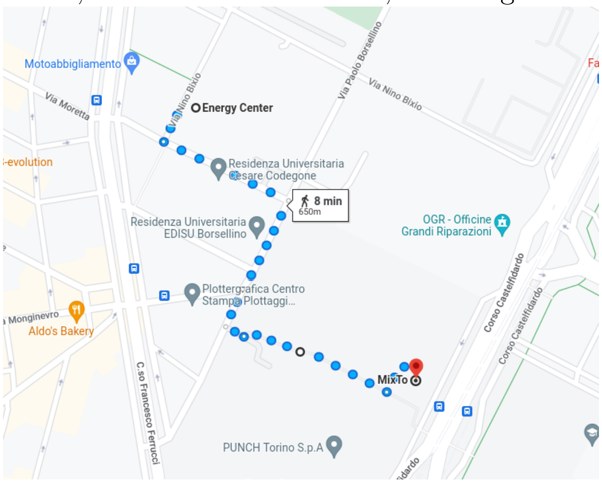
FRAME2020+2
Practical Info and Conference Program

Practical info

- The conference will be held at the **Energy Center** Auditorium, via Paolo Borsellino, 38 int. 16, Torino.

- **Lunches on Wed 18th and Thu 19th** are offered to registered attendees by the conference organization at MixTo, Corso Castelfidardo, 34/A at 12:45. Following the path in the map below, it's a ten minutes walk, following a shortcut within the Politecnico Campus.



- **Social dinner**: is offered by the conference organization to registered attendees and will take place at *Porto di Savona* restaurant, P.za Vittorio Veneto, 2, in the city center on **Wed 18th, 8:00 pm**.

Program summary

	Tue 17th	Wed 18th	Thu 19th	Fri 20th
		Chair: G. Pichot	*Chair: B. Noetinger*	*Chair: R. Masson*
9:00–10:00		(9:10 – 10:00) B. Noetinger	B. Flemisch	L. Formaggia
10:00–10:25		W. Boon	M. Ferronato	G. Etangsale
10:25–10:55		*Coffee break*	*Coffee break*	*Coffee break*
10:55–11:20		S. Bednářová	M. Ruf	F. Della Santa
11:20–11:45		F. Patacchini	A. Fumagalli	D. Zegarra Vasquez
11:45–12:10		(11:45 – 12:45) R. Le Goc	F. Vicini	P. Devloo
12:10–12:35			A. Franceschini	D. Grappein

12:35–14:30		*Lunch*	*Lunch*	

14:15–14:30	*Opening*			
	Chair: S. Berrone	*Chair: B. Flemisch*	*Chair: A. Paluszny*	
14:30–15:30	A. Paluszny	R. Masson	J. Hyman	
15:30–15:55	K. Kumar	F. Bonaldi	F. Ballarin	
15:55–16:25	*Coffee break*	*Coffee break*	S. M. Mallikarjunaiah	
16:25–16:50	J. Stebel	M. Busetto	*Coffee break*	
16:50–17:15	D. Lee	N. Pillardou	D. Losapio	
17:15–17:40	O. Duran	Y. Jia	E. Ballini	
17:40–18:05		*Round table*	M. Fuchs	
18:05–18:30		*Round table*	T. Sorgente	

Conference Program

Tuesday 17th, Afternoon session

14:15–14:30 **Opening**

14:30–15:30 Invited speaker
Adriana Paluszny, *Numerical modelling of thermo-poroelastic deformation and fracture growth in three-dimensional rock masses at the field scale*

15:30–15:55 **Kundan Kumar**, *Coupling of flow and geomechanics in a fractured subsurface setting*

15:55–16:25 **Coffee break**

16:25–16:50 **Jan Stebel**, *Continuum-fracture model of hydro-mechanics with contact conditions and nonlinear fracture permeability*

16:50–17:15 **Dongwon Lee**, *Hydro-mechanical coupling in single fractures with asperities*

17:15–17:40 **Omar Duran**, *A mixed-dimensional model for fracture mechanics using the linear theory of Cosserat continuum*

Wednesday 18th, Morning session

9:10–10:00 **Benoit Noetinger**, *About non-linear diffusion in porous and fractured media: Early- and late-time regimes*

10:00–10:25 **Wietse M. Boon**, *A solution technique for Darcy flow systems that ensures local mass conservation by using exact discrete complexes*

10:25–10:55 **Coffee break**

10:55–11:20 **Sabina Bednářová**, *Numerical methods for nonlinear flow in fractured porous medium*

11:20–11:45 **Francesco Patacchini**, *Adaptation of velocity-pressure model in fractured porous media: well-posedness and numerical strategies*

11:45–12:45 Invited speaker
Romain Le Goc, *Challenges in Field Site Modelling with DFN*

12:45–14:30 **Lunch**

Wednesday 18th, Afternoon session

14:30–15:30 Invited speaker
Roland Masson, *Nodal discretizations of two phase Darcy flows in fractured porous media*

15:30–15:55 **Francesco Bonaldi**, *Energy-stable discretization of two-phase flows in deformable porous media with frictional contact at matrix-fracture interfaces*

15:55–16:25 **Coffee break**

16:25–16:50 **Martina Busetto**, *Virtual Elements for the two-phase flow equations in porous and fractured porous media*

16:50–17:15 **Nicolas Pillardou**, *Numerical simulation of CO2 storage in geological formations based on a coupled Thermal-Hydraulic-Chemical model*

17:15–17:40 **Yunzhong Jia**, *A study of hydraulic stimulation and induced seismicity in fractured rocks as a function of pre-existing fracture network properties*

17:40–18:30 **Round table**

20:00 **Social dinner**

Thursday 19th, Morning session

9:00–10:00 Invited speaker
Bernd Flemisch, *Verification and Validation Benchmarks in Fractured Porous Media*

10:00–10:25 **Massimiliano Ferronato**, *Scalable preconditioning of coupled contact mechanics and fluid flow in discrete fracture networks*

10:25–10:55 **Coffee break**

10:55–11:20 **Matthias Ruf**, *Characterization of different thermally treated Carrara marble core samples by acoustic waves and micro X-Ray Computed Tomography*

11:20–11:45 **Alessio Fumagalli**, *Uncertainty quantification for reactive transport in fractured porous media*

11:45–12:10 **Fabio Vicini**, *Highly-efficient implementations for flow simulations on DFNs and DFMs*

12:10–12:35 **Andrea Franceschini**, *A Reverse Augmented Constraint Preconditioner for contact mechanics*

12:35–14:30 **Lunch**

Thursday 19th, Afternoon session

14:30–15:30 Invited speaker
Jeffrey De'Haven Hyman, *The Interplay of Multiple Scales in Fractured Media on Flow and Transport Properties*

15:30–15:55 **Francesco Ballarin**, *Data-driven reduced order modeling of natural convection in porous media using convolutional autoencoders and Barlow Twins self-supervised learning*

15:55–16:20 **S. M. Mallikarjunaiah**, *On Quasi-static Crack Evolution in Nonlinear Strain-limiting Elastic Bodies: Phase-Field Regularization and Finite Element Discretization*

16:20–16:50 **Coffee break**

16:50–17:15 **Davide Losapio**, *A local embedded method for flow in fractured porous media with upscaling and Artificial Neural Networks (ANNs)*

17:15–17:40 **Enrico Ballini**, *Surrogate models for flows in porous media with faults*

17:40–18:05 **Marco Fuchs**, *Fracture surface imaging methods and their application for numerical flow simulations*

18:05–18:30 **Tommaso Sorgente**, *Mesh quality optimization for the virtual element method and applications to discrete fracture networks*

Friday 20th, Morning session

9:00–10:00 Invited speaker
Luca Formaggia, *Some modeling and numerical issues for modeling underground flows*

10:00–10:25 **Grégory Etangsale**, *A Hybridizable Discontinuous Galerkin methods for fractured groundwater flow problems*

