Nash equilibria of the pay-as-bid auction with supply functions

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Workshop "Algorithmic game theory, mechanism design, and learning"
8-11 November 2022

Outline

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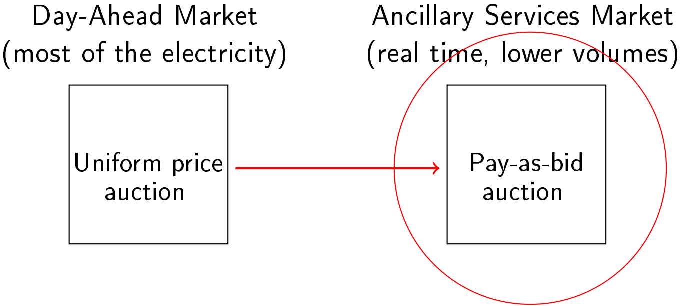
Electricity Markets

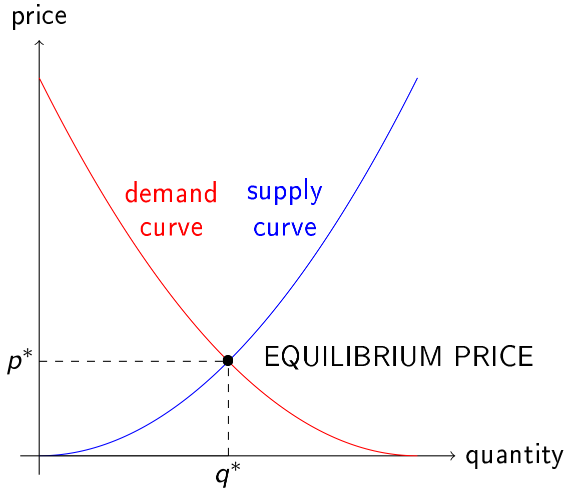
- **Electricity** cannot be stored (if not in a negligible way)
- **Dispatching**: instant by instant management of the electricity flows that pass through the transmission network (continuous balance)



Ancillary services (only producers participate):

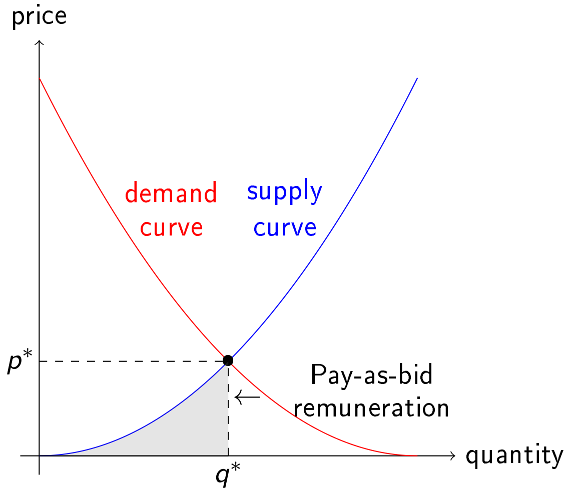
- Resolution of congestion in the network
- Creation of reserve margin
- Real-time balancing

Uniform price auction vs pay-as-bid auction



- **Uniform-price auction**: everybody pays/is paid the equilibrium price p^* (marginal price)

Uniform price auction vs pay-as-bid auction



- **Uniform-price auction**: everybody pays/is paid the equilibrium price p^* (marginal price)
- **Pay-as-bid auction:** bidders pay/are paid the bid price.

Motivation for the study of the pay-as-bid auction

1. Used in ancillary services markets (only producers)
2. Very little literature
3. Alternative to the uniform-price auction

"For many experts, one of the strategic mistakes made by Europe was that of having validated a pricing system based on the marginal price [...]. There would be an **alternative**, and it is represented by the **pay-as-bid system** [...]. Pay-as-bid is not a mechanism unknown to the electricity markets: in various countries it governs the **balancing markets** [...]."
(1st September 2022)


SEZIONI CERCA HUFFPOST ABBONATI ACCEDI

La crisi energetica: una drammatica partita a scacchi tra nemici. Tutto quel che c'è da sapere perché sia l'Europa a vincere

