Perturbed Cross-flow Boundary Layer: nontrivial effects of the obliquity angle

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Introduction

- The transient as well the long time behaviors of arbitrary three-dimensional disturbances acting on a cross-flow boundary layer are investigated.
- The cross flow boundary layer is one of the most important boundary layer in the engineering application (aerospace, mechanical, wind,..)

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- The cross flow boundary layer is one of the most important boundary layer in the engineering application (aerospace, mechanical, wind,..)
- The role of the direction of the perturbation in respect to the base flow has been poorly investigated (Breuer & Kuraishi (Phys. Fluids 1994), Taylor & Peake (jfm 1998))

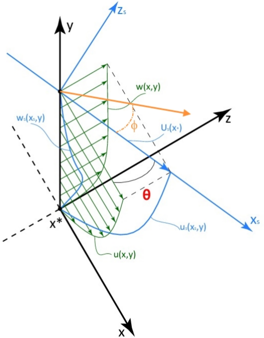
We present an exploratory analysis of the perturbation life, where a major focus is put on the obliquity of the perturbation in respect to the base flow

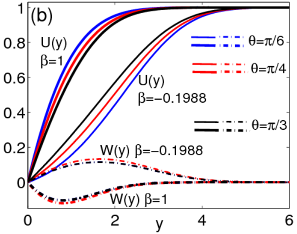
the stability property

wall perturbed pressure

Base Flows

Falkner-Skan-Cooke (FSC) velocity profiles (*Cooke, J. C. 1950 Proc. Camb. Phil. Soc.*)





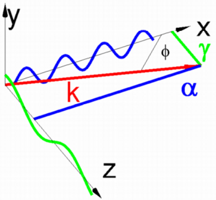
Base flow parameters:

- *Re* Reynolds number (based on $\delta *$)
- β Hartree parameter (pressure gradient)
- θ Cross-Flow angle (between the the streamwise direction and the chordwise direction)

Formulation

- Linearized Navier Stokes equation
- Velocity and vorticity formulation (*Criminale & Drazin, Stud. Appl. Math., 1990*);

- Laplace-Fourier transform in x and z directions:
  - α streamwise wavenumber
  - γ spanwise wavenumber
  - k polar wavenumber
  - ϕ obliquity angle between the perturbation and the streamwise direction
  - $v(y, t = 0) = y^2 \exp(-y^2), \quad \omega_y(y, t = 0) = 0$



Measure of the transient

- Kinetic energy density *e*:

$$e(t;\alpha,\gamma) = \frac{1}{2}\int_{-y_d}^{+y_d}(|\hat{u}|^2+|\hat{v}|^2+|\hat{w}|^2)dy$$

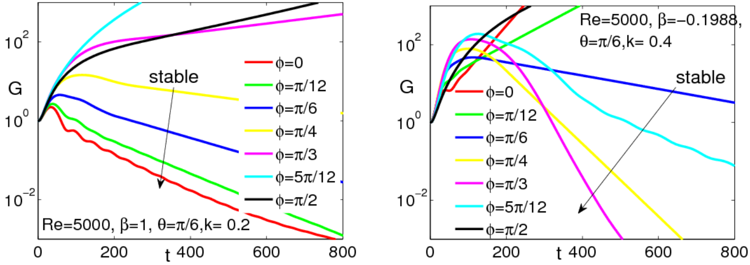
- Amplification factor *G*:

$$G(t;\alpha,\gamma) = \frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

- Temporal growth rate *r*:

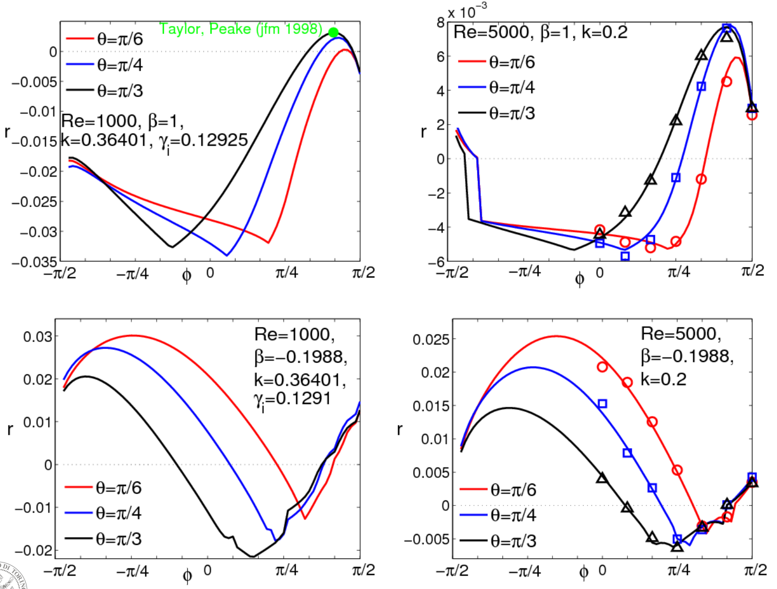
$$r(t;\alpha,\gamma) = \frac{log(e)}{2t}$$