10:25–10:55 **Coffee break**

10:55–11:20 **Francesco Della Santa**, *Uncertainty Quantification in Discrete Fracture Networks via Graph Informed Neural Networks*

11:20–11:45 **Daniel Zegarra Vasquez**, *Simulation of single-phase flows in fractured porous media using the mixed hybrid finite element method*

11:45–12:10 **Philippe Devloo**, *A multiscale hybrid method for simulating porous media flow in fractured media*

12:10–12:35 **Denise Grappein**, *A three-field optimization based approach for DFN simulations and 3D-1D coupled problems*

Abstracts

Invited speakers

Bernd Flemisch, bernd.flemisch@iws.uni-stuttgart.de, Universität Stuttgart
Verification and Validation Benchmarks in Fractured Porous Media

Computational Models need to be verified and validated in order to be trustworthy. Providing benchmark scenarios and conducting benchmark studies greatly facilitates verification and validation tasks for researchers. In this talk, existing and planned benchmarking efforts for flow, transport and deformation processes in fractured porous media will be presented. Benefits and shortcomings of comparisons against other computational models, reference solutions or experimental data will be highlighted. Openness with respect to participation and data access will be discussed. In addition, a Bayesian framework will be presented which considers uncertainties both in the modeling results and the reference data.

Luca Formaggia, luca.formaggia@polimi.it, Politecnico di Milano, Italy
Some modeling and numerical issues for modeling underground flows

The importance of modeling underground flows, and in general underground mechanics, is growing due to the role underground may have in the energy transition and climate change mitigation (geothermal, gas storage, CO2 sequestration). In this talk, I describe some of the activities carried out within the compgeo@mox to address some of the mathematical and numerical issues arising from this important application.

Jeffrey De'Haven Hyman, jhyman@lanl.gov, Los Alamos National Laboratories
The Interplay of Multiple Scales in Fractured Media on Flow and Transport Properties

In low-permeability fractured media, such as granites and shales, flow and the associated transport of dissolved solutes is controlled primarily by fractures embedded within the rock matrix. The geometry of individual fractures, size and aperture, as well as the network structure determine the structure of the fluid flow field. However, the relevant lengths scales within a fracture network range several orders of magnitude and it is unclear which features of the network influence which flow and transport properties. One tool to investigate the interplay and influence of these multiple scales are discrete fracture network (DFN) models. In this talk, I'll discuss recent studies that use high-fidelity DFN models that attempt to link flow and transport attributes to physical structures of a fracture network ranging in-fracture aperture variability to network-scale connectivity.

Romain Le Goc, r.legoc@itasca.fr, Itasca Consultants SAS
Challenges in Field Site Modelling with DFN

Discrete Fracture Networks (DFN) are a modeling framework for hydrogeological and geomechanical applications in fractured rocks. The rock mass is modeled as a network of discrete fractures embedded in a (usually) elastic impervious matrix, where the fractures are statistical or deterministic surfaces, with the dual advantages of easily incorporating geological observations and not assuming a homogenizing scale. There are critical challenges in making the DFN approach operational for applications, that can be summarized into two categories: The modelling challenges: transforming field observations - multi-scale fracture networks, flow channeling and anisotropy, depth-dependent flow, etc. - into numerical analogues. This challenge is based on the principle of "simplexity": modeling complex structures and constitutive relationships in the simplest possible way while remaining relevant to geology. This includes the stochastic generation of fracture networks, the degree of heterogeneity of structures and properties, and the parameterization of phenomenological laws. The computational challenges: developing efficient numerical methods to simulate physical properties. This includes fracture generation methods, meshing of millions of fractures and uneven domains, numerical schemes, and solvers. The challenge is to define the tradeoff between the ability to model complex physical processes under realistic conditions, provide accurate results while keeping the simulations manageable with reasonable computational efficiency. We illustrate these challenges with theoretical concepts and applications from numerical experiments and case studies.

Roland Masson, roland.masson@univ-cotedazur.fr, Université Côte d'Azur
Nodal discretizations of two phase Darcy flows in fractured porous media

This talk reviews the nodal Vertex Approximate Gradient (VAG) discretization of two-phase Darcy flows in fractured porous media for which the fracture network is represented as a manifold of co-dimension one with respect to the surrounding matrix domain. Different types of models and their discretizations are considered depending on the transmission conditions set at matrix fracture interfaces accounting for fractures acting either as drains or both as drains or barriers. Difficulties raised by nodal discretizations in heterogeneous media are investigated and solutions to solve these issues are discussed.

Adriana Paluszny, apaluszn@imperial.ac.uk,
Numerical modelling of thermo-poroelastic deformation and fracture growth in three-dimensional rock masses at the field scale

Finite element-based simulation of coupled thermo-hydro-mechanical processes are combined with the detailed assessment of fracture growth and damage. The approach uses the Imperial College Geomechanics Toolkit, which is an in-house C++ 3D simulator that captures multi-physics deformation while accounting for variable apertures and permeabilities on the fracture surface as a function of deformation. Simulations combine fluid flow and fracture growth, but also damage and fracturing, under static and dynamic loads. Validations are application specific and are the cornerstone of our publications in this area. Simulations yield channeling, fracture patterns, and the relationship between fracture interaction and permeability structures. Permeability structures are observed to be less scale dependent when incorporating heterogeneities and poroelastic effects. This talk will be an overview of our work in this area.

Contributed talks

Francesco Ballarin, francesco.ballarin@unicatt.it, Department of Mathematics and Physics, Catholic University of the Sacred Heart, Brescia, Italy
Data-driven reduced order modeling of natural convection in porous media using convolutional autoencoders and Barlow Twins self-supervised learning

Natural convection in porous media is a highly nonlinear multiphysics problem relevant to many engineering applications (e.g., the process of CO2 sequestration). In recent works [1,2,3] we have proposed a range of data-driven reduced order models (ROMs), starting from simple ROMs which approximate the solution manifold by a linear subspace (e.g., built through POD), to variants which build instead a nonlinear manifold, and which may be based on either deep convolutional autoencoders (DC-AE ROM) or a combination of an autoencoder and Barlow Twins self-supervised learning (BT-AE ROM).
Advantages and disadvantages of each proposed ROM will be discussed through a series of benchmark problems of natural convection in porous media. In the simplest test cases, a linear subspace is enough to construct an accurate ROM, and our results will show that BT-AE provides comparable accuracy to POD-based approaches. In more complex test cases, a representation based on a nonlinear manifold is required, and our results will show that a BT-AE ROMs not only outperforms POD-based approaches, but also DC-AE ROMs.
This is joint work with T. Kadeethum, Y. Choi, D. O'Malley, H. Yoon, N. Bouklas.

References
[1] T. Kadeethum, F. Ballarin, and N. Bouklas, Data-driven reduced order modeling of poroelasticity of heterogeneous media based on a discontinuous Galerkin approximation, GEM – International Journal on Geomathematics, 12, 12, 2021.
[2] T. Kadeethum, F. Ballarin, Y. Choi, D. O'Malley, H. Yoon, and N. Bouklas, Non-intrusive reduced order modeling of natural convection in porous media using convolutional autoencoders: comparison with linear subspace techniques, submitted, 2021, arXiv:2107.11460.
[3] T. Kadeethum, F. Ballarin, D. O'Malley, Y. Choi, N. Bouklas, and H. Yoon, Reduced order modeling with Barlow Twins self-supervised learning: Navigating the space between linear and nonlinear solution manifolds, submitted, 2022, arXiv:2202.05460.

