On the frequency of hydrodynamic perturbations.
From the early transient through the intermediate term to the asymptotic state

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We present recent findings concerning the frequency in the transient evolution of three-dimensional perturbations in sheared flows.
We adopt the initial-value problem formulation, proposed by Criminale and Drazin [1], that was recently used to carry out exploratory studies on the perturbation transient dynamics [2,3].
We consider two typical shear flows: the plane Poiseuille flow, as the archetype of wall flows, and the bluff-body wake flow, as an example of unbounded flow. We show evidence of a discontinuous behaviour in the frequency inside the transient life of three-dimensional travelling perturbation waves.

To describe the travelling waves evolution we define

the kinetic energy density

$e(t)=\int(|u|^2+|v|^2+|w|^2)\, dy$

the amplification factor

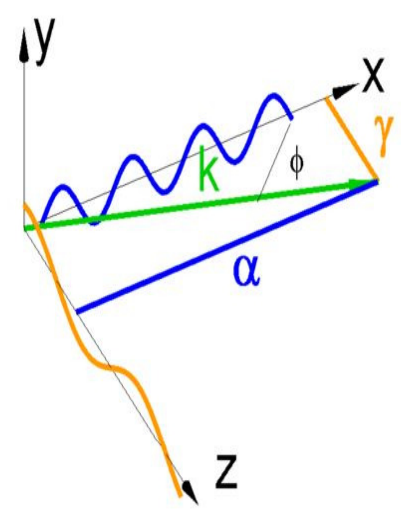
$G(t)=e(t)/e(0)$

the wrapped phase

$\theta_w(y,t)=\arg(v(y,t))$

the angular frequency

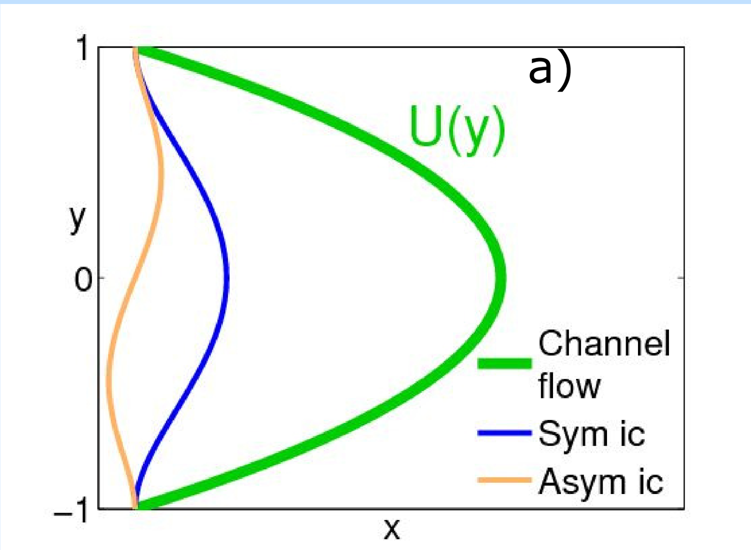
$(y_0,t)=d\theta(y=y_0,t)/dt$



Perturbation geometry scheme.
y is the shear direction. α and β are the streamwise (x) and spanwise (z) wavenumbers, respectively.
Perturbations propagate in the direction of the polar wavenumber.
ϕ is the angle of obliquity with respect to the basic flow U=U(y).
Boundary conditions:
(u,v,w) → 0 as y → ± ∞ and at walls.