Transient behavior



- It is difficult to come across a general trend
- Counter-intuitive behavior: there are configuration in which longitudinal and orthogonal waves are unstable while oblique waves are stable

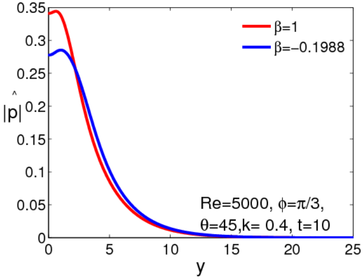
Asymptotic Growth Rate

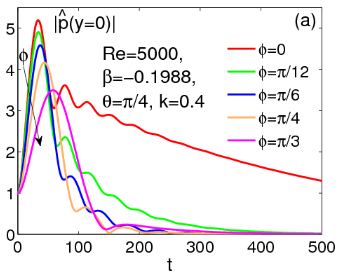


Pressure

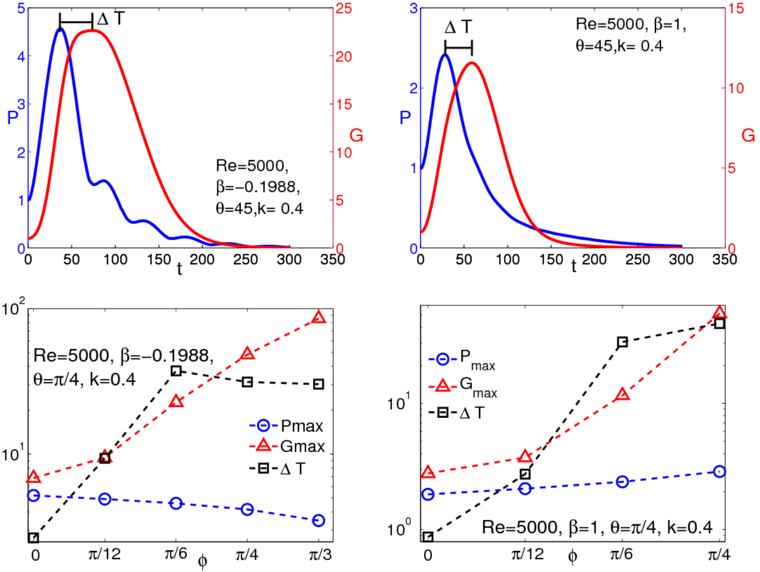
The pressure is computed a posterior by the Poisson equation.
A Pressure Amplification is defined as

$$P = |\hat{p}(y = 0, t)| / |\hat{p}(y = 0, t = 0)|$$



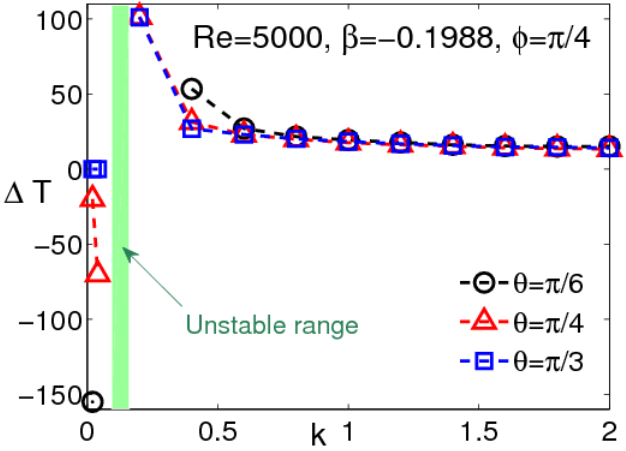


Pressure Transient



Wall Pressure Delay

$$\Delta T = t(G = G_{max}) - t(P = P_{max}))$$



Conclusion

Effects of the obliquity angle

- Let's define ϕ_{min} and ϕ_{max} as the obliquity angle for which the growth rate reaches its minimum/maximum value. They both decreases a θ and k increase
- With $\beta = 1$ the most unstable waves has about $\phi = 5\pi/12$
- With $\beta = -0.1988$ the most unstable waves has negative obliquity
- The perturbed pressure measured at the wall shows to be anticipated (short waves) o retarded (long waves) with respect to the kinetic energy evolution.
  These times increase with the obliquity angle