Stability and Learning in Strategic Queueing Systems

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In recent years, economists and computer scientists have collaborated with mathematicians, operations research experts, and practitioners to improve the design and operations of real-world marketplaces. Such work relies on robust feedback between theory and practice, inspiring new mathematics closely linked – and directly applicable – to market and mechanism design questions. This cross-disciplinary program seeks to expand the domains in which existing market design solutions can be applied; address foundational questions regarding our ways of developing and evaluating mechanisms; and build useful analytic frameworks for applying theory to practical marketplace design.

Motivation: Games and Price of Anarchy

Consider agents interacting in some game

- Main questions: What kinds of outcomes arise? How "good" are they?
- Price of Anarchy: worst-case gap between social welfare at Nash compared to social optimum

In many/most interactions are repeated interactions and participants use learning to figure out what works

- What do we mean by learning?
- What form of learning guarantees good outcome?
- What can we say about outcome of learning?

Example 1: traffic routing

- Traffic subject to congestion delays
- cars and packets follow shortest path
- Congestion game =cost (delay) depends only on congestion on edges

High Social Welfare: Price of Anarchy in Routing

Theorem (Roughgarden-T'02):

In any network with continuous, non-decreasing cost functions and very small users

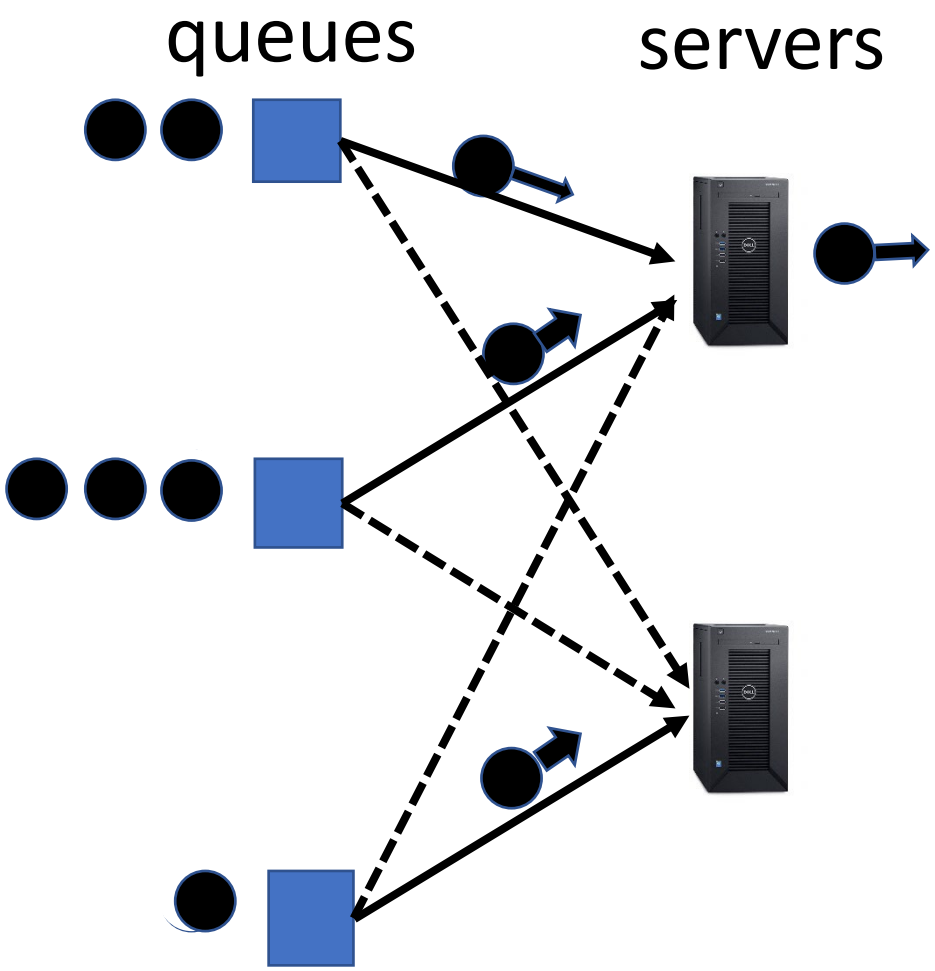
cost of Nash with rates r_i for all i	$\leq$	cost of opt with rates $2r_i$ for all i

Nash equilibrium: stable solution where no player had incentive to deviate.

Extra resource needed to guarantee good outcome at Nash

$$\text{Price of Anarchy} = \frac{\text{cost of worst Nash equilibrium}}{\text{"socially optimum" cost}}$$

Example 2: serving packets



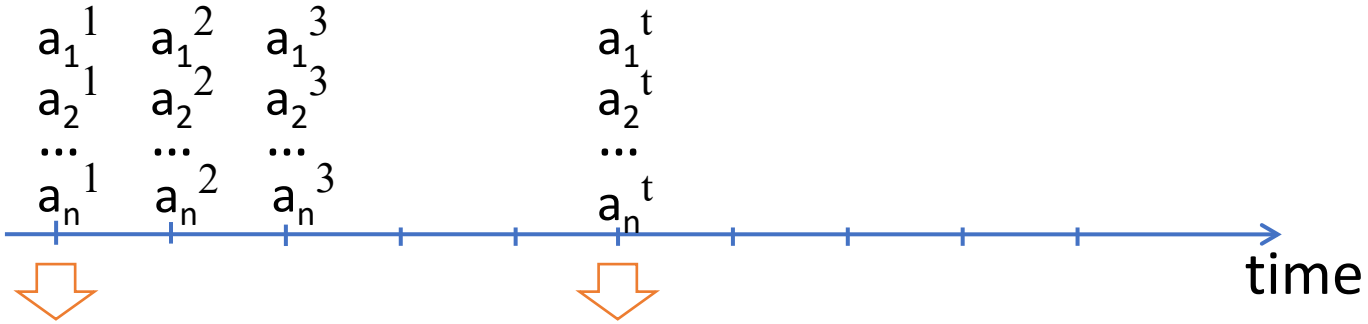
- Stream of packets that need serving
- servers have limited capacity

