TRANSIENT DYNAMICS OF SMALL THREE DIMENSIONAL PERTURBATION IN THE CROSS-FLOW BOUNDARY LAYER

Francesca De santi[1], Stefania Scarsoglio [1], William O. Criminale [2] & Daniela Tordella [1]
[1]*Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Torino, Italy 10129*
[2]*Department of Applied Mathematics, University of Washington, Seattle, Washington 98195-2420, USA*

Abstract We carry out exploratory linear studies on the transient dynamics of arbitrary three dimensional perturbation acting on the cross-flow boundary layer. In particular we find that the sign of the pressure gradient can significantly affects the transient dynamics. We also show evidence of a discontinuous behavior in the frequency inside the transient life the perturbation.

INTRODUCTION

Generally the laminar boundary layers are analyzed in the simplified case of a two-dimensional base flow. Such simplification is often useful since the transition phenomena are very complex, but some engineering applications exist where the three-dimensionality of boundary layers cannot be neglected. The flow over a swept wing is a classic example of a three-dimensional boundary layer which is of aerodynamics interest. A cross-flow velocity component can develop inside the boundary layer, caused by a spanwise pressure gradient, and can modify the flows stability characteristics. Indeed, in addition to the viscous instability mechanism observed in the two dimensional boundary layer, the three dimensional base flow is susceptible to an inviscid instability arising from an inflection point created by the cross flow component. This instability is related to oblique modes and can grows much faster than the standard Tollmien-Schlichting modes, in other words three dimensional perturbation can lead to turbulence without any observed Tollmien-Schlichting waves.
The cross flow boundary layer has been analyzed using modal theory [1], in the context of receptivity and transient optimal perturbations [2, 3] and experimentally [4]. Here we present a general three dimensional initial-value problem in order to study the linear stability and make an explorative analysis on the transient behaviors.

THE INITIAL-VALUE PROBLEM

The transient behaviors of arbitrary three dimensional disturbances acting on the FSC Cross-flow boundary layer are investigated. Instead of using the modal stability approach or the more recent techniques involving eigenfunction expansions, we have considered the velocity vorticity formulation and we have Fourier transformed the governing disturbance equations in the streamwise and spanwise directions only. The resulting partial differential equations are numerically solved by the method of lines [5]. This approach offers an alternative means for which arbitrary initial conditions can be specified and its full temporal behavior, including both early-time transients and the long-time asymptotics, can be observed. As initial condition we have chosen a Gaussian, namely $v(0) = y^2 \exp(-y.^2),\ \omega_y(0) = 0$.
We have considered sub- and supercritical flows (Reynolds number computed on the displacement thickness equal to 100 and 5000) subject to adverse and favorable pressure gradient (Hartree parameter, β, equal to -0.1988 and 1). The cross flow angle, θ, between the stream-wise direction and the chord-wise direction is taken equal to $\pi/6, \pi/4, \pi/3$. Finally, as regards the perturbations we vary both the polar wavenumber, k, and the angle of obliquity with respect to the streamwise direction, ϕ.

RESULTS AND CONCLUSIONS

The transient behaviors of the perturbative waves are observed in term of amplification factor, $G(t)$(the kinetic energy density normalized over its initial value), temporal growth rate, r (defined as $log(E(t))/2t$, where $E(t)$ denotes the total kinetic energy) and frequency, ω, defined as the temporal derivative of the unwrapped phase at a specific spatial point along the y direction.
In Figure 1 we show the role of the perturbation wavenumber on their transient lives and asymptotic state. In general, there exist a wavenumber range $k \in [0.06, 1.2]$ where perturbations are more unstable. The flow configuration considered has unstable Reynolds number, favorable pressure gradient and high cross flow angles. The amplification factor is observe for orthogonal waves and it is interesting to note that all these waves in a Blasius boundary layer would be stable.
Figure 2 shows the effect of the obliquity angle of the perturbations in term of amplification factor (panel a) and frequency (panel b). We can observe that waves with a negative obliquity angle are in general more unstable. Moreover independently on the stable or unstable behavior of the perturbation, the frequency undergoes a sudden variation of its value, as a jump. We interpret this jump as the transition between the early transient and the beginning of an intermediate term that reveals itself for times large enough for the influence of the fine details of the initial condition to disappear [6].