Enrico Ballini, enrico.ballini@polimi.it, Politecnico di Milano, Dipartimento di Matematica
Surrogate models for flows in porous media with faults

To transition towards a greener economy, the exploitation of the subsoil by injection of fluid, as performed in the technology of carbon capture storage, or thermal energy storage, may give a great contribution. However, fluid injection alters the local stress, possibly causing the reactivation of faults and consequently microseismicity phenomena. The risk of triggering microseismicity should be carefully assessed before starting injection operations, with the help of numerical simulations. These simulations are, however, very costly from the computational point of view due to the complexity of the domain and the time-dependent nature of the problem. Moreover, some physical and geometrical properties of the subsoil are usually unknown or uncertain, leading to the need to evaluate different scenarios and quantify the uncertainties. A key strategy for the computation of many different scenarios is the use of a model reduction technique to obtain a surrogate model that should be both reliable and fast. In this work, we compare two different data-driven techniques: a linear and a non-linear one. We consider a test problem consisting of a flow governed by the Darcy equation in a 2D porous medium with a fault that crosses the domain. The aim is to assess the pressure changes associated with some physical parameters that could be affected by uncertainties, such as the permeability of the porous medium and the fault, and geometrical features of the domain. First, we assess the proper orthogonal decomposition (POD), which is a linear technique, implemented both in a monolithic approach, so computing a unique set of basis for the representation of the reduced space, and a block approach, that means computing as many subsets of basis as the number of physical variables. Then, we assess a non-linear technique based on the sole use of Neural Networks (NNs), called DL-ROM (Deep Learning based Reduced Order Model). We consider different NNs architectures as fully connected NN or convolutional NN, in a framework based on a connection of multiple NNs. With this work, we expect to provide the ground for the creation of specific ROM methodologies for poromechanics problems with the presence of faults.

Sabina Bednářová, sabina.bednarova@tul.cz, Technical University of Liberec
Numerical methods for nonlinear flow in fractured porous medium

This presentation concerns numerical methods used to solve nonlinear Darcy – Forchheimer equation in fractured porous medium. Porous media flow is usually described by the Darcy equation, where there is a linear relationship between the flux and the pressure gradient. However, it is well established that this relationship becomes nonlinear at sufficiently large Reynolds numbers. For these cases, the flow is supposed to follow the Forchheimer equation, where we find an extra term that depends quadratically on the flow rate. The Forchheimer regime is more likely to occur in fractures, as the fractures have a significant impact on the flow. Solution of the nonlinear Darcy-Forchheimer equation becomes much more involved and its efficiency depends on the chosen linearization strategy. We derive the mixed-hybrid formulation of the equation and present 3 linearization methods: Picard iterations, Newton's method and L-scheme method. All three of them have been implemented into the software Flow123d. The methods were compared and verified on a simple analytical solution and on a problem of two perpendicular fractures. Finally, one linearization method was chosen and tested on a benchmark case.

Francesco Bonaldi, francesco.bonaldi@umontpellier.fr, Université de Montpellier
Energy-stable discretization of two-phase flows in deformable porous media with frictional contact at matrix–fracture interfaces

We address the discretization of two-phase Darcy flows in a fractured and deformable porous medium, including frictional contact between the matrix–fracture interfaces. Fractures are described as a network of planar surfaces leading to the so-called mixed- or hybrid-dimensional models. Small displacements and a linear elastic behavior are considered for the matrix. Phase pressures are supposed to be discontinuous at matrix–fracture interfaces, as they provide a better accuracy than continuous pressure models even for high fracture permeabilities. The general gradient discretization framework is employed for the numerical analysis, allowing for a generic stability analysis and including several conforming and nonconforming discretizations. We establish energy estimates for the discretization, and prove existence of a solution. To simulate the coupled model, we employ a Two-Point Flux Approximation (TPFA) finite volume scheme for the flow and second-order (P2) finite elements for the mechanical displacement coupled with face-wise constant (P0) Lagrange multipliers on fractures, representing normal and tangential stresses, to discretize the frictional contact conditions. This choice allows to circumvent possible singularities at tips, corners, and intersections between fractures, and provides a local expression of the contact conditions. We present numerical simulations of benchmark examples and one realistic test case based on a drying model in a radioactive waste geological storage structure.

Wietse M. Boon, wietsemarijn.boon@polimi.it, Politecnico di Milano
A solution technique for Darcy flow systems that ensures local mass conservation by using exact discrete complexes

Constructing fast solution schemes often involves deciding which errors are acceptable and which approximations can be made for the sake of computational efficiency. Herein, we consider a mixed formulation of Darcy flow and take the perspective that the physical law of mass conservation is significantly more important than the constitutive relationship, i.e. Darcy's law. Within this point of view, we propose a three-step solution technique that guarantees local mass conservation.
In the first step, an initial flux field is obtained by using a locally conservative method such as the TPFA Finite Volume Method. Although this scheme is computationally efficient, it lacks consistency and therefore requires a suitable correction. This correction will be divergence-free and so the Helmholtz decomposition ensures us that it can be described as the curl of a potential field. The second step therefore employs an H(curl)-conforming discretization to compute the correction potential and update the flux field. The pressure field is computed in the final step by using the same TPFA system from the first step.
We present our technique in the context of structure-preserving discretization methods and use finite element and virtual element methods of lowest order as our leading examples. In terms of computational cost, each of the three steps involves solving an elliptic problem posed on either the cell centers or the edges of the mesh.
The procedure guarantees local mass conservation regardless of the quality of the computed correction. Thus, we relax this computation using tools from reduced order modeling. In particular, we introduce a reduced basis method that is capable of rapidly producing a potential field for given permeability fields. By applying the curl to this field, we ensure that the correction is divergence-free and mass conservation is not impacted.
Finally, we extend the method to solving Darcy flow in fractured porous media. We rewrite the equations in terms of the mixed-dimensional divergence and identify the problem as a mixed-dimensional Darcy flow system. In turn, the solution procedure directly applies. The mixed-dimensional curl ensures that the correction step is divergence-free and we show the performance of our technique in the context of flow in three-dimensional, fractured media.

Martina Busetto, martina.busetto@polito.it, Politecnico di Torino
Virtual Elements for the two-phase flow equations in porous and fractured porous media

The design of numerical discretization techniques able to handle general polygonal tessellations of the computational domain is gaining considerable interest in the field of geological flow simulations. Indeed, the geometric flexibility offered by polygonal elements allows to design efficient computational grids and this is of primary importance especially when dealing with complex and realistic problems. In this framework, the Virtual Element Method (VEM) represents a promising numerical scheme allowing approximation on arbitrarily shaped grids. The VEM has been successfully tested on geophysical models characterized by high geometric complexity. However, so far, simple linear diffusion models have been mainly considered. The investigation of complex non-linear flow models has only very recently began. Consequently, there is still a lot to be explored. In the present contribution, we propose a virtual element approach to tackle the simulation of the two-phase flow equations of immiscible fluids in porous media. The model problem involves two coupled non-linear time-dependent partial differential equations. We discretize the governing equations in time and in space combining an iterative Implicit-Pressure-Implicit-Saturation method with a primal conforming virtual element discretization. Furthermore, we extend the proposed approach to poro-fractured media modelled by a Discrete Fracture Network (DFN). We impose suitable matching conditions at the fracture intersections and we use an original local-global non-standard polygonal conforming mesh to preserve a fracture-independent meshing approach. Numerical tests show the potentialities of the resulting scheme in handling general polygonal tessellations of the domain. We consider both regular problems and realistic imbibition problems that are of interest for engineering applications.

Francesco Della Santa, francesco.dellasanta@polito.it, Politecnico di Torino

Uncertainty Quantification in Discrete Fracture Networks via Graph Informed Neural Networks

Flow and transport characterization through underground fractured media is a crucial issue in many engineering and industrial applications (e.g., oil and gas extraction, water resources analyses, etc.). Discrete Fracture Network (DFN) models are often used since they allow to perform accurate flow simulations through networks of subsurface fractures. However, only statistical distributions of position, size, orientation, and hydrogeological properties of the fractures are given. This probabilistic nature of DFNs, make Uncertainty Quantification (UQ) a key issue in the framework of flow and transport characterization in a real fractured medium. Since UQ methods often require thousands of DFN simulations, it is worth considering the application of Neural Networks (NNs) [1, 2] to perform flux regression tasks and speed up the UQ process. In this work, we describe a recently developed NN layer to build NN architectures that take advantage of the DFN's graph representation. In particular, we introduce extra knowledge about the DFN geometry and relationships between fractures, building a Graph-Informed NN (GINN) [3], where the layers are characterized by the adjacency matrix of the DFN's graph. The flux regression performances are analyzed showing the advantages of using GINN in the UQ framework.