- Drop (or return) unsent packets, that need to get resend

Example 3: ad-auctions

- Repeatedly bidding for impression
- Limited by a budget for longer period

Learning outcome: No-regret without stability (Hannan consistency)



Simple behavioral assumption: no-regret learning:

do at least as well as any single action with hindsight

for each player i and each fixed action x:

$$\sum_t u_i\left(x, a_{-i}^t\right) \leq \sum_t u_i(a^t) + o(T)$$

Quality of Learning Outcomes: Price Anarchy No-regret/Hannan consistency as a behavioral assumption

Price of Anarchy [Koutsoupias-Papadimitriou'99]

$$PoA = \min_{a\ Nash} \frac{\sum_i u_i(a)}{Opt}$$

Assuming no-regret learners in stable game: [Blum, Hajiaghayi, Ligett, Roth'08, Roughgarden'09]

$$PoA = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} \sum_i u_i(a^t)}{T\ Opt}$$

Extends to even to repeated games with **dynamically changing population** assuming no-regret learners : [Lykouris, Syrgkanis, T'16]

→ No regret is a natural and strong enough assumption on what learners achieve

Social Welfare of Learning Outcomes

Critical Assumption: new copy of the same game is repeated (no carryover effect between rounds other than through learning)

Is this reasonable?

Large population games: traffic routing

Morning rush-hour traffic

✔ No carryover effect (except through the learning of the agents)

Second-by-second packet traffic

✖ Packets take time to clear, dropped packets need to be resent in the next round

Price of Anarchy in Stateful Systems

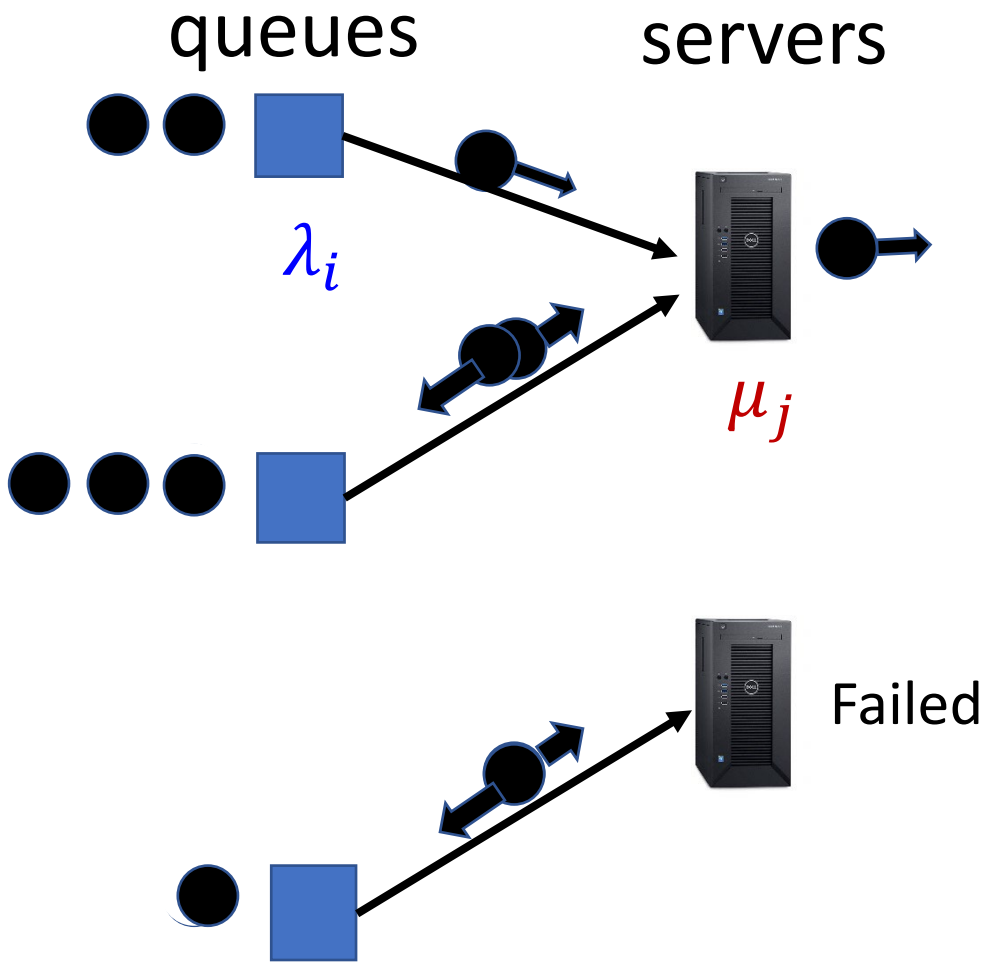
- Not as well understood: do PoA-style bounds still hold with dependence between games in each round?

Questions for queuing application:

- How much extra capacity ensures good system performance despite selfish users
- Is no-regret learning a good enough way to learn in presence of dependence between rounds

Simple Model of Queuing

- Queue i gets new packets with a Bernoulli process with rate λ_i
- Server j succeeds at serving a packet with probability μ_j
- Each time step: each queue can send **one** packet to one of the servers to try to get serviced
- **Server can process at most one packet and unserved packets get returned to queue**
- Servers attempt to serve oldest packet