/ di Renato Brunetta

La Russia si è dimostrata abilissima, si comporta come il gatto con il topo, mentre l'Ue sembra aver compreso soltanto negli ultimi giorni l'urgenza di cambiare schema. Si riparta dalla lezione di Gerry Kasparov: "La più grande capacità negli scacchi risiede nel non consentire all'avversario di mostrarti ciò che è capace di fare"

01 Settembre 2022 Aggiornato alle 10:44

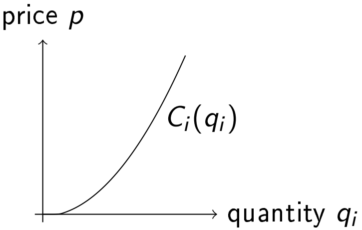
Segui i temi

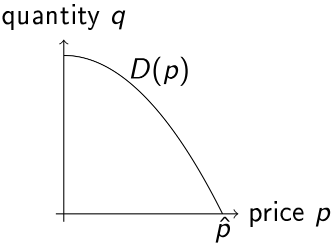
gas russia energia

Outline

Markets as games

- *Agent set:* n asymmetric producers
- ⋆ production costs C_i with $C_i' \geq 0$, $C_i'' \geq 0$
- ⋆ aggregate demand function D with $D' < 0$, $D'' \leq 0$
- Fundamental assumption: production costs and demand function are common knowledge





- *Strategy space*:
- ⋆ Supply Function model: producers bid a *supply function* relating price to quantity.

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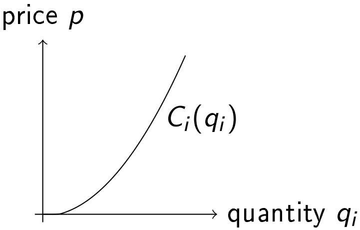

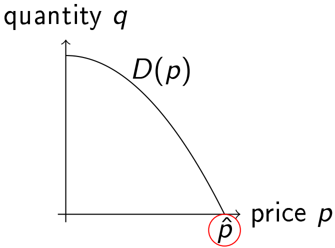

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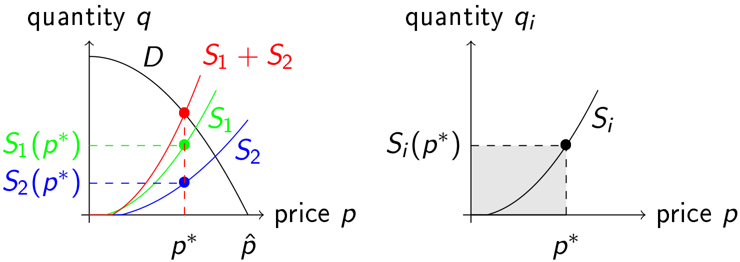
Supply Function model for the uniform-price auction

- Producers bid supply functions $S_i(p)$, S_i continuous and non-decreasing, $S_i(0) = 0$
- Equilibrium price:

$$D(p^*) = \sum_i S_i(p^*)\,.$$

- Utility with **uniform price:**:

$$u_i(S_i(\cdot), S_{-i}(\cdot)) = p^* S_i(p^*) - C_i(S_i(p^*))\,.$$



Related literature: Klemperer and Meyer (1989) *Supply function equilibria in oligopoly under uncertainty*, Green and Newbery (1992), *Competition in the british electricity spot market*.

Supply function model for the pay-as-bid auction

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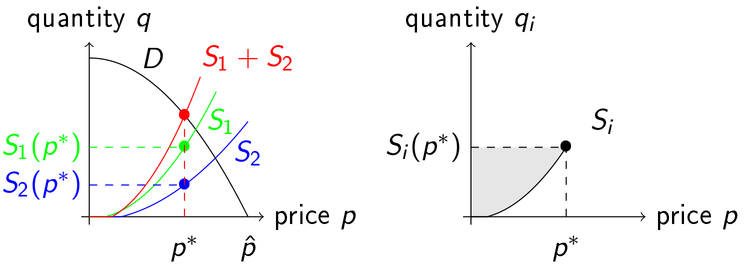

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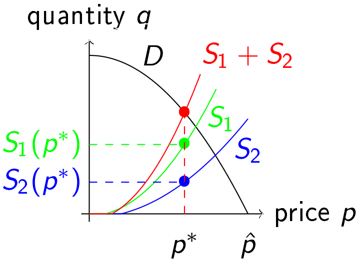