References

[1] Stefano Berrone, Francesco Della Santa, Sandra Pieraccini, and Francesco Vaccarino. "Machine learning for flux regression in discrete fracture networks". In: GEM - International Journal on Geomathematics 12.1 (2021), p. 9. DOI: 10.1007/s13137-021-00176-0.
[2] Stefano Berrone and Francesco Della Santa. "Performance Analysis of Multi-Task Deep Learning Models for Flux Regression in Discrete Fracture Networks". In: Geosciences 11.3

(2021), p. 131. DOI: 10.3390/geosciences11030131.
[3] Stefano Berrone, Francesco Della Santa, Antonio Mastropietro, Sandra Pieraccini, and Francesco Vaccarino. “Graph-Informed Neural Networks for Regressions on Graph-Structured Data”. In: Mathematics 10.5.786 (2022). DOI: https://doi.org/10.3390/math10050786

Philippe R. B. Devloo, phil@fec.unicamp.br, LabMeC-FEC-Universidade Estadual de Campinas, Campinas-SP, Brazil
A multiscale hybrid method for simulating porous media flow in fractured media

This presentation reports on recent advances to develop a 3-D multiscale methodology to simulate porous media flow in fractured reservoirs. Flow in the porous media is modeled with traditional Darcy's equations and the coupling between flow in the porous media and fractures is based on the conceptual Discrete- Fracture-Matrix representation, where the fractures are idealized as lower-dimensional elements at the interface of matrix elements. Local features are modeled using the Multiscale Hybrid Method with H(div)- confirming flux approximations [1]. In this method, normal fluxes and piecewise potentials are computed over macro elements. Local features are solved in separate local problems based on fine representations of the macro elements. Mixed Finite Elements are adopted to solve the local problems, which inherently leads better flux accuracy and local mass conservation. Since these local problems are independent, this method is particularly appealing for parallel computing environments. The methodology is compared with benchmark examples from [2] to demonstrate its robustness, accuracy, and efficiency.

References
[1] Durán, O., Devloo P.R.B., Gomes S.M., Valentin, F. A multiscale hybrid method for Darcy's problems using mixed finite element local solvers, Computer Methods in Applied Mechanics and Engineering (2019) 354: 213-244.
[2] Berre, I., Boon, W., Flemisch, B., Fumagalli, A., Gläser, D., Keilegavlen, E., Scotti, A., Stefansson, I., Tatomir, A., Brenner, K., Burbulla, S., Devloo, P., Duran, O., Favino, M., Hennicker, J., Lee, I- Hsien, Lipnikov, K., Masson, R., Mosthaf, K., Nestola, M.G.C., Ni, Chuen-Fa, Nikitin, K., Schädle, P., Svyatskiy, D., Yanbarisov, R., Zulian, P. Verification benchmarks for single-phase flow in three- dimensional fractured porous media, Advances in Water Resources (2021) 147.

Omar Duran, omar.duran@uib.no, University of Bergen
A mixed-dimensional model for fracture mechanics using the linear theory of Cosserat continuum

This research was inspired and intended as an extension to the mixed-dimensional poromechanical models for fractured media presented in [1]. There the classical continuum mechanics is considered in the context of finite and infinitesimal strain. We addressed fracture mechanics from a different perspective by considering the linear theory of the Cosserat (micropolar, asymmetric) continuum. Extra regularity can be achieved on the asymmetric component of the stress tensor and fracture tractions through the angular momentum balance defined on surfaces. We address the second- and first-order systems and present numerical results for the adopted model.

References
[1] W.-M. Boon and J.-M. Nordbotten. Mixed-dimensional poromechanical models of fractured porous media, arXiv:2112.05038 2021.

Grégory Etangsale, gregory.etangsale@univ-reunion.fr, University of La Réunion, France
A Hybridizable Discontinuous Galerkin methods for fractured groundwater flow problems

Modeling flows in fractured porous media has received tremendous attention in environmental, energy and engineering applications during the last decades. In these applications, the presence of fractures strongly influenced the fluid flow, and the assignment of accurately modeling the interaction between the fracture network and the porous matrix is particularly challenging. Here, we focus on the hybrid-dimensional description that considers the fracture as an encapsulated object of lower dimension, i.e., (d-1)-dimension. As a result, the flow process is governed by distinctive equations in the matrix region and fractures, respectively. Thus, coupling conditions are added to close the problem. This mathematical description of the fractured porous media has been initially introduced by Martin et al. in 2005 and is referred to as the Discrete Fracture-Matrix (DFM) model.
Modeling fluid flow in fractured domains with the DFM model is challenging. Several numerical methods have been used in the literature. Recently, extensions of non-conforming discretization to the coupled matrix-fracture problem have been accomplished with Interior Penalty Discontinuous Galerkin (IPDG), Enriched Galerkin (EG), and Extended Finite Element (XFEM) methods. However, the applications of the Hybridizable DG (HDG) methods to the aforementioned problem are scarce, and only the HHO scheme addresses this topic. In the present work, we propose a numerical strategy to solve the coupled matrix-fracture problem. We develop a families of IP Hybridizable Discontinuous Galerkin (IP-HDG) formulation in the matrix-region coupled with a Conforming Galerkin (CG) approximation inside the fracture network. HDG methods inherit the advantages of the DG schemes (i.e., local conservation, eligibility to hp-refinement strategies, discontinuous piecewise polynomial approximation). Moreover, they are known to be more robust, accurate, and competitive than standard DG schemes for many challenging problems. The application of the static condensation procedure reduces significantly the number of degrees of freedom and the CPU time. Numerical experiments are presented to assess the validity of the discretization method in terms of stability, accuracy, and efficiency on the simplified case (single fracture). We also investigate the ability of the HDG scheme to handle more complex geometries, including intersecting and immersed fractures, by comparing it with commercial software (COMSOL) and existing benchmarks.

Marwan Fahs, fahs@unistra.fr, ITES University of Strasbourg
Modeling variably saturated flow and transport in fractured porous media with the mixed finite element method

Modeling variably saturated flow and transport in fractured domains is computationally challenging due to the nonlinearity of the governing equations, sharpness of the solution and preferential flows. The mixed finite element (MFE) method is superior to the standard finite element method in handling heterogeneity and fractures. MFE method has been used in the literature for variably saturated flow in fractured porous media. But this method has been rarely used for solving the transport equation as it can suffer from unphysical oscillations for convection-dominated problems. This is the case in high permeable fractures, where, due to high velocity, the transport processes are dominated by convection. In this work, we develop a full MFE numerical scheme for the simulation of flow and transport in unsaturated fractured porous media. The MFE method is employed for the spatial discretization of both flow and transport on the 2D-matrix elements as well as on the 1D-fracture elements. To avoid unphysical oscillations encountered with the standard MFE in the case of convection dominated problems, we develop a new upwind scheme of the MFE. Furthermore, while most existing numerical schemes are based on the first order Euler scheme, we use the method of line to improve time integration by using high-order methods and efficient automatic time-stepping schemes. Several numerical examples and comparison with standard finite element method and Discontinuous Galerkin confirm the robustness and efficiency of the developed MFE scheme.