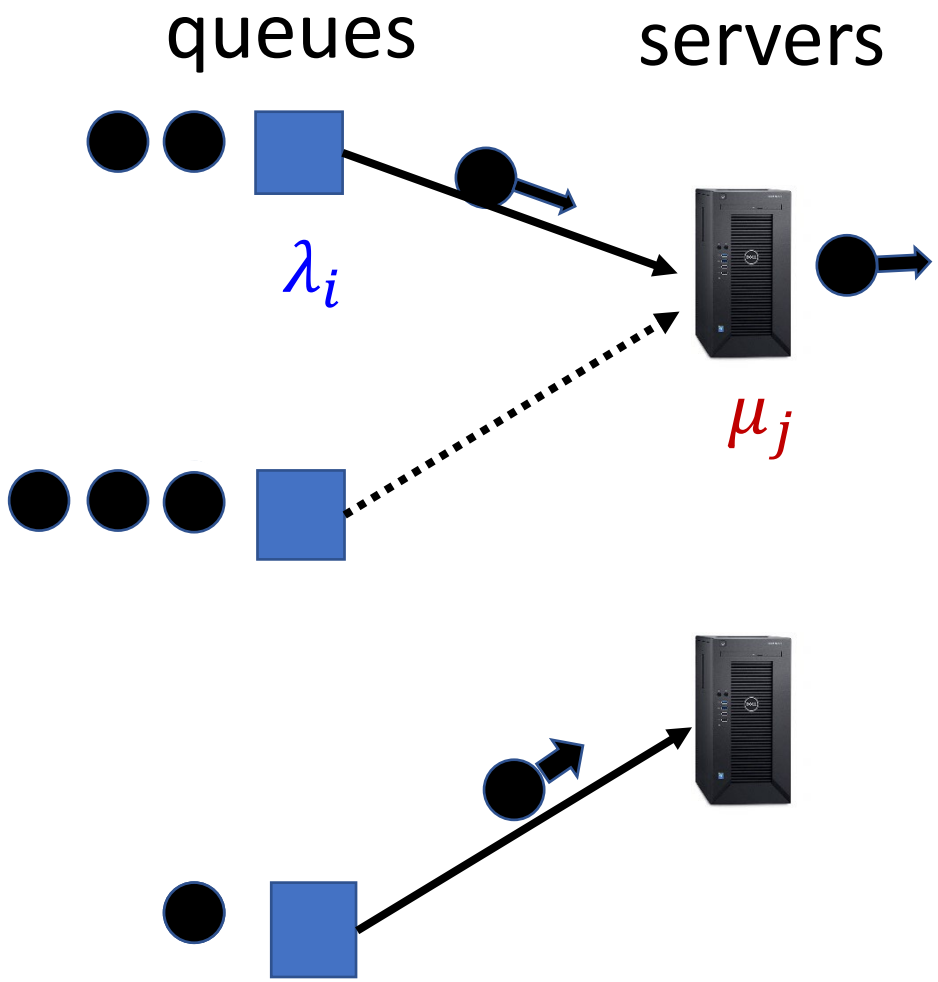


Baseline Measure: Coordinated Queues

Assume queues and servers are sorted:

$$1 > \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$$

$$1 \geq \mu_1 \geq \mu_2 \geq \cdots \geq \mu_m > 0$$



Theorem 0: necessary/sufficient condition for centralized stability: for all k,

$$\sum_{i=1}^{k} \lambda_i < \sum_{i=1}^{k} \mu_i$$

(Recall can only send one packet each round)

Selfish Queuing: Price of Anarchy

Theorem 1 [Gaitonde-T]: if queues use no-regret algorithms to select servers and for all k,

$$\sum_{i=1}^{k} \lambda_i < 0.5 \sum_{i=1}^{k} \mu_i$$

Then queue lengths/ages remain bounded in expectation.

Theorem 2 [Gaitonde-T]: If queues choose servers patiently, and for all k

$$\sum_{i=1}^{k} \lambda_i < 0.63 \sum_{i=1}^{k} \mu_i$$

then in every equilibrium, queue lengths/ages grow sublinearly. $0.63 = (e-1)/e$

Plan for rest of the time

- Outline of the proof of Theorem 1:
  why is no-regret a useful condition
- Limitation of no-regret:
  example showing no-regret can be too myopic
- Theorem 2 comments:
  worst case example
- Directions of further research

Theorem 1 Proof Idea (using no-regret)

Technical tool: [Pemantle, Rosenthal '04]: random process satisfying

i. Sufficiently regular
ii. Negative drift when large

remains bounded in expectation for all times

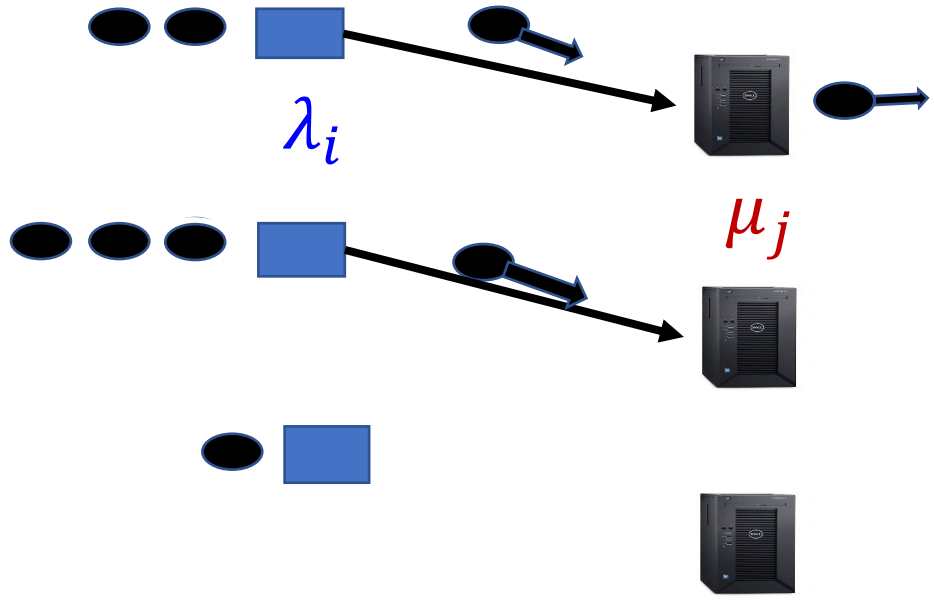
Use potential function $\sqrt{\Phi}$ where $\Phi \approx \sum_\tau \Phi_\tau$
with $\Phi_\tau =$ # packets aged τ or older in the system