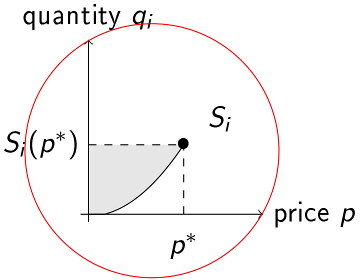

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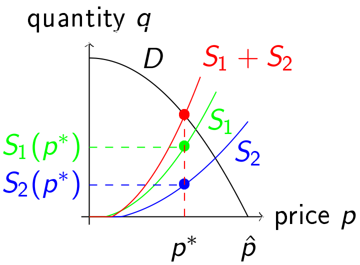

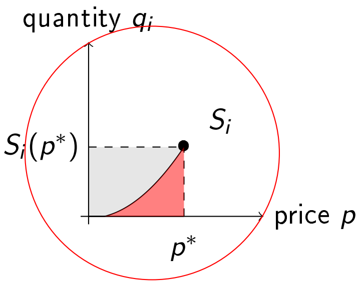

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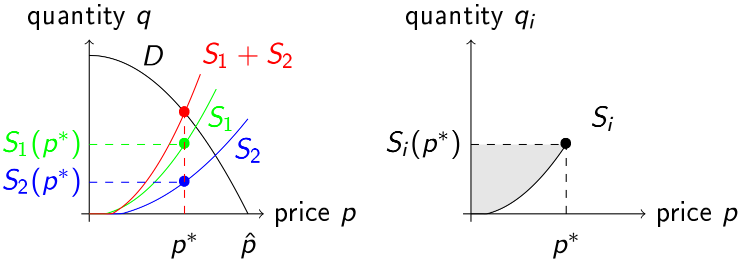


Goal: existence and characterization of Nash equilibria of the pay-as-bid auction

The choice of the strategy space

Proposition

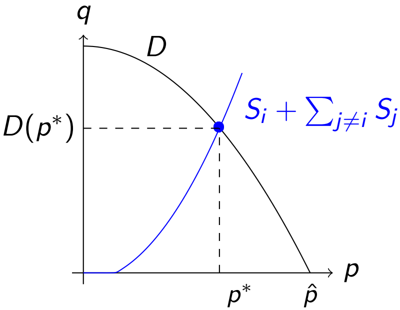
- *Pay-as-bid (PAB) auction*
- $\mathcal{A} = \{S \in C^0, S_i(0) = 0,\ S \text{ non-decreasing}\}$.

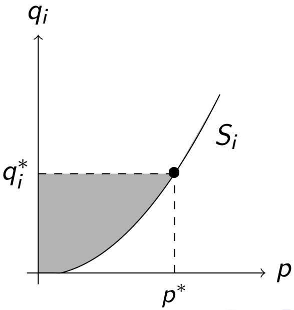
Then, for every S_{-i},

$$\mathcal{B}_i(S_{-i}) = \emptyset\,.$$

No best-response ⇒ No Nash equilibria

Idea of proof: we fix S_{-i} and consider any $S_i \in \mathcal{A}$





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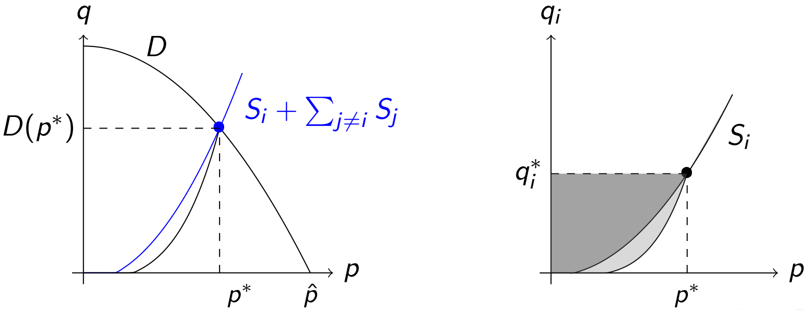
No best-response $\Rightarrow$ No Nash equilibria

Idea of proof: we fix S_{-i} and consider any $S_i \in \mathcal{A} \rightarrow \exists \tilde{S}_i \in \mathcal{A}$ leading to higher utility.