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Scalable preconditioning of coupled contact mechanics and fluid flow in discrete fracture networks

In many subsurface applications, a major role is played by the coupled simulation of frictional contact mechanics and fluid flow in fractured porous media. Large domains with high resolution representations of the geological structures and their heterogeneous properties are usually required to achieve the desired accuracy. These aspects naturally reflect on the growing demand for better performance of sophisticated and computationally expensive models.

In this talk, the focus is on the linear solver, which is the most time consuming component of a full simulation, and in particular on the design of a scalable and robust preconditioning framework for the coupled contact mechanics and fluid flow problem in discrete fracture networks. The model is based on a Lagrange-multiplier approach used to impose the contact constraints. Low order finite elements are used for the mechanics, while a cell-centered finite volume scheme has been adopted for the fluid flow in the fracture network. The selected discretization is stabilized as proposed in [1] to meet the LBB inf-sup condition. The Jacobian matrix arising from the proposed approach is generally non-symmetric and indefinite with a 3x3 block structure, where displacements on the continuous medium, tractions on the contact surfaces and pressure within the fractures are the main variables.

We design a scalable preconditioning framework for the linear problem by exploiting an aggregation-based inner multigrid solver [2]. In essence, the global multi-physics problem is first reduced to a single-physics problem and approximately solved, then the solution is prolonged back to the full system size. Two different approaches are derived according to the reduction sequence, with the identification of theoretical bounds for the eigenvalue distribution of the preconditioned matrices. The two strategies are tested in different numerical examples to prove the algorithmic scalability, the influence of the relative weight of fracture-based unknowns, and the performance on a realistic application.

Joint work with Andrea Franceschini and Laura Gazzola

References

[1] A. Franceschini, N. Castelletto, J.A. White, H.A. Tchelepi, Algebraically stabilized Lagrange multiplier method for frictional contact mechanics with hydraulically active fractures, Comput. Methods in Appl. Mech. Eng. 368 (2020), 113161.

[2] A. Franceschini, L. Gazzola, M. Ferronato, A scalable preconditioning framework for stabilized contact mechanics with hydraulically active fractures, ArXiv: 2112.12397 [math.NA].

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A Reverse Augmented Constraint Preconditioner for contact mechanics

The accurate simulation of fault and fracture behavior is of great importance in the context of geomechanics. While several phenomena need to be captured, such as micro-seismicity and fracture propagation, their physical description is quite complex, due to the strong coupling between fractures and mechanical deformation. The problem becomes even more complex when flux is considered.
The major difficulties in this kind of simulations arise from frictional contact, that is one of the most challenging problems in computational mechanics. Typically, it produces a stiff non-linear problem requiring several Newton iterations to converge, where each iteration consists of an ill-conditioned linear system solution with the Jacobian matrix. By enforcing the constraint using Lagrange multipliers, the associated Jacobian matrix is saddle-point and indefinite independently of the specific discretization and needs a special treatment for a robust and fast numerical solution.
The focus of this presentation is on preconditioning strategies for the saddle-point Jacobian matrices arising from the discretization of fractured media by Lagrange multipliers. A well-established way to address such a problem relies on approximating the inverse of the leading block and the related Schur complement, e.g. [1]. However, finding accurate and consistent approximations for the inverse of the leading block is not trivial and, in any case, such an approach fails whenever the leading block is singular. In this work, we propose a constraint preconditioner based on the elimination of the Lagrange multipliers unknowns, hence the name reverse. A suitable augmentation [2] is introduced, which produces a primal Schur complement resembling a standard stiffness matrix for the continuous problem, such that state-of-the-art multigrid techniques for structural matrices, e.g. [3], are very effective. Numerical evidence of the robustness and efficiency of the proposed approach are provided by solving large size problems from various applications.
Joint work with Matteo Frigo, Carlo Janna and Massimiliano Ferronato.

References
[1] A. Franceschini, N. Castelletto, M. Ferronato, Block preconditioning for fault/fracture mechanics saddle-point problems, Comp. Meth. Appl. Mech. Eng., 344 (2019), pp. 376-401.
[2] A. Franceschini, M. Ferronato, M. Frigo, C. Janna, A Reverse Augmented Constraint preconditioner for Lagrange multiplier methods in contact mechanics, Comp. Meth. Appl. Mech. Eng., accepetd for publication.
[3] G. Isotton, M. Frigo, N. Spiezia and C. Janna, Chronos: A general purpose classical AMG solver for High Performance Computing, SIAM J. Sci. Comput., 43(5) (2021), pp. C335-C357.

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Fracture surface imaging methods and their application for numerical flow simulations

A good understanding of fracture properties and related flow processes is important for many geoscience applications, such as nuclear waste disposal, geothermal energy and hydrocarbons.

In recent years, hydro-mechanical numerical models have been established in parallel to experimental methods to estimate single fracture parameters like the hydraulic aperture controlling the distribution of flow. A crucial issue for numerical modeling is the accurate imaging of the fracture surfaces providing the geometrical information of the fracture. Hence, in this work we apply three different fracture surface imaging methods, (1) handheld laser scanner (HLS), (2) mounted laser scanner (MLS), and (3) Structure from Motion (SfM) to a bedding joint in a sandstone block and compare the results. The surfaces are matched assuming a contact area of 1 % and used for detailed single fracture flow simulations. From this, the hydraulic aperture is calculated and compared with laboratory measurements averaging 82 $\pm$ 12 μm using a handheld air permeameter. Results reveal that the resolution of the fracture surface obtained from HLS is insufficient for any numerical simulations. While MLS and SfM provide suitable datasets for flow simulations, the resulting hydraulic apertures of 176 μm and 221 μm significantly exceed the measurements. The flow simulations reveal preferential flow through major channels that are structurally and geometrically predefined. In order to minimize the gap between simulation and measurement, the contact area was gradually increased in 1 % steps up to 15 %. It is found that a contact area of 8 % results in a hydraulic aperture of 80 μm from MLS and a contact area of 9 % results in a hydraulic aperture of 82 μm from SfM, respectively. Conclusively, it is demonstrated that the quality of the results of a flow simulation strongly depends on the resolution and accuracy of the imaging devices and that SfM provides a promising, flexible and low-cost method for fracture imaging on cores or outcrops.

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Uncertainty quantification for reactive transport in fractured porous media

CO2 sequestration is one of the promising technology to mitigate climate change in the next decades, by using depleted wells to inject it into the subsurface where long term storage is granted by reliable cap rocks. An assessment of the reliability of selected storage sites should account for the effect of highly permeable and impermeable fractures, as well as the chemical alteration induced by the CO2 injection. Indeed, the presence of CO2 may alter water pH and, due to dissolution and precipitation of minerals, change the physical properties of the porous medium and the fractures, and, therefore, the stability of the system.
In the underground uncertainty is ubiquitous and could compromise the predictive capability of numerical simulations; moreover, the measurement of some of the physical parameters may be rather challenging. In this study, we consider an uncertainty quantification analysis on the coupled problem of flow and mineral rections in fractured porous media to estimate, with the Sobol indices, the most relevant parameters that affect the safety of the sealing mechanism. To speed up the process, we have employed the sparse grid technique to sample the parameter space, substantially reducing the computational burden with respect to the Monte Carlo approach.

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A three-field optimization based approach for DFN simulations and 3D-1D coupled problems

A new numerical approach is proposed for the simulation of fluid flows in complex discrete fracture networks (DFNs). The method is based on a three- field domain decomposition strategy, in which proper variables are introduced at fracture intersections, allowing to completely decouple

the problems defined on the different fractures. The coupling conditions, continuity of the hydraulic head and flux conservation, depend indeed uniquely on these variables. The problem is then recast into a PDE-constrained optimization problem, in which a cost functional expressing the error in the continuity of the solution is minimized constrained by the equations written in the three-field formulation. This strategy allows to handle non-conforming meshes and ensures local mass conservation at fracture intersections. The possibility to work independently on each fracture makes the method suitable for parallelization, and thus ready to be applied to networks with a very large number of fractures.
The same kind of strategy is proposed also for the coupling of three-dimensional and one-dimensional elliptic equations (3D-1D coupling). Possible applications concern all those cases in which small tubular inclusions are embedded in a much wider three-dimensional porous matrix. One of the advantages of working with the three-field optimization based approach lies once again in the possibility of using non-conforming meshes: the partitions on the 1D domains are built independently from the surrounding 3D mesh and no remeshing is required in case the set of the inclusions evolves in time.

