Communication on Networks and Strong Reliability

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Algorithmic game theory, mechanism design, and learning
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Introduction

We study sender-receiver games on communication networks, where the sender and receiver are distant nodes.

- There is a sender S and a receiver R.
- The sender knows a payoff-relevant state $\omega \in \Omega$.
- The receiver takes a decision $a \in A$.
- The sender and receiver are two distinct nodes in a communication network $\mathcal{N}$ (an undirected graph), with I a set of intermediaries.
- The payoff to $i \in I \cup \{S, R\}$ is $u_i(a, \omega)$.

The Main Theoretical Question

We ask when it is possible to implement the equilibrium outcomes of the direct communication games as perfect Bayesian equilibrium (PBE) outcomes of indirect communication games, i.e., when the information flows from the sender to the receiver via intermediaries.

Remarks:

- Implementing Bayes-Nash equilibrium outcomes but also communication equilibrium outcomes of direct communication games.
- When does indirect communication helps in achieving outcomes, which wouldn't be achievable with direct and unmediated communication?
- In other words, when can intermediaries emulate the Forges-Myerson mediator?

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Motivation: Organization Theory

In large organizations, such as public administrations, multinational corporations, armed forces, information typically flows through the different layers of the organizations, from engineers, sale representatives, accountants to executives.

Communication is thus indirect.

While indirect communication is necessary in large organizations, this creates the opportunity for individuals to suppress, distort or delay the flow of information so as to achieve their own goals.

Are they organizational arrangements, which mitigate these issues?

Motivation: Matrix Organization

Matrix organizations (or arrangements) mitigate these issues.

Matrix organizations consist in organizing activities along several dimensions, e.g., functions (R&D, accounting, sales), geography (Europe, US, Asia), or products.

Firms such as NASA, IBM, Starbuck, or Siemens have adopted this mode of organizations.

A central feature of matrix organization is *multiple reporting*, i.e., information flows via different channels, mirroring the dimensions along which the organization is structured.

The multiplicity of reporting lines (paths) explains why matrix organization facilitates effective communication.

Model

- There is a sender S and a receiver R.
- The sender knows a payoff-relevant state $\omega \in \Omega$.
- The receiver takes a decision $a \in A$.
- The payoff to $i \in \{S, R\}$ is $u_i(a, \omega)$.

Direct communication game: The sender sends a message $m \in M$ to the receiver, prior to the receiver taking an action $a \in A$.

Let $\mathcal{E}^d$ be the set of (Bayes-Nash) equilibrium distributions over $A \times \Omega$.

Mediated communication game

- The sender sends a message $m \in M$ to a mediator. The mediator then sends a message $r \in R$, possibly randomly, to the receiver, who then takes an action $a \in A$.
- A strategy for the sender is $\sigma : \Omega \to \Delta(M)$.
- A strategy for the receiver is $\tau : R \to \Delta(A)$.
- The mediator follows a recommendation rule: $\varphi : M \to \Delta(R)$.
- A communication equilibrium is a communication device $\langle M, R, \varphi \rangle$ and an equilibrium (σ^*, τ^*) of the mediated game induced by the communication device.
- Thanks to the revelation principle (Forges, 1986 and Myerson, 1986), we can restrict attention to canonical communication equilibria, where $M = \Omega$, $R = A$ and the sender has an incentive to be truthful, and the receiver has an incentive to be obedient.

Let $\mathcal{CE}^d$ the set of communication equilibrium distributions.

Communication equilibria might be Pareto improving

Farrell's example (1988), where both states are equally likely:

(u_S, u_R)	a	b	c
ω	$2, 3$	$0, 2$	$-1, 0$
ω'	$1, 0$	$2, 2$	$0, 3$

- The receiver takes b in all equilibria of the direct communication game (sender's payoff is then 0 or 2, while the receiver's is 2)
- As argued by Myerson (1991), there exists a communication equilibrium where both the sender and the receiver are better off
- The mediator recommends action b at ω' and randomizes uniformly between a and b at ω
- Upon observing a (resp. b), the receiver infers that the state is ω with proba 1 (resp. $\frac{1}{3}$) $\Rightarrow$ incentive to be obedient
- Incentive for the sender to be truthful
- Sender's payoff is 1 or 2, while the receiver's is $\frac{9}{4}$

Indirect communication game

The sender and receiver are two distinct nodes in a communication network $\mathcal{N}$ (an undirected graph).