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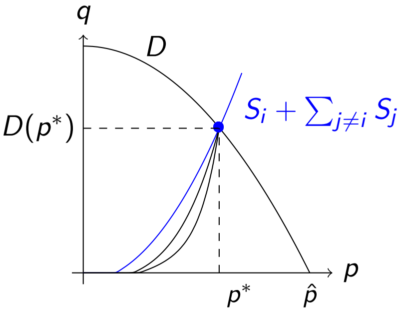

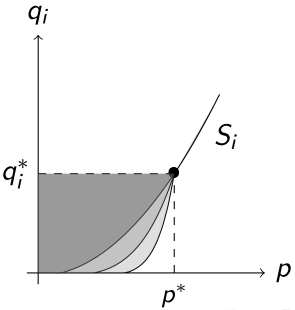

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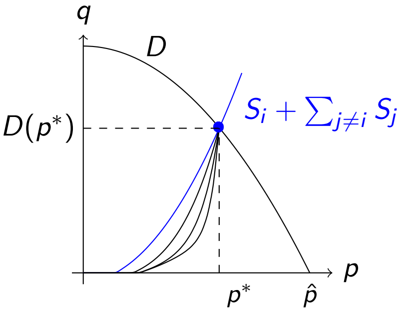

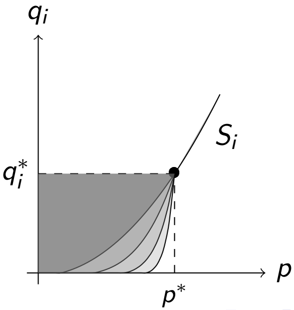

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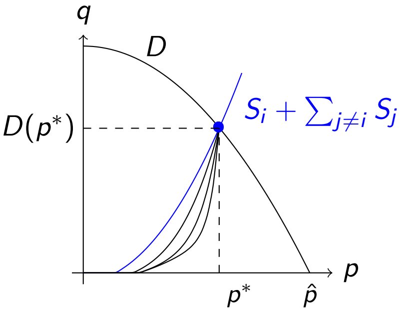
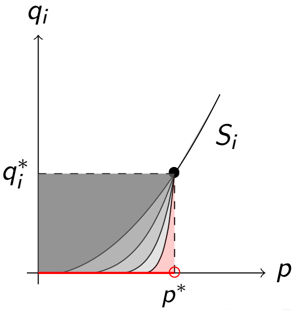




Outline

Main result

- Pay-as-bid (PAB) auction
- ⋆ production costs C_i with $C_i' \geq 0$, $C_i'' \geq 0$
- ⋆ aggregate demand function D with $D' < 0$, $D'' \leq 0$
- $\mathcal{A} = \{S \in C^0\,,\, S(0) = 0, S$ non-decreasing , S K-Lipschitz$\}$ for some $K > 0$

Theorem (Existence and characterization of NE)

There exists at least one Nash equilibrium $\boldsymbol{S}^*$ *such that for every agent* i*:*

$$S_i^*(p) = K[p - p_i]_+$$

for some $p_i \in [0, \hat{p}]$.

Step 1: characterization of best-responses

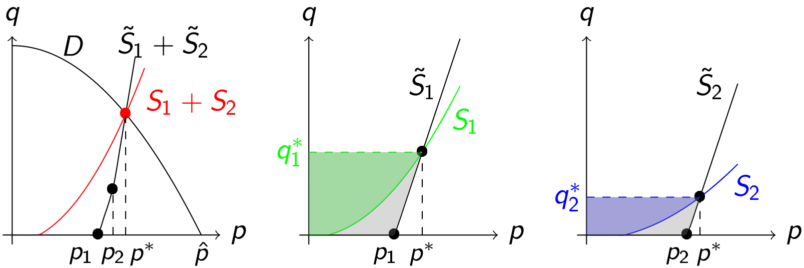
- Pay-as-bid (PAB) auction
- $\mathcal{A} = \{S \in C^0, S_i(0) = 0,\ S \text{ non-decreasing}, S \text{ is K-Lipschitz}\}$ for some $K > 0$

Lemma (Affine BR)

For every agent i:

$$S_i \in \mathcal{B}_i(S_{-i}) \quad \Rightarrow \quad S_i(p) = K[p - p_i]_+$$

for some $p_i \in [0, \hat{p}]$.