Φ remains bounded $\rightarrow$ all queues remain bounded (in expectation)

- No-regret + factor 2 slack $\rightarrow$ negative drift when queues have large backup

Why Φ and How No-Regret Helps

- Look at queues with packets at least τ-old; they have priority
- Fix long window and look at best servers
- Either: i) many τ-old queues send there throughout window ➔ Φ_τ decreases by a lot



Why Φ and How No-Regret Helps

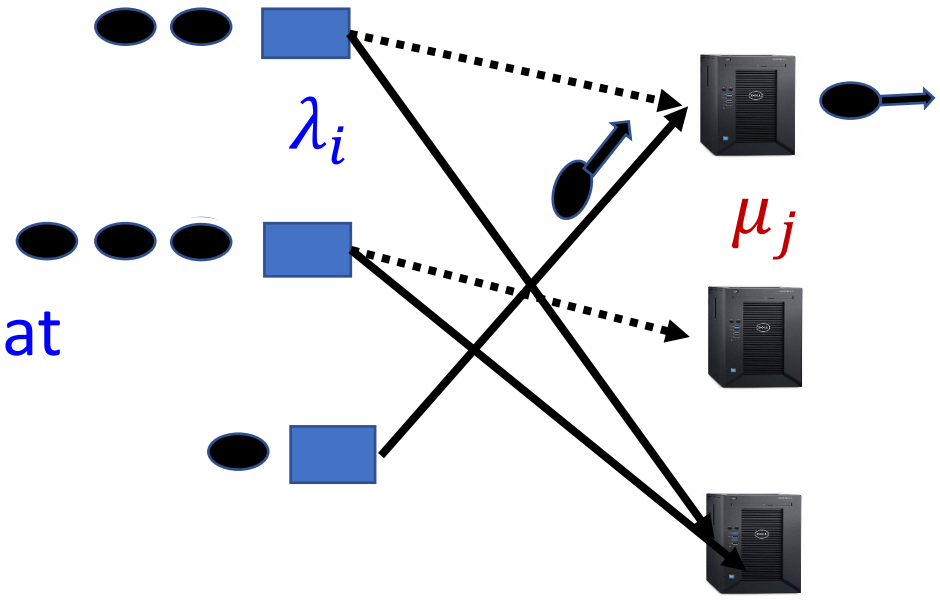
- Look at queues with packets at least τ-old; they have priority
- Fix long window and look at best/fastest servers
- Either: i) many τ-old queues send there throughout window ➔ decrease in queue size, OR

ii) they do not:

had priority there so no-regret kicks in:

Any queue with τ-old packets would have regret, unless it managed to get service for at least this much!

Apply at all thresholds τ simultaneously to get no-regret at all scales ➔ implies negative drift

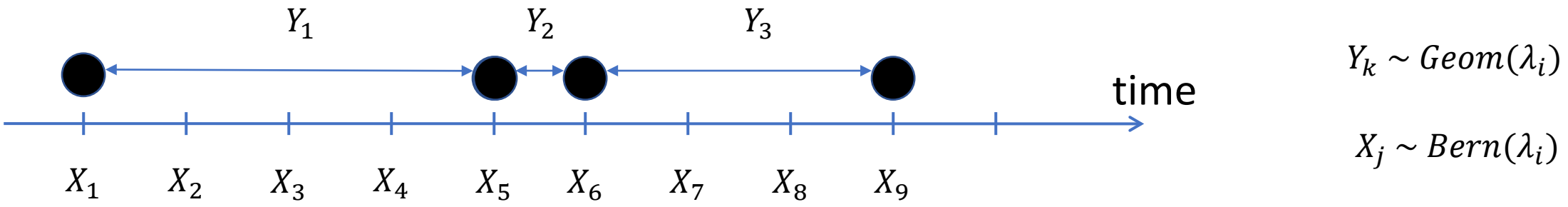


Extra Technical Details

- No-regret needs to hold with high-probability

  unlikely bad situations **will** happen, need to be able to recover

- Pemantle/Rosenthal needs "sufficiently regular" = bounded moments:
- use model with deferred decisions, study *ages* instead of *sizes:* age of oldest packet T_i^t in queue i



Extra Technical Details (3)

Potential: $\sqrt{\Phi}$ where $\Phi = \sum_\tau \Phi_\tau$
with $\Phi_\tau =$ # packets aged τ or older in the system

With deferred decision: $\boldsymbol{\Phi_\tau = \sum_i \lambda_i (T_i^t - \tau)}$
= expected # of packets age τ or older, given oldest in each queue

Contribution of one que to $\Phi = \sum_{\tau < T_i^t} \lambda_i \left(T_i^t - \tau\right) \approx \lambda_i (T_i^t)^2/2$

Clearing one packets decreases T_i^t by $Y \sim Geom(\lambda_i)$,
expected decrease is $1/\lambda_i$

Selfish Queuing: Price of Anarchy

Theorem 1 [Gaitonde-T '20]: if queues use no-regret algorithms to select servers and for all k,