The set of nodes, other than S and R, is $I := \{1, \ldots, n\}$.

A communication game on the network $\mathcal{N}$ is a multi-stage game, with $T < \infty$ stages.

At each stage, a player can broadcast messages to each possible subset of their neighbors.

Broadcasting: A player broadcasts a message to a subset of neighbors if all receive the same message, and it is a common belief among them.

E.g., face-to-face group meetings, online meetings via Zoom or Microsoft Teams, or Whatsapp groups.

A communication mechanism, denoted $\boldsymbol{\mathcal{M}}$, is the sets of messages players can send to each others.

PBE implementation of direct communication

Definition

PBE implementation of all communication equilibrium (resp. Bayes-Nash equilibrium) outcomes of direct communication games is possible on the network $\mathcal{N}$ if there exists $\boldsymbol{\mathcal{M}}$ on $\mathcal{N}$ such that for all $(u_i)_{i \in I}$, for all distributions $\mu \in \mathcal{CE}^d$ (resp. $\mu \in \mathcal{E}^d$) of all direct communication games, there exists a perfect Bayesian equilibrium σ of the indirect communication game satisfying:

$$\mathrm{marg}_{A \times \Omega} \mathbb{P}_\sigma = \mu.$$

Strong reliability

Alternative question: the sender wishes to transmit the message $m \in M$, a realization of the random variable $\mathbf{m}$ with distribution $\boldsymbol{\nu}$, to the receiver, through the network $\mathcal{N}$.

The transmission of the message is **strongly reliable** on the network $\mathcal{N}$ if we can construct a protocol, i.e., a communication mechanism and a profile of strategies, such that the receiver correctly "learns" the message sent at all terminal histories consistent with at most one intermediary deviating at every stage.

Remark: in computer science, the adversary controls the same k nodes throughout the execution of the protocol

Histories consistent with unilateral deviations

Let $\sigma = (\sigma_{i,t})_{i,t}$ be a profile of strategies, where $i \in \{S, R\} \cup I$.

Define $\Sigma(\sigma)$ as the subset of strategies consistent with at most one intermediary deviating at each stage from σ, that is,

$\sigma' \in \Sigma(\sigma)$ if there exists a sequence of intermediaries $(i_1, \ldots, i_t, \ldots)$ such that

$$\sigma'_t = (\sigma'_{i_t,t}, \sigma_{-i_t,t}),$$

for all t.

Strong reliability

Definition

The transmission of messages is strongly reliable on the network $\mathcal{N}$ is possible if there exist a protocol and a decoding rule $\mathbf{m}_d : H_R^{T+1} \to M$ such that

$$\mathbb{P}_{\sigma'}\left(\left\{h_R^{T+1} : \mathbf{m}_d(h_R^{T+1}) = m\right\}\Big|\mathbf{m} = m\right) = 1,$$

for all $\sigma' \in \Sigma(\sigma)$, for all m.

Main Theoretical Result

Assume that the sender and receiver are not directly connected in the network $\mathcal{N}$.

Theorem

The following statements are equivalent.

(i) *PBE implementation of all communication equilibrium outcomes of direct communication games is possible on the network $\mathcal{N}$.*

(ii) *PBE implementation of all Bayes-Nash equilibrium outcomes of direct communication games is possible on the network $\mathcal{N}$.*

(iii) *The transmission of messages is strongly reliable on the network $\mathcal{N}$.*

(iv) *There are (at least) two disjoint paths of communication between the sender and the receiver in the network $\mathcal{N}$.*

Related Literature

Computer Science: Resiliency or Reliability on (wired) networks. Dolev et al (1993) or Franklin and Wright (2002).

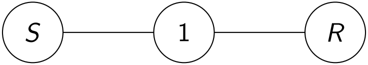
Do not require our strong reliability, but do not allow for our rich communication possibilities (broadcast, unicast and anything in between).