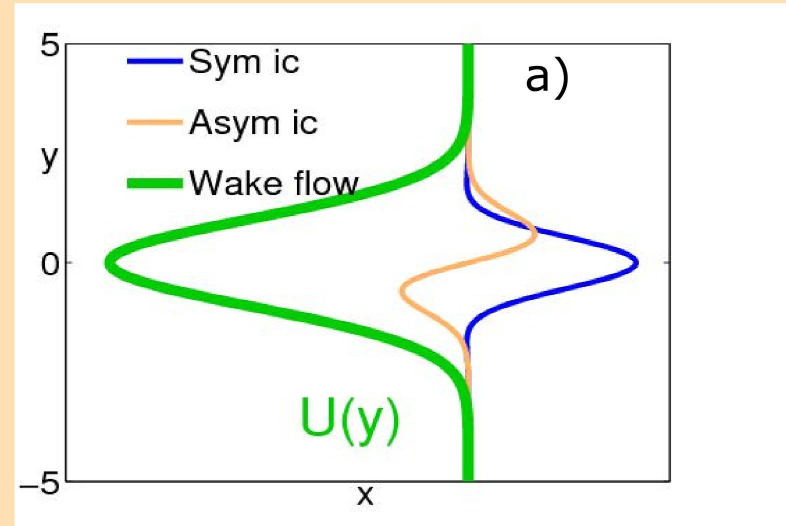
Plane Poiseuille Flow

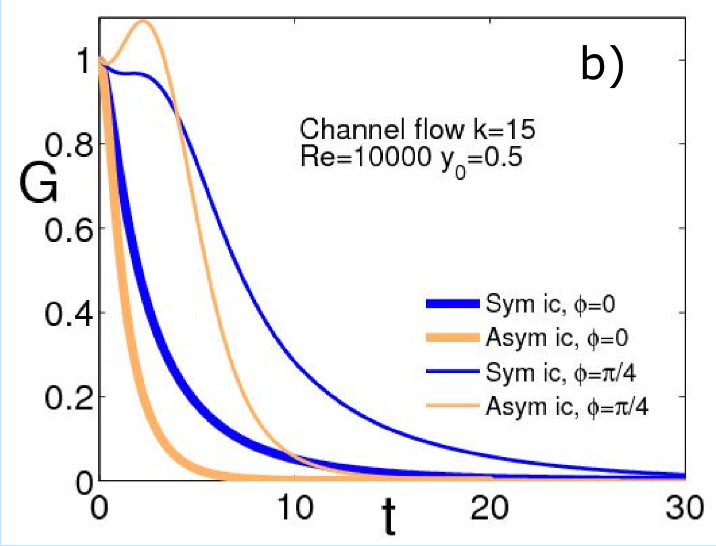
Bluff-Body Wake Flow

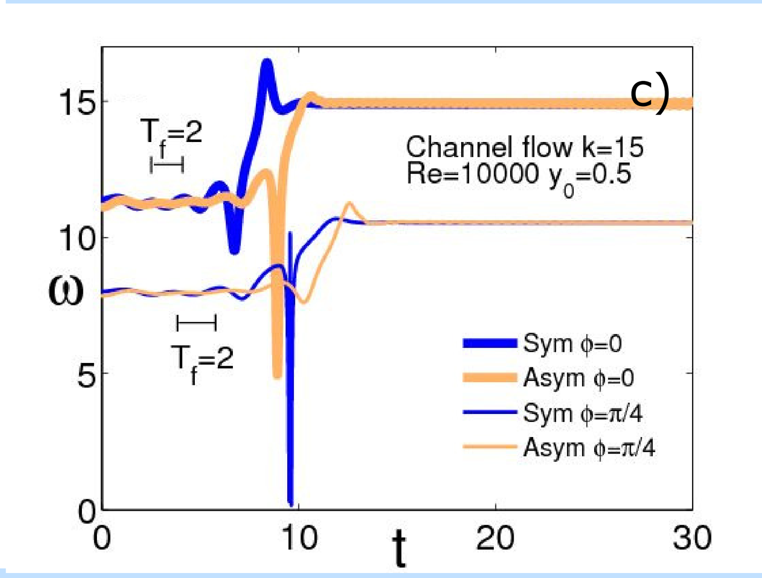


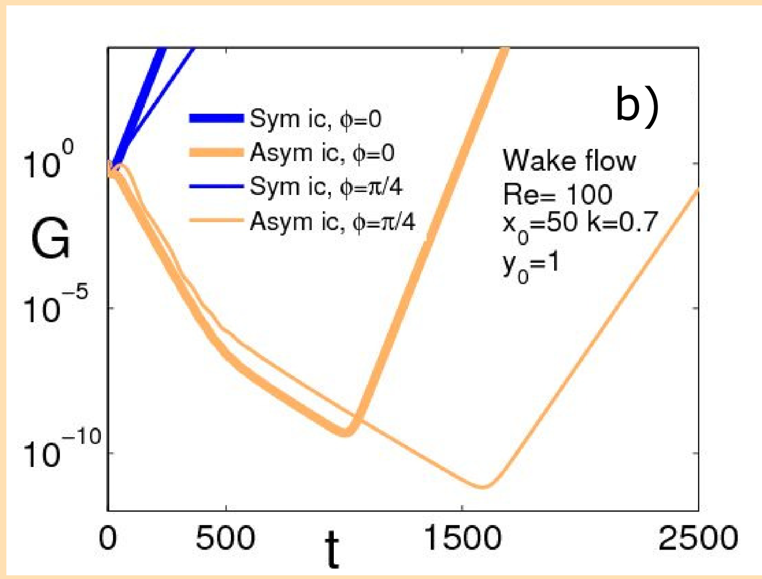
a) Base flow velocity profiles, U(y) (green curves) and symmetric and antisymmetric initial conditions of the perturbation transversal velocity.

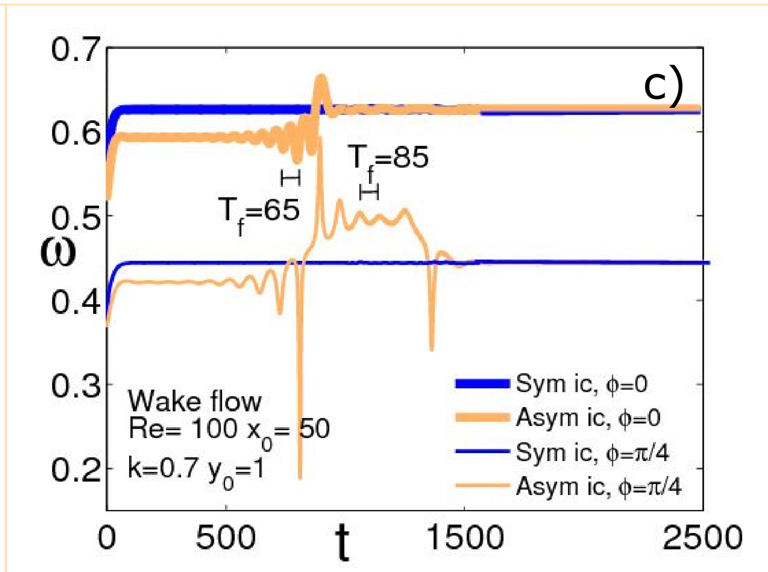
b-c) transient lives of the perturbations observed through the amplification factor, G (b panels) and the angular frequency, w (c panels). The angular frequency is computed at a distance from the wall equal to 0.25 the channel width and at a distance equal to one body length from the centre of the wake flow.
The quantity T_f in panel c indicates the temporal periodicity related to the frequency fluctuations observed in the early and intermediate dynamics.

