Noise-induced spatial pattern formation

Stefania Scarsoglio[1]
Francesco Laio[1] Paolo D'Odorico[2] Luca Ridolfi[1]

[1]Department of Hydraulics, Politecnico di Torino, Torino, Italy
[2]Department of Environmental Sciences, University of Virginia, Charlottesville, Virginia, USA

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Outline

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  - the landscape's susceptibility to desertification (*von Hardenberg et al. 2001*, *D'Odorico et al. 2005*).
- Deterministic models have been studied for quite a long time (*Turing 1952*, *Cross & Hohenberg 1993*) with a number of applications to environmental processes (*Borgogno et al. 2009*, *von Hardenberg et al. 2010*, *Manor & Shnerb 2008*, *Couteron & Lejeune 2001*, *Rietkerk & Van de Koppel 2008*, *Kefi et al. 2007*, *Lefever et al. 2009*).

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- Since these models use complicated non-linear terms for the local dynamics and the multiplicative noise terms, their process-based interpretation is often not straightforward.

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  - Rich literature (unlike Gaussian colored or dichotomous noise).
- We call patterned a field that exhibits an ordered state with organized spatial structures. This definition is often adopted in the environmental sciences, where the concomitance of many processes can prevent the organization of the system with a clear dominant wavelength.

Stochastic modeling: general framework

Temporal evolution of the state variable ϕ at any point $\mathbf{r} = (x, y)$:

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- $h(\phi)F(t)$: time-dependent forcing term, which can be modulated by a function, $h(\phi)$, of the local state of the system $\Rightarrow$ seasonal phenomena (*phreatic aquifer*).

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  - Structure function (SF): prognostic tool able to assess the presence of a selected wavelength in the spatial field;

Scarsoglio, Laio, Ridolfi, D'Odorico, submitted *Phys. Rev. Lett.* 2010.

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- Numerical simulations:
  - Heun's predictor corrector scheme, 2D square lattice with 128x128 sites;
  - periodic BCs, ICs given by uniformly distributed random numbers between [-0.01, 0.01].

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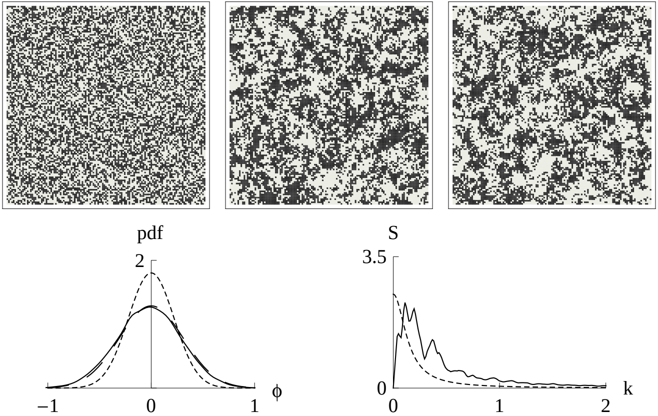
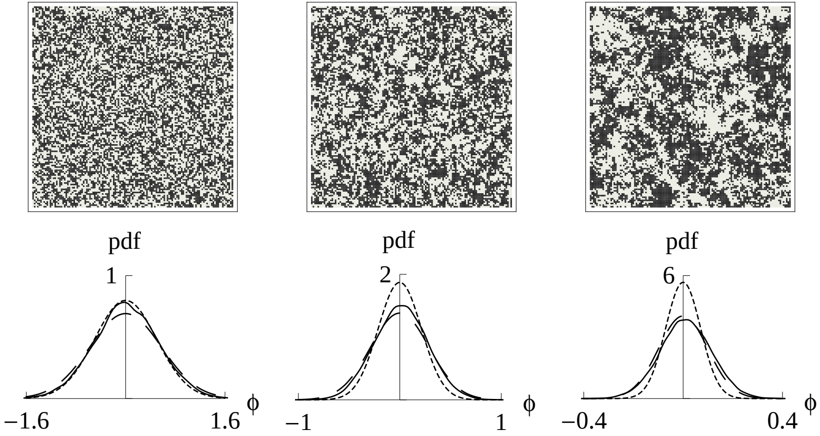
Steady and multiscale patterns