Game Theory: Repeated Games on Networks. Ben-Porath and Kahnemann (1996), Renault and Tomala (2004, 2008), Laclau (2012, 2014), Wolitzky (2015), among others.

Game Theory: Mediated and Unmediated Games. Aumann (1974), Barany (1983), Forges (1986), Myerson (1986), Ben-Porath (1998), Gerardi (2004), among others.

Either no networks or cannot deal with sequential rationality.

Necessity of (iv): Idea



Intermediary 1 controls all the information received by the $R \Rightarrow$ for some utility functions, he will always prevent R from learning the message from S.

Main Theoretical Result

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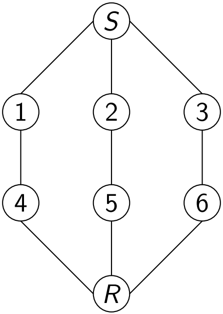
(iv) *There are (at least) two disjoint paths of communication between the sender and the receiver in the network $\mathcal{N}$.*

A First Look at the Difficulties (iv) $\Rightarrow$ (iii)

The sender wants to truthfully report the state.

Naive idea: use a majority argument.

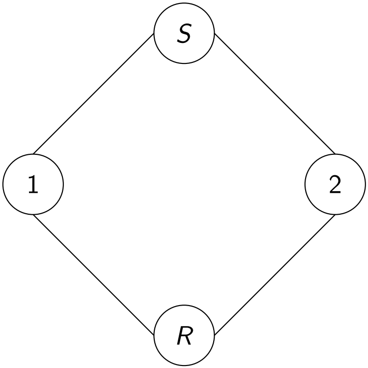


Problem: inconsistent messages.

If S sends ω, 1 reports ω' and 5 reports ω'', R observes $(\omega', \omega'', \omega)$.

If S sends ω', 3 reports ω and 5 reports ω'', R also observes $(\omega', \omega'', \omega)$.

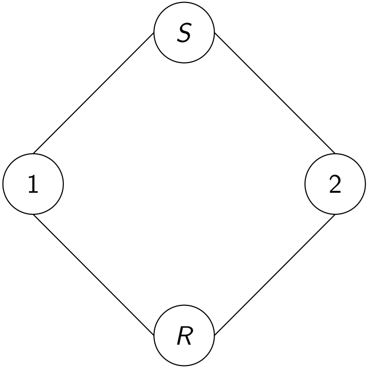
Strong Reliability: A (Simplified) Protocol



Communication protocol in 6 stages:

- At $t = 1$, S broadcasts m.
- At all stages $t = 2, \ldots, 6$:
  - 1 and 2 broadcast m,
  - 1 and 2 broadcast x_1^t and x_2^t uniformly drawn in $[0, 1]$: authentication keys,
  - if 1 (resp., 2) does not broadcast m at t, S broadcasts $(1, t, x_1^t)$ (resp., $(2, t, x_2^t)$) at stage $t + 1$,
  - 1 and 2 broadcast the keys received by S,
  - if S, 1 or 2 does not broadcast the key at stage t when he was supposed to so, he broadcasts it at stage $t + 1$.

Strong Reliability: A (Simplified) Protocol



Decoding rule of R:

- If R receives $m_1^t = m_2^t$ at some stage $t \geq 2$, then he decodes it as the correct message, and takes an action accordingly.

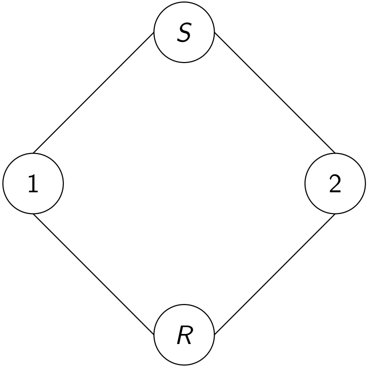
- Otherwise, at the end of stage 6, R analyzes his messages:

  - if at stages $t_1 < t_2 < t_3$, he receives three times the (same) message m_i from $i \in \{1, 2\}$,

  - and if he did not receive before t_3 the authentication key $x_i^{t_1}$ of i,

$\Rightarrow$ then m_i is decoded as the correct message, and R takes his action accordingly.