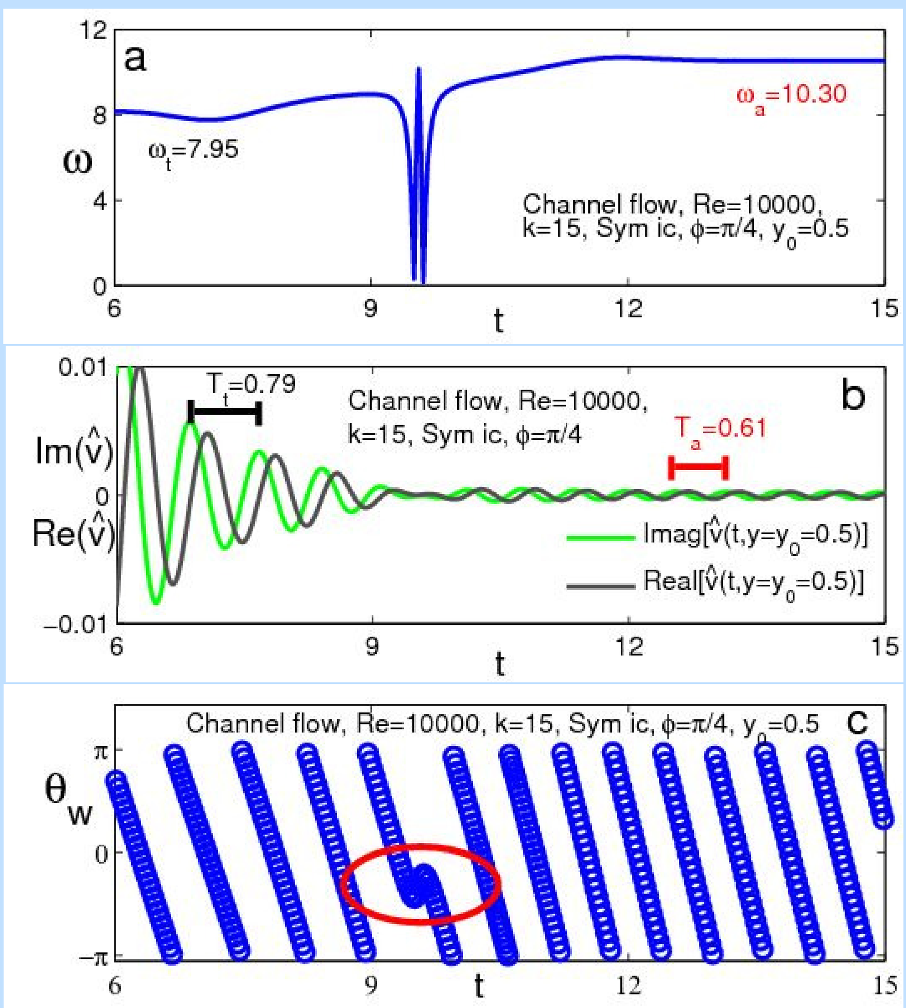








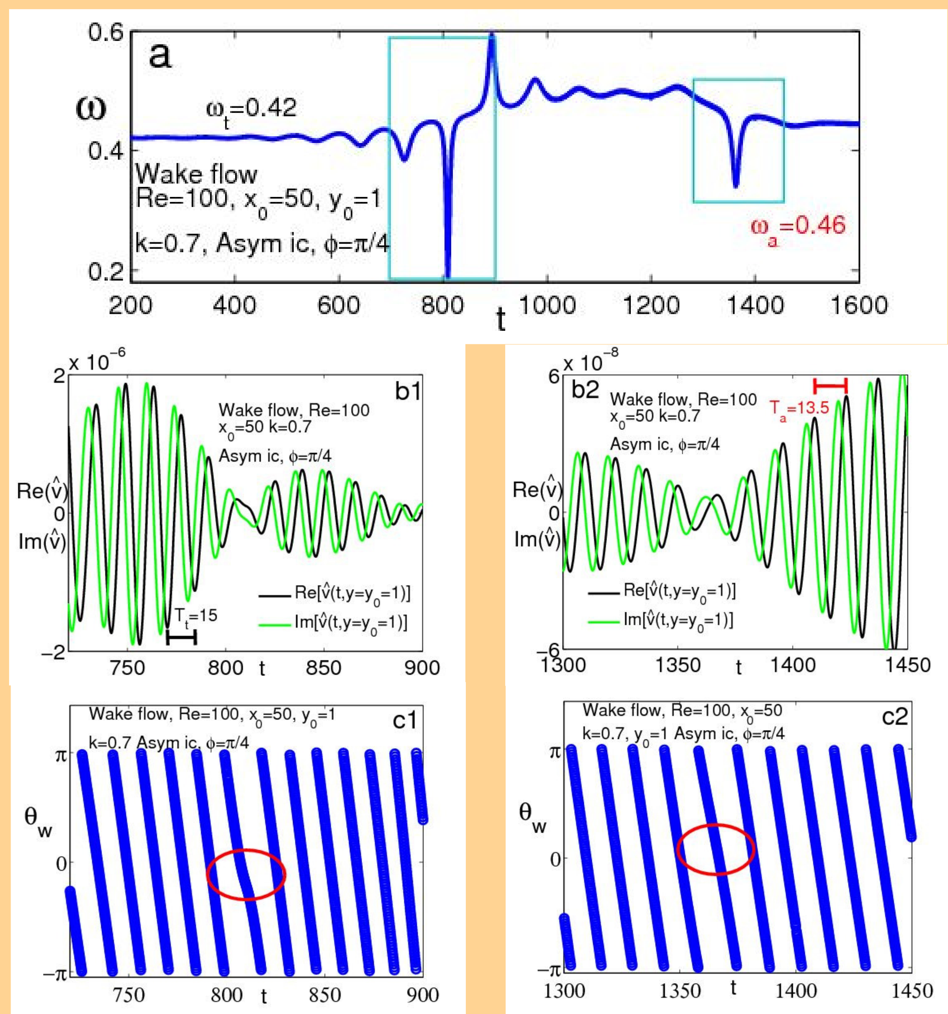




(a) Frequency temporal evolution: w_t is the value in the early transient while w_a is the asymptotic one.

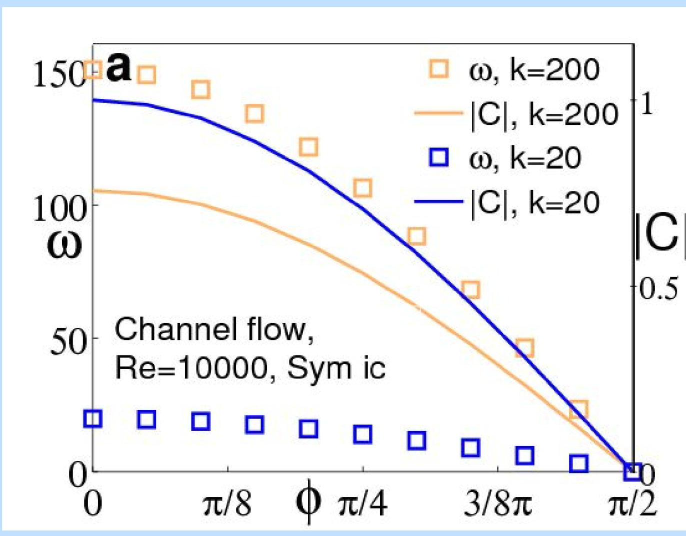
