Temporal network-based analysis of turbulent mixing

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1 Introduction

Complex network-based tools have been recently attracting a lot of interest in the analysis of fluid flows. So far, most of the research has focused on geophysical flows [1], two phase flows [2], as well as turbulent flows [3, 4]. In particular, the investigation of fluid flows from a Lagrangian point of view – i.e., by tracing particle trajectories during the motion – has recently been explored [5, 6]. However, particle trajectories are typically exploited to build static networks, thus losing transient information on the flow dynamics. In this work, we propose a temporal network-based approach [7–9] for the study of turbulent mixing, that fully captures the temporal evolution of particle dynamics.

2 Data description and network building

A turbulent channel flow was simulated through a direct numerical simulation at $Re_\tau = Hu_\tau/\nu = 950$ and performed for a time $T^+ = Tu_\tau/\nu = 15200$ with a time step $\Delta t^+ = 4.75$, where H is the half-channel height, u_τ is the friction velocity and ν is the kinematic viscosity [10]. A set of 100×100 fluid particles were initially arranged at $x^+ = 0$ on a uniformly spaced grid in the plane (y^+, z^+) (see Fig. 1(a)), where (x^+, y^+, z^+) are the streamwise, wall-normal and spanwise coordinates, respectively. Particles were grouped into $N = 100$ levels, l_i, according to their y^+ value at the initial time $t^+ = 0$ (see Fig. 1(a)). Trajectories of the particles are then computed over time.

If two particles are sufficiently close to each other at a given time, then a connection is established between them. Specifically, a particle A is connected to a particle B if A lies inside a reference ellipsoid centred in B, and *vice versa* (by symmetry). For example, in Fig. 1(b) the trajectories of two particles A and B and their corresponding reference ellipsoids are shown. The corresponding time-sequence of connections between A and B is illustrated in Fig. 1(c). The ellipsoid was geometrically anchored and centred to each particle location and it was chosen as reference geometry to take into account the anisotropy of the flow. Since the average Euclidean distance between particles increases in time (mainly due to the streamwise dispersion), the semi-axes of the ellipsoid were set to grow proportionally to the average distance in each direction.

As particles follow the turbulent motion, their relative distance changes in time, thus particles belonging to different levels generate different time-sequences of connections.

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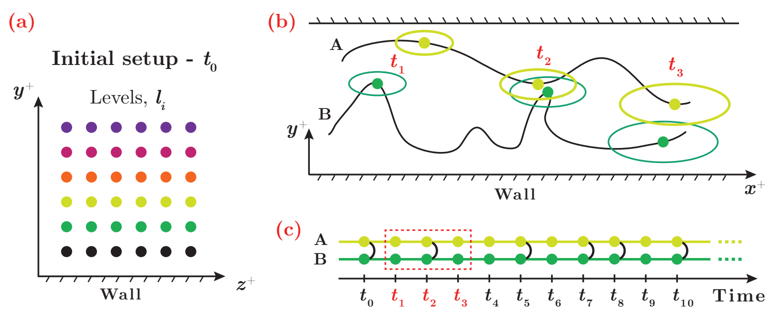

Fig. 1. (a) Sketch of the initial arrangement of particles at $x^+ = 0$. Different colors highlight different levels. (b) Example of trajectories (black lines) of two particles A and B and their reference ellipsoids. (c) Time-sequence of the connections between particles A and B shown in panel (b) (the corresponding sampling time is highlighted by the red dashed box).

Therefore, we modelled the turbulent particle dynamics by means of a weighted time-varying network [9], $W_{i,j}(t^+)$, in which nodes correspond to levels, l_i (i.e., groups of particles initially at the same y^+). At each time, the weight of a link between two nodes (i, j) – i.e., two levels – is equal to the total number of connections shared by particles belonging to levels l_i and l_j. By doing so, a time-varying network of $N_T = 3200$ undirected weight matrices (each one of size 100×100) is obtained, that is able to capture the temporal effects of turbulent mixing and flow advection on particle dynamics.