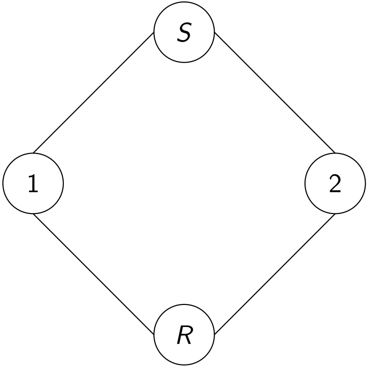
Strong Reliability: A (Simplified) Protocol



Why does it work?

- Assume that m_1 is sent by 1 at stages t_1, t_2 and t_3
- If m_1 is not the correct message sent by S, then
  - 1 deviated at stages t_1, t_2 and t_3
  - Hence, S and 2 were not deviating at stages t_1, t_2 and t_3.
  - Therefore, S broadcasts $(1, t_1, x_1^{t_1})$ at or before stage t_2, and 2 broadcasts it at or before stage t_3.
  - Therefore, R does not decode m_1 as the correct message

Strong Reliability: A (Simplified) Protocol



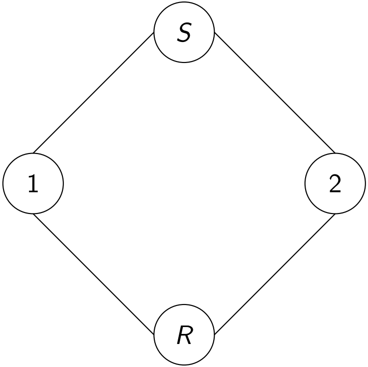
Why does it work?

- If m_1 is the correct message sent by S, then
  - either S never deviates and R does not receive $x_1^{t_1}$ before t_3, since the probability of 2 guessing correctly the key $x_1^{t_1}$ is zero.

    $\Rightarrow$ R decodes m_1 as the correct message
  - or S deviates at some stage t, but then neither 1 nor 2 deviates at stage t under unilateral deviations and both send $m_1 = m_2 = m$

    $\Rightarrow$ R decodes m_1 as the correct message

Strong Reliability: A (Simplified) Protocol



Why does it work?

- If S deviates at some stage $t \Rightarrow R$ learns the correct message at stage t.
- If S never deviates, there must exist a sequence of three stages (t_1, t_2, t_3), where 1, or 2, broadcast three times the correct message, since
  - whenever 1 (resp., 2) does not broadcast the correct message, 2 (resp., 1) does,
  - and there are 5 stages of reporting.

Strong Reliability: The General Case

The two disjoints paths: $(S, i_1, \ldots, i_k, \ldots, i_{\bar{k}}, R)$ and $(S, j_1, \ldots, j_\ell, \ldots, j_{\bar{\ell}}, R)$.

$n_C := 2 + \bar{k} + \bar{\ell}$ nodes on these two paths.

Do blocks of length $2n_C - 3$. (In the above example, $n_C = 4$, so $2n_C - 3 = 5$.)

At each block, apply the same logic as above, with roughly the same protocol, i.e., checking sequences of messages of length $n_C - 1$.

But, at each block, the role of the "receiver/decoder" is played by a different node, started initially with i_2 or j_2.

We show at the end of a block a new node necessarily learns the message sent by the sender.

After sufficiently many blocks, R then learns the message.

Emulating the Mediator (iii) $\Rightarrow$ (i)

- We can use our protocol to (PBE) implement all distributions over actions and states of the *mediated* communication games between the sender and the receiver, as follows:

  Phase I: Let S truthfully report the state ω to intermediaries 1 and 2.

  Phase II: Replication of the communication device: Let S and 1 simultaneously draw random numbers x and y. Use our protocol to guarantee that 1 transmit y to 2 (and S). S transmits x to 1 and 2.

  Phase III: Use x and y to output a recommendation a; S, 1 and 2 do it simultaneously and then transmit it to R, using three times our protocol in parallel. Use a majority rule for R to take a decision.

Thank you!