$$\sum_{i=1}^{k} \lambda_i < 0.5 \sum_{i=1}^{k} \mu_i$$

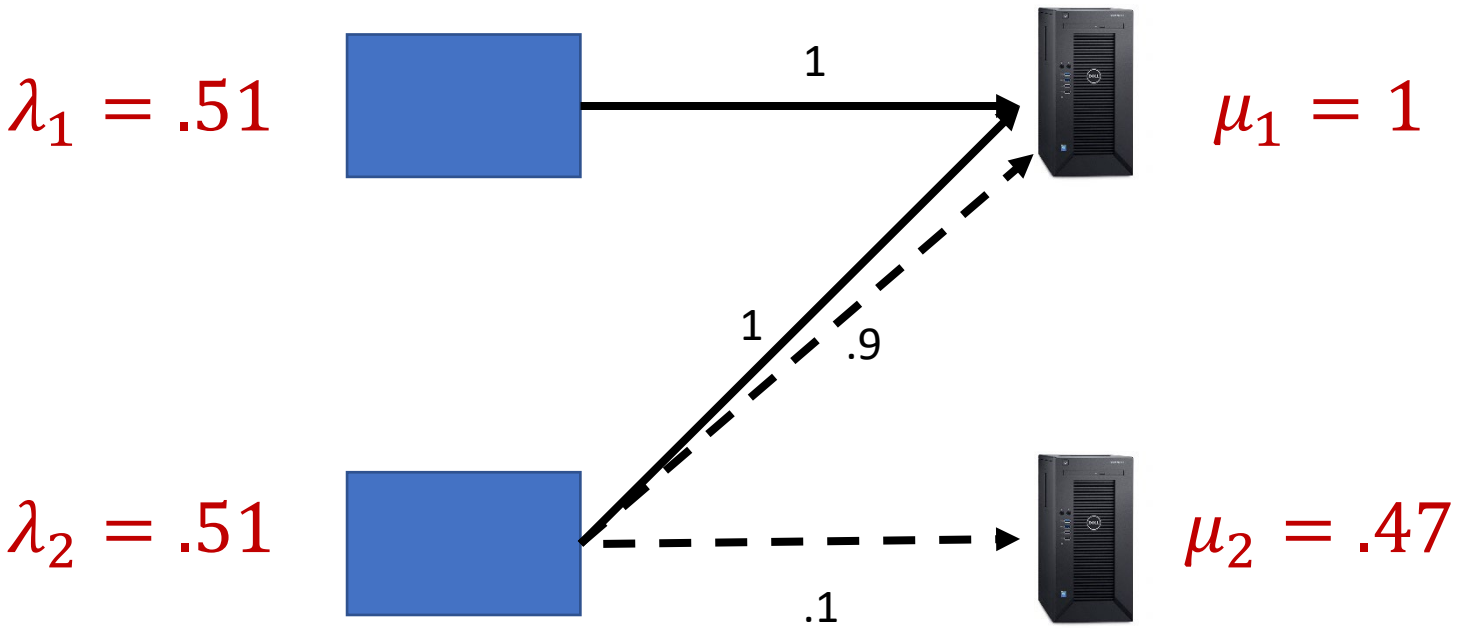
Then queue lengths/ages grow sublinearly, **and this bound is tight**.

Theorem 2 [Gaitonde-T'21]: If queues choose servers patiently, and for all k

$$\sum_{i=1}^{k} \lambda_i < 0.63 \sum_{i=1}^{k} \mu_i$$

then in every equilibrium, queue lengths/ages grow sublinearly. $0.63 = (e-1)/e$

No-regret is myopic: Example



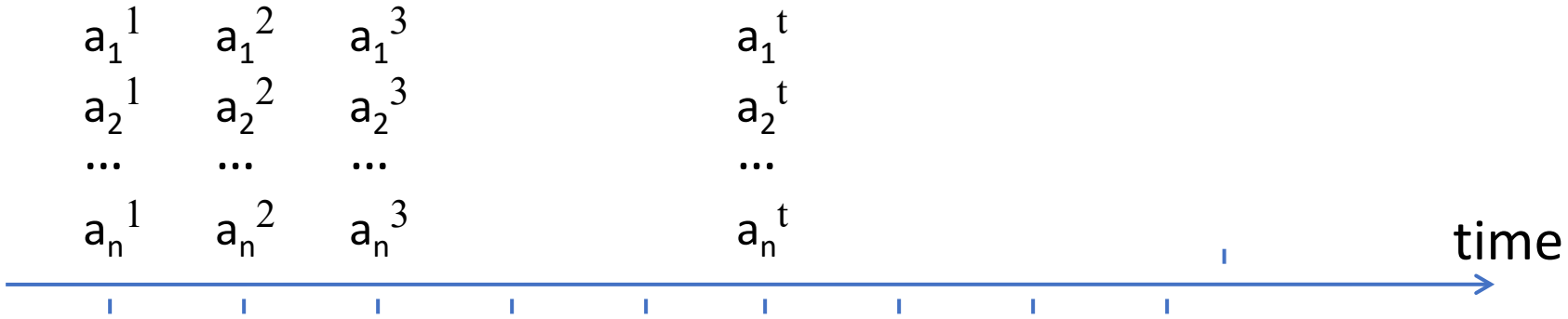
- Both sending to top server gets .5 rate and so has no-regret
- Age/split top server equally → linear growth
- Deviating gives regret

- **Moving to inferior server selfishly helps:**

  Why? Helps top queue clear, indirectly helping both queues clear!

But the 0.47 rate causes regret!

No-regret learning (Hannan consistency)



No regret: for any fixed action x:

regret

$$\sum_t u_i(a^t) \geq \sum_t u_i(x, a_{-i}^t) \; -error$$

Algorithmic idea for no-regret: explore-exploit

If outcome also depends on history:

$$\sum_t u_i(a^{1:t}) \geq \sum_t u_i((a_i^{1:t-1}, x), a_{-i}^{1:t}) - \mathrm{o(T)}$$

What's Going On?