(top) Numerical simulation of ϕ at $t = 0, 10, 100$, $D = 50$, $s = 5$. (below) Pdf (solid: numerical simulation, dotted: classic MFA, dashed: corrected MFA) and azimuthal-averaged power spectrum S (solid: numerical simulation, dotted: SF) of ϕ at $t = 100$.

Role of D



(top) Numerical simulation of ϕ at $t = 100$, $s = 1$, $D = 1, 10, 100$ (left to right). (below) Pdf of ϕ (solid: numerical simulation, dotted: classic MFA, dashed: corrected MFA).

Comparison with vegetation pattern

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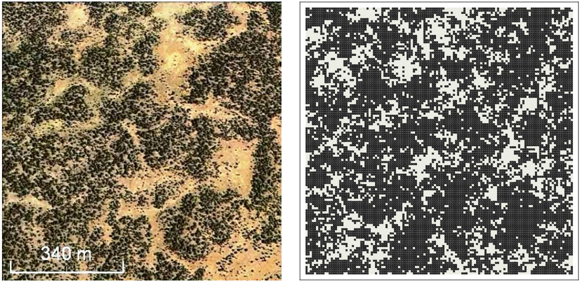

(left) Aerial photograph of vegetation pattern in New Mexico (34°47'N, 108°21'O) and (right) numerical simulation at $t = 100$, $a = -1$, $D = 80$, $s = 2$, $\mu = 0.1$.

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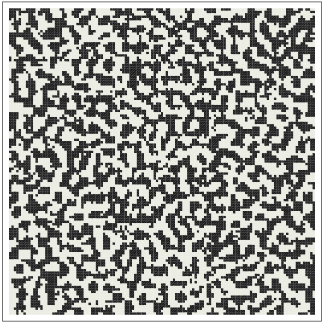

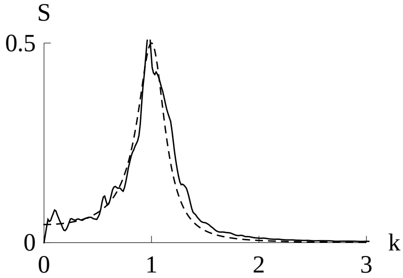

(left) Numerical simulation of ϕ at $t = 100$, $s = 1$, $D = 10$, $k_0 = 1$. (right) Azimuthal-averaged power spectrum S (solid: numerical simulation, dotted: SF).

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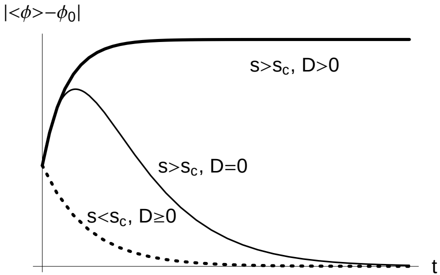

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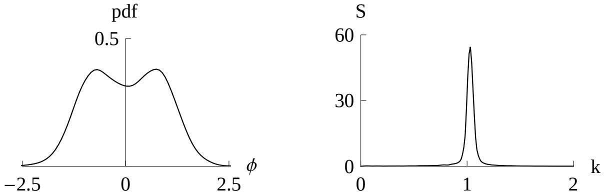

(top) Numerical simulation of the spatial field ϕ at $t = 0, 10, 100$, with $D = 15$, $s = 5$, $k_0 = 1$. (below) Pdf and azimuthal-averaged power spectrum S at $t = 100$.