(b) Perturbation transversal velocity (real and imaginary parts). Temporal periods (T_t : transient value, T_a : asymptotic value).

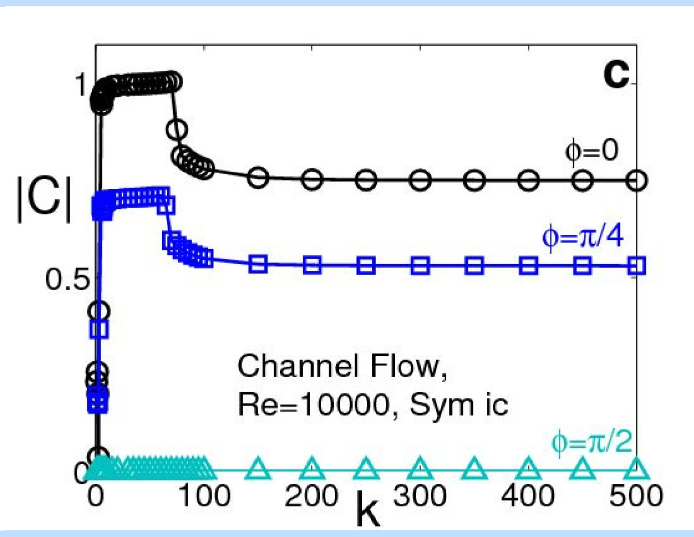
(c) Wrapped wave phase, $\theta_w(t)$.