- What we do: evaluate alternate outcome without considering long-term effect of the change

$$\sum_t u_i(a^{1:t}) \geq \sum_t u_i\left(\left(a_i^{1:t-1}, x\right), a_{-i}^{1:t}\right) - o(T)$$

Too myopic: not patient enough to see long term benefit of "bad" servers:

- What we may want (?):

$$\sum_t u_i(a^{1:t}) \geq \sum_t u_i\left(x^{1:t}, a_{-i}^{1:t}\right) - o(T)$$

- We study the patient queuing game with stationary strategies

Patient Queuing Game

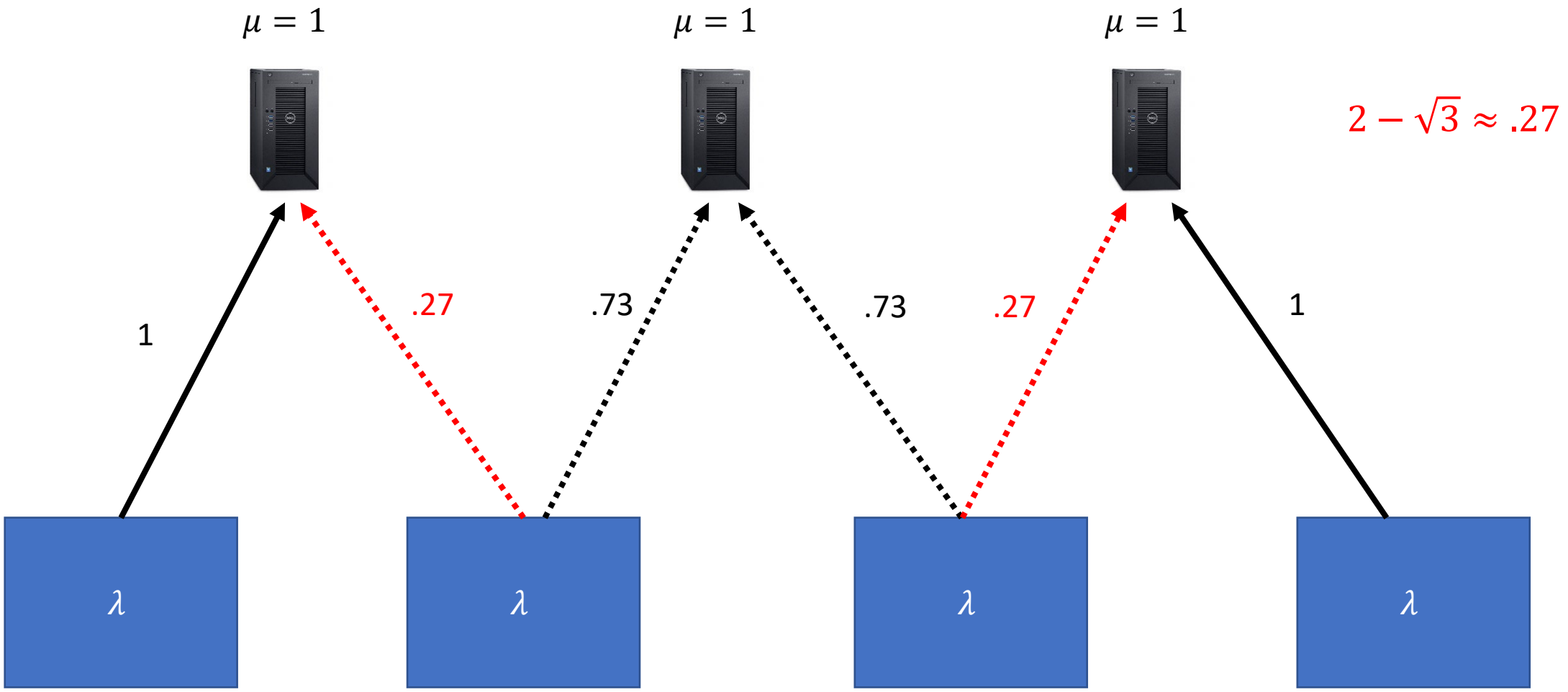
- Each queue i picks fixed randomization over servers, $p_i \in \Delta^{m-1}$ to be played in every round it has a packet
- Induces a Markov chain of queue ages
- Each queue i aims to minimize expected asymptotic aging rate, i.e.

$$\limsup_{T \to \infty} \frac{\mathbb{E}[Age\ of\ Queue\ i\ at\ Time\ T]}{T}$$

  when running Markov chain with randomizations $p_1, \ldots, p_n$

- [Gaitonde-T'21]: Nash equilibria of this game

Example of a Nash equilibrium



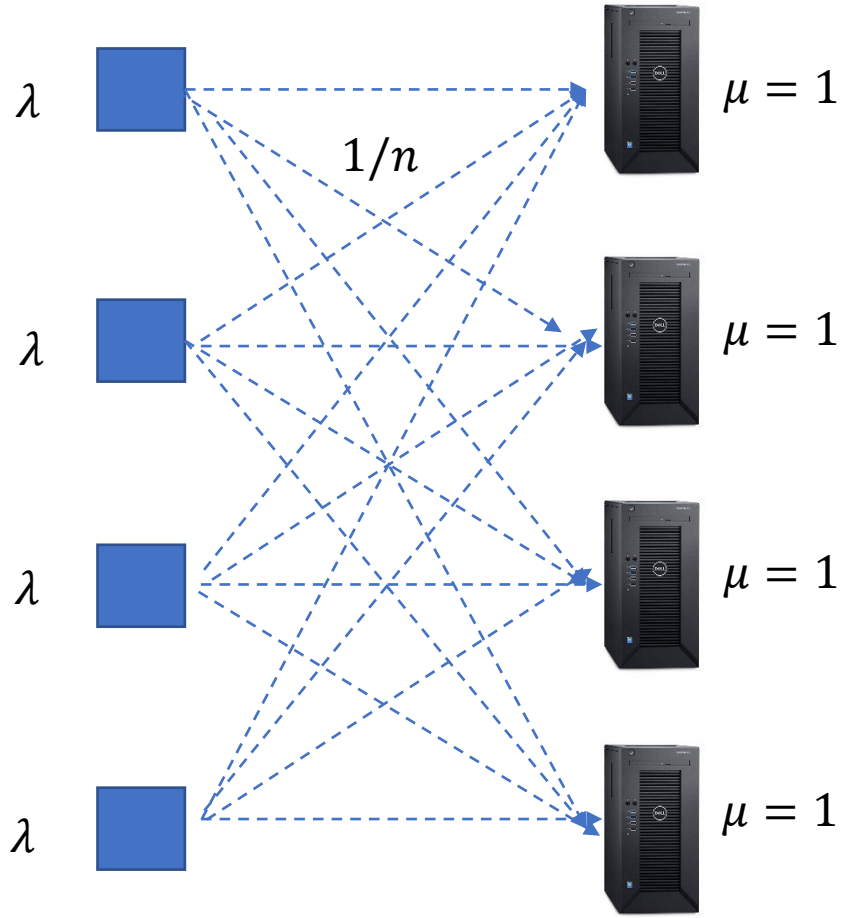
All ques clear if $\lambda \leq \frac{3-\left(2-\sqrt{3}\right)^2}{4} \approx .73$

Patient Queuing Game

- Immediate problems:
  - What are the asymptotic aging rates?
  - Why should there exist an equilibrium?
  - Price of anarchy?