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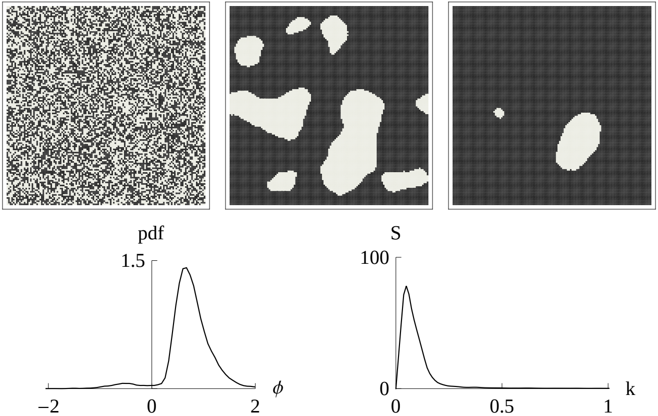

(top) Numerical simulation of the spatial field ϕ at $t = 0, 10, 40$, with $D = 20$, $s = 4$.
(below) Pdf and azimuthal-averaged power spectrum S at $t = 40$.

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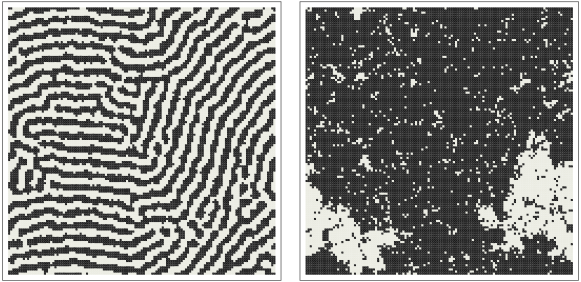

Numerical simulation of ϕ. (left) Swift-Hohenberg spatial coupling at $t = 100$, $D = 15$, $s = 5$, $k_0 = 1$, and (right) Laplacian spatial coupling at $t = 200$, $D = 20$, $s = 4$.

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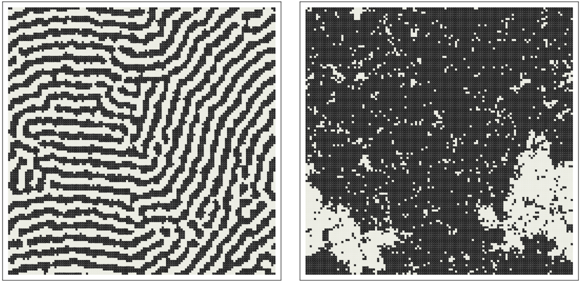

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Non-linearities do not change the pattern scenario, provided that the interplay between short-term instability and spatial coupling remains the same.

Time oscillating patterns

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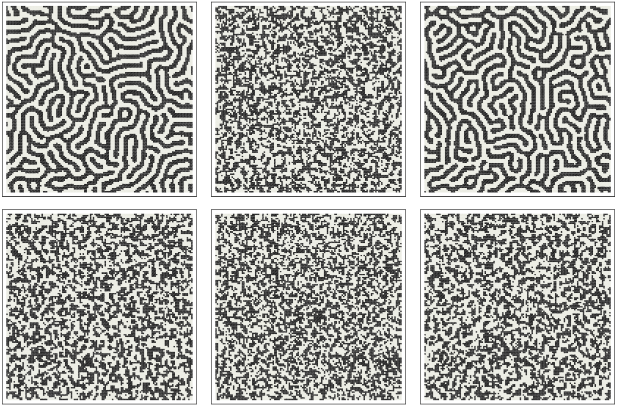
$s = 0.025$

$s = 2.5$



$t = 110$ $t = 140$ $t = 195$

Numerical simulation of ϕ with $\alpha = k_0 = 1$, $k = 0.1$, $\omega/2\pi = 0.012$, and $D = 1$.

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- For high enough multiplicative noise intensity, the spatial coupling exploits the initial instability giving rise to ordered structures;
- The presence of a temporal periodicity promotes oscillating patterns which periodically emerge and disappear;
- Since noisy fluctuations are always present in real systems and pattern formation, here described, is completely noise-induced, randomness can actually promote spatial coherence in different environmental processes.