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A study of hydraulic stimulation and induced seismicity in fractured rocks as a function of pre-existing fracture network properties

Hydraulic stimulation is commonly used to increase the target rock permeability and heat extraction area during the development of Enhanced Geothermal Systems (EGS). The fluid injection will create new tensile fractures and induce shear slip of pre-existing natural fractures. This may also cause seismic activity. In this study, we develop a coupled hydro-mechanical model by combining finite element method and cohesive element technique to investigate the fracture propagation, activation of the pre-existing natural fractures, damage evolution, and seismic response due to fluid injection in fractured rock. The fractured rock is assumed to have a pre-existing network with different flow percolation levels, average fracture lengths and natural fracture orientations. Seven different cases are implemented and investigated. Results indicate that the flow percolation level appears to control the seismic response in terms of the total number of seismic events, while average fracture length determines the maximum magnitude of the events. These results can be understood through the fact that seismic events are associated with the shear slip of pre-existing natural fractures, which is more likely to occur for high percolation levels with more connected fractures. Further, the radiated energy is proportional to the fracture length so that larger seismic magnitude can be found in fracture networks with larger average lengths. On the other hand, the orientations of natural fractures relative to the maximum stress direction are found to have little effect on seismic behaviour. They largely determine the topography of the fracture networks after hydraulic stimulation. Our results imply that, before any hydraulic simulation in EGS, a comprehensive geological survey, hydraulic testing, and borehole imaging are necessary to gain knowledge of pre-existing natural fractures' properties (numbers, lengths and orientations), as well as estimates of in situ stress states. These may prove to be important input for optimal hydraulic stimulation design for an EGS project.

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Coupling of flow and geomechanics in a fractured subsurface setting

We study coupling of flow and geomechanics in a fractured porous medium setting. We present a mixed dimensional model for a fractured poro-elasic medium. The fracture is a lower dimensional surface embedded in a bulk poro-elastic matrix. The flow equation on the fracture is a Darcy type model that follows the cubic law for permeability. The bulk poro-elasticity is governed by fully dynamic Biot equations. The resulting model is a mixed dimensional type where the fracture flow on a surface is coupled to a bulk flow and geomechanics model.
We extend the Biot equations to include fracture flow model and complex friction and contact mechanics. We further consider different iterative and time schemes (multirate schemes) for each of the Multiphysics models. We consider finer time steps for the flow and coarser time steps for the mechanics. We prove the well-posedness of the model and the numerical analysis of the multirate time discrete schemes.
This is a joint work with Vivette Girault, Mary F Wheeler, Tameem Almani, and Maarten de Hoop.

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Hydro-mechanical coupling in single fractures with asperities

Hydro-mechanical coupling of fractures in reservoirs is an important reservoir phenomenon and has a major influence on the interpretation of data obtained from stimulation or pumping experiments. Since the presence of fractures changes the effective permeability and storativity, fractures eventually affect the productivity of geothermal and/or a gas or oil reservoirs.
In addition, fracture deformation in terms of aperture changes interacts with the local fluid pressure and the mechanical stress state within the reservoir. Traditionally, fractures are characterized by consistent analysis of pressure and flow transients which are induced by pumping and stimulation experiments. Given that most of the reservoirs are placed in the deep subsurface and therefore the detailed fracture topology including the small-scale fracture roughness are unknown, the inverse analysis requires a proper physic-based model which is able to account for aperture changes triggered by the interaction of pressure and deformation transients.
Since the fractures' deformation characteristic depends on the geometrical stiffness, which is mainly determined by the shape, stiffness of the surrounding rock bulk material and the contact stiffness induced by the rough topology of the fracture surfaces, it leads to a high number of Hertzian contact zones and a strongly non-linear behavior. Thus, predictive models allowing for the physics-based characterization of fractures need to account for the effective stiffness of fractures as a function of the current stress state.
Therefore, we characterize the effective stiffness of a single fracture in a natural porous rock (Bentheim sandstone). We set-up a lab-based experimental workflow which consists on two steps. First, under triaxial stress conditions, a hydraulically-induced tensile fracture is created. The fracture experiment was conducted on cylindrical rock cores (D=30mm, H=75mm) where pore pressure exceeds the least compressive stress by the tensile strength of the sandstone sample. Second the effective stiffness of the induced crack is characterized and related to the image-based analysed fracture roughness. Finally, we show how the experimentally obtained

results could be implemented in a non-linear hybrid-dimensional simulation model.

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Hydromechanical simulation of fault reactivation induced by subsurface fluid injection from laboratory mesoscale to field reservoir-scale

Injecting large amounts of fluid into the subsurface space is an inevitable step in many subsurface operations such as geoenergy extraction, underground natural gas storage and geologic carbon dioxide sequestration, and has the potential risk of inducing fault activation. The injection operation involves complex hydromechanical coupling issues that are difficult to solve directly by analytical methods. Therefore, numerical methods are becoming an indispensable toolbox to simulate the complicated physical process of aforementioned subsurface operations, and the most commonly used method is the finite element method. Due to the unique advantages in solving complex engineering problems, finite element software is widely developed and popularly used around the world. However, only a few finite element simulators permit large-scale, complex analyses and prediction of fluid injection-induced fault reactivation problems. Therefore, we have carefully chosen the commercial finite element software, ABAQUS, for our research, and are focusing on the development of user-defined subroutines to improve the hydromechanical coupling analysis of fluid injection-induced fault reactivation. According to the effective stress law and dynamic friction law, the effective normal stress and shear displacement of the fault were defined as the field variables varying with time step in the user subroutines. Through the exchange and update of data between the mechanical module and the hydraulic module in each step of the interpolation calculation, hydromechanical coupling analysis of fluid injection-induced fault reactivation was successfully realized in ABAQUS. Then, a series of verification was performed to demonstrate the capabilities and reliabilities of the user-defined subroutine programs. Four cross-scale examples ranging from laboratory mesoscale (centimeter level) to field reservoir-scale (hundred-meter level) show that the subroutine programs can be an effective toolbox for analyzing subsurface fluid injection-induced fault reactivation in geological engineering applications.

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A local embedded method for flow in fractured porous media with upscaling and Artificial Neural Networks (ANNs)

The study of flow in fractured porous media is a key ingredient for many geoscience applications, such as reservoir management. Modelling and simulation of these highly heterogeneous and geometrically complex systems require the adoption of non-standard numerical schemes. The Embedded Discrete Fracture Model (EDFM) is a simple and effective way to account for fractures with coarse and regular grids, but it suffers from some limitations: the expression for the flux interaction terms between porous matrix and fractures comes from the assumption of linear pressure distribution around fractures, which holds true only far from the tips and fracture intersections, and it can be employed for highly permeable fractures only. We propose an improvement of EDFM, i.e. the Local Embedded Discrete Fracture Model (LEDFM), which aims at overcoming both its limitations computing an improved coupling between fractures and the surrounding porous medium by a) relaxing the linear pressure distribution assumption, b) accounting for impermeable fractures modifying near-fracture transmissibilities. These results are achieved by adopting local flow-based upscaling methods to compute new transmissibilities for matrix-fracture and near-fracture matrix-matrix connections. Here the coarse model coincides with an embedded model, whereas in the numerical upscaling techniques for fracture networks found in the literature the coarse model typically belongs to the family of Continuum Fracture Models (CFM), that are not capable of explicitly representing fractures. The definitions of the local fine scale problems for transmissibility computation are inspired from the aforementioned techniques, and a conforming method is used to solve them. In some cases, a higher accuracy for the description of the near fracture flow is needed, so that the local problems for the computation of matrix-matrix transmissibilities are replaced with a multiscale approach. Generally, a high number of local problems should be solved. Hence, to speed up an otherwise very costly procedure, neural networks are integrated in the model to provide a fast evaluation of the local flow problems. Indeed, these are solved in an offline stage, where different fracture configurations and matrix-fracture permeability contrasts are examined. The results obtained are then used to train two feedforward neural networks, whose goal is that of learning the transmissibility functions relative to the two different local problems. The transmissibilities are then obtained by evaluating the networks instead of solving the local problems directly, resulting in a dramatic reduction of the computational cost of the method. The results obtained from several numerical tests, comparing the solutions of different embedded methods, including the newly developed LEDFM, with the reference ones, show that the local method overcomes both the limitations mentioned before pertaining to the classic EDFM method.