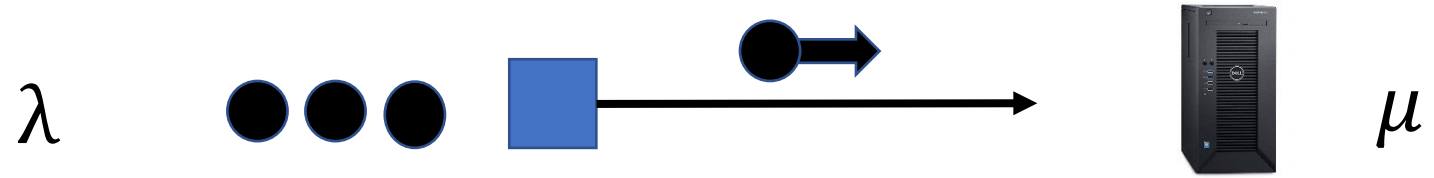
Price of Anarchy

- Worst-case (intuitively): n equal queues, n servers with rate 1, uniform mixing → worst case needs at least $\frac{e}{e-1}$ slack
- In general: fastest-aging queue cannot benefit from deviation at equilibrium, but not clear *why*
- *Queue incentives can come from many tight subsets!*



Long-Run Limits: One Queue

- Example: one queue, one server (no strategies, no competition)



- Ages by one deterministically
- Clears packet with probability μ
- Expected decrease in age $1/\lambda$ if server succeeds

→ Long-run aging rate should be

$$max\left\{0\,,1\,-\,\frac{\mu}{\lambda}\right\}$$

Long-Run Limits: General Case

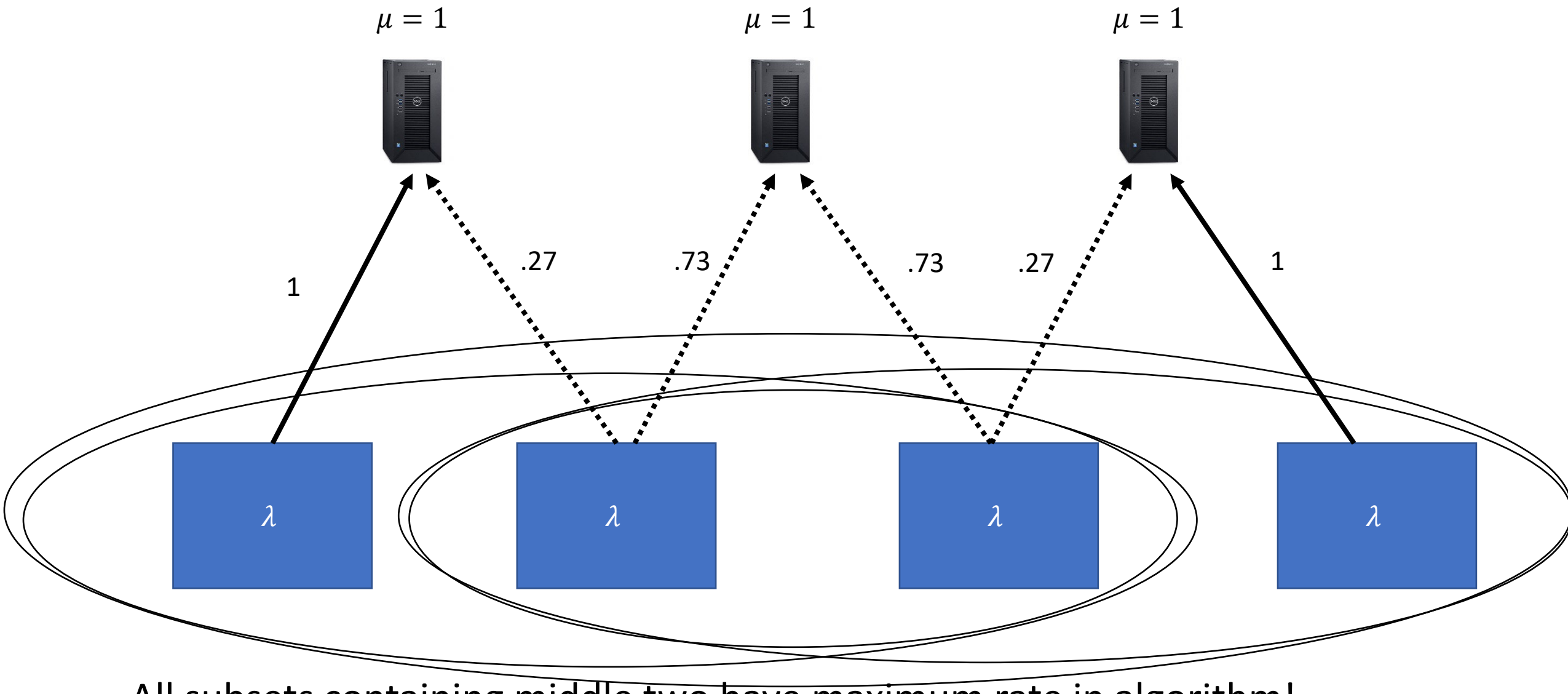
- Theorem [Gaitonde-T]: For fixed $p_1,\ldots, p_n$, queues almost surely cluster into groups with same long-run aging rate
- Fastest aging subset S_1 given by

$$S_1 = argmax_{S\subseteq[n]}\left[1 - \frac{\mu(S)}{\lambda(S)}\right]$$

where $\lambda(S) = \sum_{i\in S}\lambda_i$ = combined arrival rate of subset S,

$\mu(S) =$ expected # packets cleared by S when having priority

- Recurse on rest of queues by discounting servers by probability any queue in S_1 sends to server