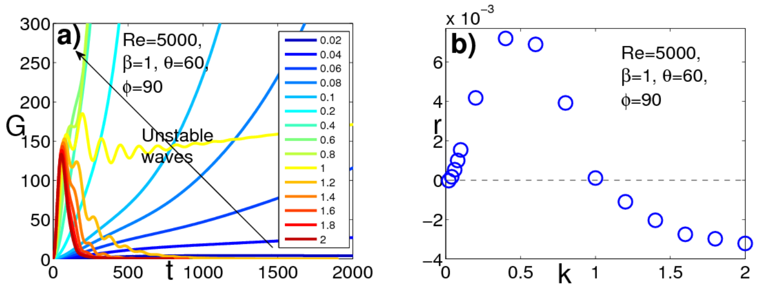

Figure 1. Role of the wavenumber on the temporal evolutions of the amplification factor, (a), and on the temporal growth rate (b). $Re = 5000$, $\beta = 1$, $\theta = \pi/3$, $\phi = \pi/2$,$k \in [0.02, 2]$.

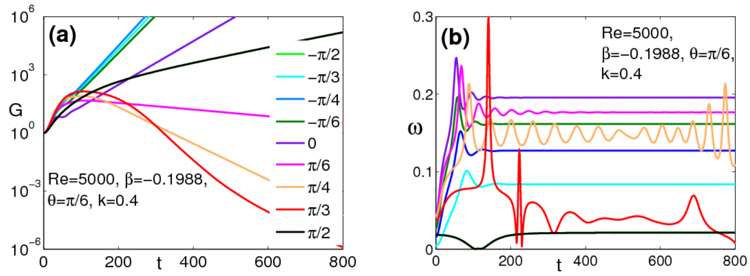

Figure 2. Temporal evolutions of the amplification factor, G(t), and the pulsation $\omega(t)$. These figure highlight the role of the obliquity angle, ϕ, for waves with wavenumber k=0.4. $Re = 5000$, $\beta = -0.1988$ and $\theta = \pi/6$.

Figure 3 shows the combined effect of the pressure gradient and the cross-flow angle. We observe that for positive pressure gradient the waves are more instable if the cross flow angle in the base flow is small, on the other hand for negative pressure gradient the perturbation are more instable when the angle between the stream-wise direction and the chord-wise direction is large.

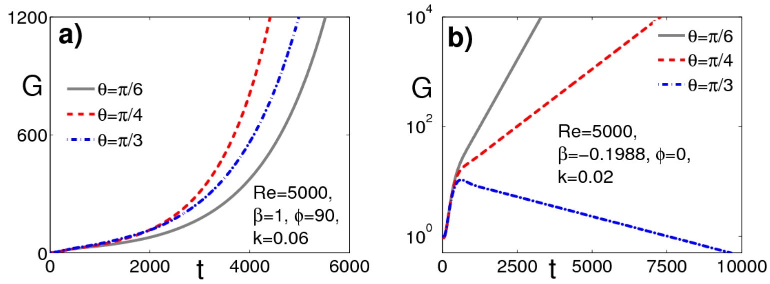

Figure 3. Temporal evolution of the amplification factor for different base flow configuration. In both panel three different value for the cross flow angle are considered, $\theta = \pi/6, \pi/4, \pi/3$.(a) $Re = 5000$, $\beta = 1$, $\phi = \pi/2$, $k = 0.06$. (b) $Re = 5000$, $\beta = -0.1988$, $\phi = 0$, $k = 0.02$.

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