Step 2: the restricted game

⋆ Nash equilibrium $\mathbf{S}^* \Rightarrow S_i^*(p) = K[p - p_i]_+$ for some p_i in $[0, \hat{p}]$

Restricted game

- Actions $p_i \in [0, \hat{p}]$
- Utility

$$u_i^r(p_i, p_{-i}) := p^* K[p^* - p_i]_+ - \int_0^{p^*} K[p - p_i]^+ dp - C_i\left(K[p^* - p_i]_+\right)$$

$$\text{s.t.:} \quad K[p^* - p_i]_+ = D(p^*) - \sum_{j \neq i} K[p^* - p_j]_+ \, .$$

Corollary

The set of Nash equilibria of the PAB auction with K-Lipschitz supplies coincides with the set of Nash equilibria of the restricted game.

Complexity reduction: from an infinite-dimensional strategy space to one dimension.

Step 3: quasi-concavity

Lemma

For every agent i, the utility u_i^r is quasi-concave in p_i for every choice of p_{-i}.

- Fix agent i and p_{-i}, utility:

$$u_i^r(x, p_{-i}) = \varphi(x)[\varphi(x) - x]_+ - \frac{1}{2}([\varphi(x) - x]_+)^2 - C_i([\varphi(x) - x]_+)\,,$$

  where $\varphi : [0, \hat{p}] \to [0, \hat{p}]$ is the unique equilibrium price when $p_i = x$.
- When $\varphi(x) > x$, $\varphi(x) = f^{-1}(x)$ with f explicit. When differentiable, $\varphi'(x) \in (0, 1)$ and $\varphi''(x) \leq 0$ (assumption on demand)
- u_i^r concave in the stationary points (assumption on costs)
- Points of non-differentiability: in case of a change of sign, from positive to negative.

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- u_i^r concave in the stationary points (assumption on costs)
- Points of non-differentiability: in case of a change of sign, from positive to negative.

Lemma + Kakutani's Theorem $\Rightarrow$ The restricted game admits pure Nash equilibria.

Outline

Supermodularity

⋆ u_i^r has *increasing differences* in (p_i, p_{-i}) if for all $p_i' \geq p_i$ and $p_{-i}' \geq p_{-i}$ it holds

$$u_i^r(p_i', p_{-i}') - u_i^r(p_i, p_{-i}') \geq u_i^r(p_i', p_{-i}) - u_i^r(p_i, p_{-i}) .$$

Remark: the restricted game is not supermodular in general:

- $K = 1$, $n = 2$, $D(p) = 100 - p$ and $C_1(q) = \frac{1}{2}q^2$.
- Let $p_2 = 0$ and $p_1 = 50$. We find $p^* = 50$ and $u_1^r(p_1, p_2) = -C_1(0) = 0$.
- Let $p_2' = 1$ and $p_1 = 50$. We find $p^* = 50.3$ and $u_1^r(p_1, p_2') \approx 16.7 \neq 0$.
- For $p_1' = 50.2$, we find $u_1^r(p_1', p_2) = 0$ and $u_1^r(p_1', p_2') \approx 10.04$.
- Then, $u_1^r(p_1', p_2') - u_1^r(p_1, p_2') \approx -6.63 \not\geq u_1^r(p_1', p_2) - u_1^r(p_1, p_2) = 0$.

Supermodularity

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If *affine demand* $D(p) = N - \gamma p$, u_i^r has increasing differences when

1. player i sells a non-zero quantity, and
2. the number of players selling in the auction does not change.

"Piece-wise" supermodular?

Conclusions

Summary

- Supply function model with pay-as-bid remuneration and asymmetric firms
- Existence and characterization of Nash equilibria with K-Lispschitz supply function

Current work

- "Piece-wise" supermodular with affine demand
- Algorithm to compute the Nash equilibria
- Conditions for uniqueness of Nash equilibria

Further work

- Validate the model with real data of the Italian electricity market
- Uncertainty in the demand
- Network in the model
- Concatenation of a uniform-price auction and a pay-as-bid one, modeled as a two-stage game

Thank you for your attention!