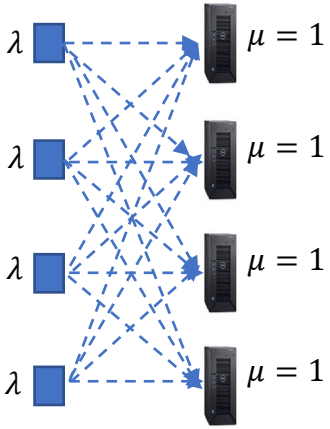
All subsets containing middle two have maximum rate in algorithm!

Structural Properties:

- set of subsets with maximum rate at each step is closed under union and intersection
- Aging rates continuous in $p_1,\ldots, p_n$
- With these costs, ∃ Nash equilibria (Kakutani's theorem)
- Proof long-run rates as given relies heavily on concentration and careful accounting of priorities

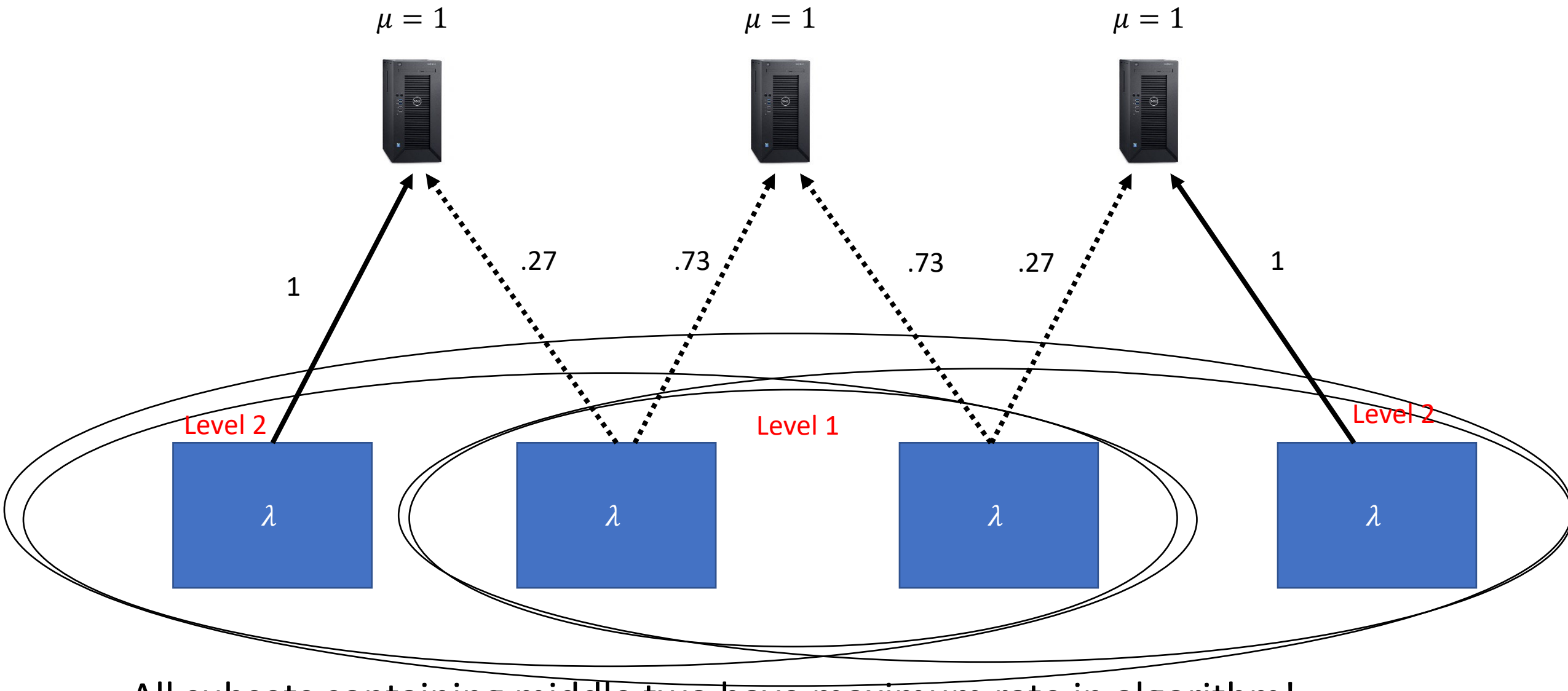
Price of Anarchy: Proof Sketch

- Idea: continuously deform a Nash profile to one with "proportional" loads on each server
  - Given total load on each server, worst case is symmetrized profile
- Want to show deformation process only increases aging rate via Nash property
- Key idea: use structural properties (closure under unions and intersection) of tight subsets



Price of Anarchy: Proof Sketch

- Tight subsets inside S_1 (fastest-aging group) form levels
- Queue incentives essentially determined by level



All subsets containing middle two have maximum rate in algorithm!

Price of Anarchy: Proof Sketch

- Tight subsets inside S_1 (fastest-aging group) form levels
- Queue incentives essentially determined by level
- Shift all highest-level queues first towards symmetrized profile

→ Succeeds directly from Nash definition

- Inductively proceed to next level

→ Can show this schedule preserves relevant incentives from Nash

- Bound aging rate at final symmetrized profile

Conclusions

Learning players achieve high social welfare many games (even some with carryover effect),

- but no-regret learning can be too simplistic: can one obtain better bounds using better learning algorithms?

What may be a good learning method for the players?

My first guess: learning with low policy regret

Sentenac, Boursier, Perchet, NeurIPS'21: not true

- Offers cooperative solution

Maybe more natural learning processes do better (even just no-regret)?