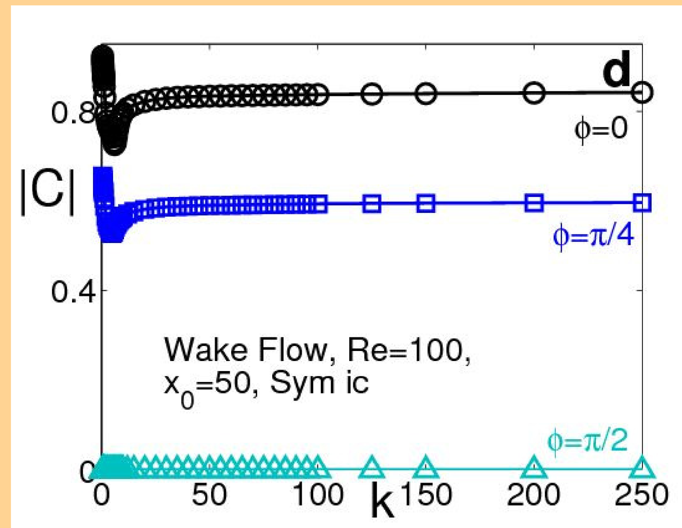
The discontinuity, never observed before, appears after many eddy turn over times have elapsed and last about the 50% of the global transient length. We interpret this phenomenon as the signature of both the end of the early transient, the part of the evolution must affected by the initial condition, and the beginning of the intermediate term, where the accomplishment of the final values of the wave characteristics take place in accordance with the modal theory.

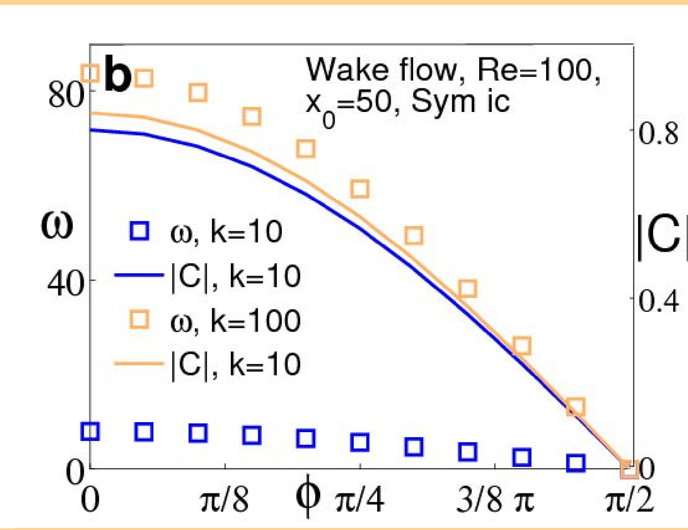
The investigation of the dispersion relation in the asymptotic regime reveals that longitudinal long waves and all the perturbations not aligned with the base flow present a dispersive behavior, while only longitudinal short waves are non-dispersive.
Moreover the frequency is proportional to the cosine of the obliquity angle. As a result, purely orthogonal waves, always stable in the long-term, are standing waves. Since any of these waves arriving in the system will have a zero phase velocity and since during the early transient orthogonal waves can present intense algebraic growth, the system in this condition faces a situation where, in principle, instability can be incentivated.





(a)-(b) Asymptotic frequency, ω, and phase speed amplitude, $|\mathbf{C}|=|\omega/k|$, as fuctions of the angle of obliquity, φ, for different perturbation wavelengths.
(c)-(d) Asymptotic spectral distribution of the phase velocity amplitude, $|\mathbf{C}|$, for three angles of obliquity (φ =





References

[1] Criminale W. O., Drazin P. G., Stud. Applied Math. 83:123-157, 1990.

[2] Scarsoglio S., et. All., Stud. Applied Math. 123:153-173, 2009.

[3] Scarsoglio S., Tordella D., Criminale W. O., Phys. Rev. E 81:036326, 2010.

[4] Scarsoglio, De Santi, Tordella submitted to NJP