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On Quasi-static Crack Evolution in Nonlinear Strain-limiting Elastic Bodies: Phase-Field Regularization and Finite Element Discretization

In this talk, we discuss a phase-field model for quasi-static crack propagation in nonlinear strain-limiting elastic bodies. Our goal is to formulate a diffuse interface approach for the initiation and growth of a single crack in an elastic material that is governed by a special constitutive relationship. Such a setting yield physically reasonable crack-tip fields which are well within the basic assumptions used in the derivation of the model. Further, a quasi-static evolution problem is formulated as a minimization of total energy functional based on Griffith's criterion. The coupled system consists of two nonlinear second order elliptic partial differential equations. The discretized coupled system is solved by a monolithic Newton-Raphson iterative algorithm coupled with continuous Galerkin finite elements. The efficacy of the proposed model is demonstrated by several numerical results.
This is a joint work with Dr. Hyun C. Yoon (KIGAM, South Korea) and Dr. Sanghyun Lee (Florida State University, Tallahassee, Florida)

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About non-linear diffusion in porous and fractured media: Early- and late-time regimes

Studying non-linear diffusion phenomena in porous media is a generic problem which is often encountered in particular for applications involving displacement of non-aqueous phase liquid (NAPL) by water. In this paper, we revisit the problem by presenting two exact asymptotic solutions valid for short and long times. In the latter, a complete analytical solution is presented. In the time domain, it involves a simple Ansatz, under the form of a power law time decay of the NAPL saturation. On the spatial domain, that solution is an eigenvector of the non-linear diffusion operator driving the saturation, with Dirichlet boundary conditions. If the diffusion coefficient varies as a power law of the NAPL saturation, the spatial variations of the solution is given analytically. The solution is in very good agreement with results of numerical simulations involving various realistic sets of input transport parameters. It agrees also with previous findings of other authors based on other approximations.

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Adaptation of velocity-pressure model in fractured porous media: well-posedness and numerical strategies

The accurate description of fractures is fundamental to obtain reliable numerical outcomes useful for, e.g., energy management. Depending on the physical and geometrical properties of the fractures, which can be strongly heterogeneous, fluid flow can behave in different ways, going from a slow Darcian regime to more complicated Brinkman or even Forchheimer regimes for high velocity. The main problem is to determine where in the fractures one regime is more adequate than others, and thus also to localize the transition zone between these regimes. To determine these low-speed and high-speed regions, we propose an adaptive strategy based on selecting the appropriate constitutive law linking velocity and pressure according to a threshold criterion on the magnitude of the fluid velocity itself. This gives rise to a velocity-pressure law which is discontinuous in velocity. Both theoretical and numerical aspects are considered and investigated, showing the potential of the proposed approach. From the analytical viewpoint, we show existence of weak solutions under reasonable hypotheses on the constitutive law. To this end, we use a variational approach identifying solutions with minimizers of an underlying energy functional. From the numerical viewpoint, we propose two algorithms: one which tracks the transition zone between the low- and high-speed regions, best adapted to one-dimensional fractures; and another which is based on the regularization of the underlying energy functional, naturally useable in higher-dimensional geometries. By running numerical experiments using both algorithms, we illustrate some interesting behaviors of our adaptive model on single fractures in one and two space dimensions and on a small network of intersecting fractures in dimension one.

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Numerical simulation of CO2 storage in geological formations based on a coupled Thermal-Hydraulic-Chemical model

Reactive two-phase flows in porous media appear in diverse energetic and environmental problematics. One can mention non-exhaustively geological gas storage (carbon dioxide, hydrogen or natural gases), nuclear waste management, petroleum engineering or geothermal energy production. In this talk, we will mainly focus on the application of CO2 storage that

appears as a promising way to reduce greenhouse gas emissions.
Nonisothermal reactive two-phase flows are modeled by mass conservation laws written for each chemical component in each phase (liquid or gaseous) and an energy balance equation. These equations are coupled with Darcy's law to characterize the flow velocity of each phase. Chemical reactions are described thanks to reaction rates that are either function of concentration in case of kinetic reactions or unknown for equilibrium reactions. Reaction rates at equilibrium can be eliminated by means of a linear transformation by using Morel's formalism. The chemical system is then closed with the help of mass action laws which are algebraic equations linking activities (or molar fractions) of the different species. Finally, the full system is closed by the capillary pressure law, equations of state and solubility laws characterizing each phase equilibrium. Thus, the unknowns of the system are pressures, saturations, molar fractions and the temperature. By consequence, the problem is modeled by a nonlinear system of degenerate partial differential equations (modeling the flow), coupled with algebraic or ordinary differential equations (provided by chemical reactions).
In this talk, we consider finite volume schemes for modeling nonisothermal two-phase flow coupled with geochemical interactions. Spatial discretization is carried out using a cell-centered finite volume scheme while the time discretization is performed by a Backward Differentiation Formula 2 (BDF2). The approach combines advantages of the TPFA (Two Point Flux Approximation) method to accurately solve fluxes and diffusive terms and upstream for advective terms. The nonlinear system is solved by a Newton method and a BiConjugate Gradient STABilized (BiCGSTAB) method with an AMG preconditioner is used to solve the linear systems. We aim at comparing two numerical strategies: a fully coupled fully implicit and a sequential scheme. In the fully implicit scheme, the nonlinear system gathering all equations above mentioned is solved at each time step. For the sequential solution approaches, a nonisothermal two-phase flow and a reactive transport problem are solved sequentially. These two THC modules have been implemented in the parallel open source platform DuMuX using High Performance Computing. Numerical results for CO2 storage in a large-scale 3D heterogeneous reservoir will be presented to demonstrate the effectiveness and efficiency of these schemes.

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Characterization of different thermally treated Carrara marble core samples by acoustic waves and micro X-Ray Computed Tomography

Thermal treatment of rocks is a frequently used method to initiate micro-cracks in intact rocks. This is done to study different physical phenomena related to fractures. In this contribution, twelve thermal treatments, distinguishing in the applied maximum temperature and the applied cooling condition (slow versus fast cooling) are experimentally studied for dry Bianco Carrara marble under ambient stress conditions. As effective quantities on the core-scale of the sample, the bulk volume, the bulk density, and the P- and S-wave velocities, including shear wave splitting, are examined. To obtain a three-dimensional (3d) insight into the physical mechanisms occurring on the grain-scale level and to link those back to the effective quantities on the core-scale of the sample, micro X-Ray Computed Tomography (μXRCT) imaging is employed. A model is presented which predicts the relative change of ultrasound velocities depending on the initiated relative bulk volume change (fracture volume). By different segmentation approaches, the full 3d fracture network was successfully segmented from μXRCT data sets.