3 Results

Since $W_{i,j}(t^+)$ embeds the information of the modelled particle dynamics, the weight matrices at three representative times are shown in Fig. 2(a), where colors indicate the intensity of the weight, i.e., the number of connections between each pair of levels. At $t^+ = 0$, the weight matrix displays a band structure corresponding to the initial arrangement, where nodes close in space are strongly linked each other. As time increases, the weight matrix changes structure: a three-square pattern of high $W_{i,j}(t^+)$ values emerges, as a consequence of turbulent dispersion, while turbulent mixing enables the activation of links between initially distant levels (e.g., levels 20 and 90). Finally, for large times ($t^+ = 15200$) the turbulent mixing dominates the particle dynamics, approaching an asymptotic behaviour, where particles are well-mixed independently from their starting level. Consequently, the resulting weight matrix does not show any clear pattern. The change in the particle dynamics is also highlighted by a different network topology of $W_{i,j}(t^+)$. This is shown in Fig. 2(b), where the change in the network topology as function of time is evident. In particular, at $t^+ = 900$ the nodes tend to cluster into three groups in analogy with the three-square pattern in Fig.2(a). This pattern is lost at $t^+ = 15200$, in which the network topology appears as a random layout. From a centrality perspective, the analysis of the average strength over time is able to highlight and discern characteristic advection-mixing regimes in a straightforward way. Hence, the main advantage of the proposed approach is its ability to fully incorporate the information about particle dynamics into the weight matrices, at any time.

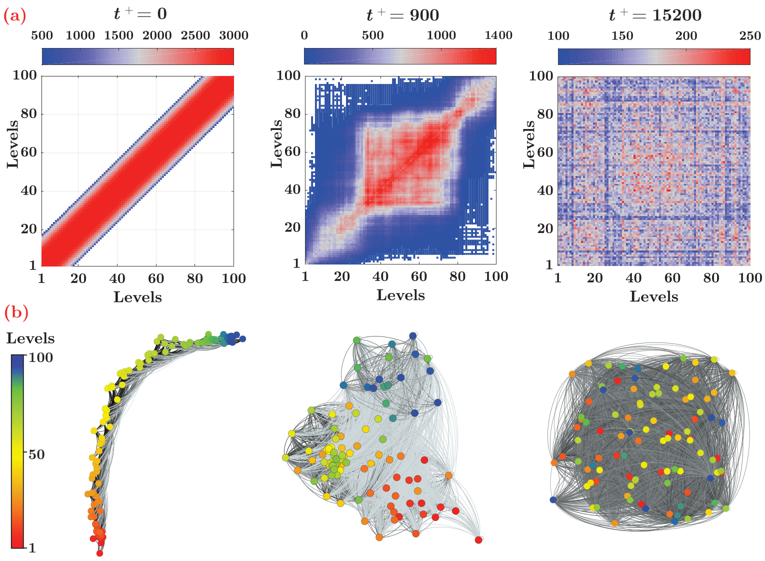

Fig. 2. (a) Weight matrices of the time-varying network at three representative times. The colorbar indicates the intensity of the links. (b) Visualization of the networks reported in panel (a). Nodes are colored with their level (see colorbar), while link intensity is colored in grey-scale (strong links are in black, weak links are in light-grey).

Summary. In this work, the turbulent mixing in a channel flow is investigated through a Lagrangian approach. A set of fluid particles is traced in time, where each particle belongs to a level corresponding to its initial wall-normal coordinate. By exploiting the pairwise distance between particles, a weighted time-varying network is built in which nodes represent levels, and the link weight is equal to the number of connections between each pair of levels. Different particle dynamics are found to result into different network topologies and link weight patterns. In this way, it is possible to entirely capture the features of the turbulent mixing and geometrize the evolution of the particle swarm.

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