However, a link back to macroscopic measurements, here, the relative bulk volume change, indicates that all segmentation approaches significantly overestimate the fracture aperture in a systematic way. The reason for this is that micro-cracks have a disadvantageous ratio between the crack aperture and the crack length. Therefore, imaging, as well as the subsequent segmentation, is challenging. In particular, the spatial resolution of μXRCT devices often comes to its limitation to reliable resolve the crack aperture. Furthermore, an inherent noise and low contrast of the resulting μXRCT data set causes difficulties to achieve an accurate image segmentation. For the outlined issues, it is not advisable to directly use such a segmented data set for further numerical studies, and it also indicates how crucial image segmentation quality control is. Although the fracture aperture cannot be quantified reliably based on μXRCT imaging, the topology of fracture networks can be considered relatively reliable. Consequently, the fracture topology in combination with other experimental quantities should be used to derive an adequate simulation model. Besides the systematical study of the effect of different thermal treatments on the acoustic waves in Carrara marble, this contribution shows the importance of combining measurements of multiple scales.

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Hybrid-dimensional Stokes-Darcy model for highly heterogeneous fractured porous media

Hybrid-dimensional models for fractured porous media are effective approximations of full dimensional models. Traditionally applied hybrid-dimensional Darcy-Darcy models fail to accurately describe flows in fractures separating rock matrices with significantly different permeabilities. We develop and analyze a hybrid-dimensional Stokes-Darcy model for flows in highly heterogeneous fractured porous media. Fractures can store and transport fluid and they are modeled as lower-dimensional entities in the surrounding porous media. The proposed Stokes-Darcy model is validated against the full-dimensional model and experimental data, and it is compared to the hybrid-dimensional Darcy-Darcy model.

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Mesh quality optimization for the virtual element method and applications to discrete fracture networks

Discrete Fracture Networks (DFN) flow simulations are characterized by complex geometries generated by the use of random probability distributions to create the computational domain in geological applications. The generation of a conforming mesh discretization in real DFNs with classic tools often leads to a large number of degrees of freedom (DOFs) to garantee the conformity on fracture intersections. To tackle this problem, the use of Virtual Element Method (VEM) has been proved to be successful in the treatment of tessellations with very generally shaped elements, and therefore highly versatile in the admissible meshes. Moreover, recent studies saw the appearance of a *mesh quality indicator* able to reduce the number of DOFs over a given mesh, maintaining a good quality of the VEM discrete solution. We investigate the application of this indicator on a given DFN, addressing a graph-cut optimization algorithm with a parameter λ regulating the entity of the optimization of the DFN mesh. The algorithm navigates the network and agglomerates neighboring elements trying to maximize the global quality, while respecting conformity constraints along boundaries and fractures. As a result,

the number of elements of the optimized mesh significantly decreases, leading to a drastic reduction of the DOFs and consequently of the VEM solution computational cost. This process is strongly effective in the computation of several solutions over the same domain with a parameter changing at every iteration, *e.g.* time dependent problems or model reduction problems. We show in our experiments how the effects of this reduction is limited on the error magnitude and the convergence order, particularly when working with high order formulations of the method.
This is a joint work with Drs Fabio Vicini, Stefano Berrone, Gianmarco Manzini, Silvia Biasotti and Michela Spagnuolo.

Jan Stebel, jan.stebel@tul.cz, Technical university of Liberec
Continuum-fracture model of hydro-mechanics with contact conditions and nonlinear fracture permeability

The interaction of flow and mechanics in fractured crystalline rocks plays an important role e.g. in the modelling of deep geothermal systems or underground repositories. We present a model of flow and mechanics in domain containing discrete fractures, which can be formally derived from the Biot equations by averaging over the fracture aperture with help of a special tangential and normal calculus. The model takes into account possible anisotropy of the hydraulic conductivity and elasticity tensors. In fractures, we consider the cubic law between the fluid flux and aperture, and the non-penetration contact conditions. The existence analysis will be shown using a fixed-point argument, based on an appropriate splitting of the flow and mechanical part of the problem. Finally we present a discretization based on the mixed-hybrid FEM for the flow part and P1-FEM for the mechanics, with elementwise contact conditions. The discrete problem is then linearized by the fixed-stress splitting. The non-smoothness of the mechanical subproblem leads to a variational inequality formulation, which is solved using quadratic programming methods. The numerical scheme was implemented in Flow123d, a simulator of THMC processes in fractured porous media. Finally, we demonstrate the results of several numerical computations.

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Highly-efficient implementations for flow simulations on DFNs and DFMs

Flow simulations on porous fractured media with Discrete Fracture Networks (DFN) and Discrete Fracture and Matrix (DFM) models require to address two challenging issues: the first, related with the complex geometries generated by the mathematical modeling, the second related to the severe size of the computational domains. In real applications, known probabilistic distributions are used to generate the network of fractures immersed in the porous rock matrix, which leads to critical geometrical features, such as multi-scale distribution of fracture intersection lengths and segments forming very narrow angles. Moreover, networks for practical simulations might count up to several thousands of fractures, making the efficient handling of computational resources mandatory. To overcome these problems we propose a constrained optimization approach which allows the use of non-conforming meshes on fracture intersections and that generates small fracture-local problems which can be solved efficiently with parallel algorithms. Darcy law is used in the numerical tests with different discretization

strategies, such as classical Finite Element (FEM) method and the modern Virtual Element (VEM) framework.

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Simulation of single-phase flows in fractured porous media using the mixed hybrid finite element method

In underground environments, fractures are very numerous and present at all scales, with very heterogeneous sizes. In particular for flows, they are preferential channels: flows are much faster there than in the neighboring rock. Indeed, the permeability of rock is generally about two orders of magnitude lower than that of fractures. This makes fractures play a vital role in a large number of industrial and environmental applications. These particularities of the fractured porous domain make the modeling and simulation of the flows passing through it a major challenge today for which it is necessary to develop dedicated, robust and efficient models and numerical methods.

The most commonly used model for representing fractures is the discrete fracture network (DFN) in which fractures are represented as structures of codimension 1. The model of single-phase flows in fractured porous media is described in [5]. The particularity of the fractured porous problem, compared to the porous-only or fractured-only problem [3], is the coupling between the flow in the fractures and the flow in the rock. Due to the difficulties encountered in taking into account the geometric complexity of large fractured networks in simulations, the test cases recently proposed in the literature are mainly 2D, or 3D with a limited number (about ten) of fractures [1].

In this talk, we will present the nef-flow-fpm solver, which solves the stationary 3D fractured porous problem using the mixed hybrid finite element method. The method developed in the solver is inspired by [4]. To mesh the domain, a first simplical and conforming 2D mesh is generated for the DFN and for the boundaries of the domain, then a second simplical and conforming 3D mesh is generated from the first mesh. The solvers integrated in nef-flow-fpm are direct solvers, like LU and Cholesky, and iterative solvers, like PCG and AMGCL [2]. We validated nef-flow-fpm on the test cases presented in [1]. We will propose new test cases with a larger number of fractures (a few thousand).

Joint work with: Michel Kern*, Géraldine Pichot*, Martin Vohralík*, *Inria SERENA & École des Ponts ParisTech

References

[1] I. Berre et al. Verification benchmarks for single-phase flow in three-dimensional fractured porous media. Advances in Water Resources, 147, 2021.

[2] D. Demidov. AMGCL : An efficient, flexible, and extensible algebraic multigrid implementation. Lobachevskii Journal of Mathematics, 40, 2019.

[3] A. Ern, F. Hédin, G. Pichot, N. Pignet. Hybrid high-order methods for flow simulations in extremely large discrete fracture networks. Pre-print https ://hal.inria.fr/hal-03480570/, 2021.

[4] H. Hoteit, A. Firoozabadi. An efficient numerical model for incompressible two-phase flow in fractured media. Advances in Water Resources, 31(6), 2008.

[5] V. Martin, J. Jaffré, J. E. Roberts. Modeling fractures and barriers as interfaces for flow in porous media. SIAM Journal on Scientific Computing, 26(5), 2005.