12th European Turbulence Conference

Linear generation of multiple time scales by three-dimensional unstable perturbations

S. Scarsoglio[#], **D.Tordella**[#] and **W. O. Criminale***

[#] *Dipartimento di Ingegneria Aeronautica e Spaziale, Politecnico di Torino, Torino, Italy*

* *Department of Applied Mathematics, University of Washington, Seattle, Washington, Usa*

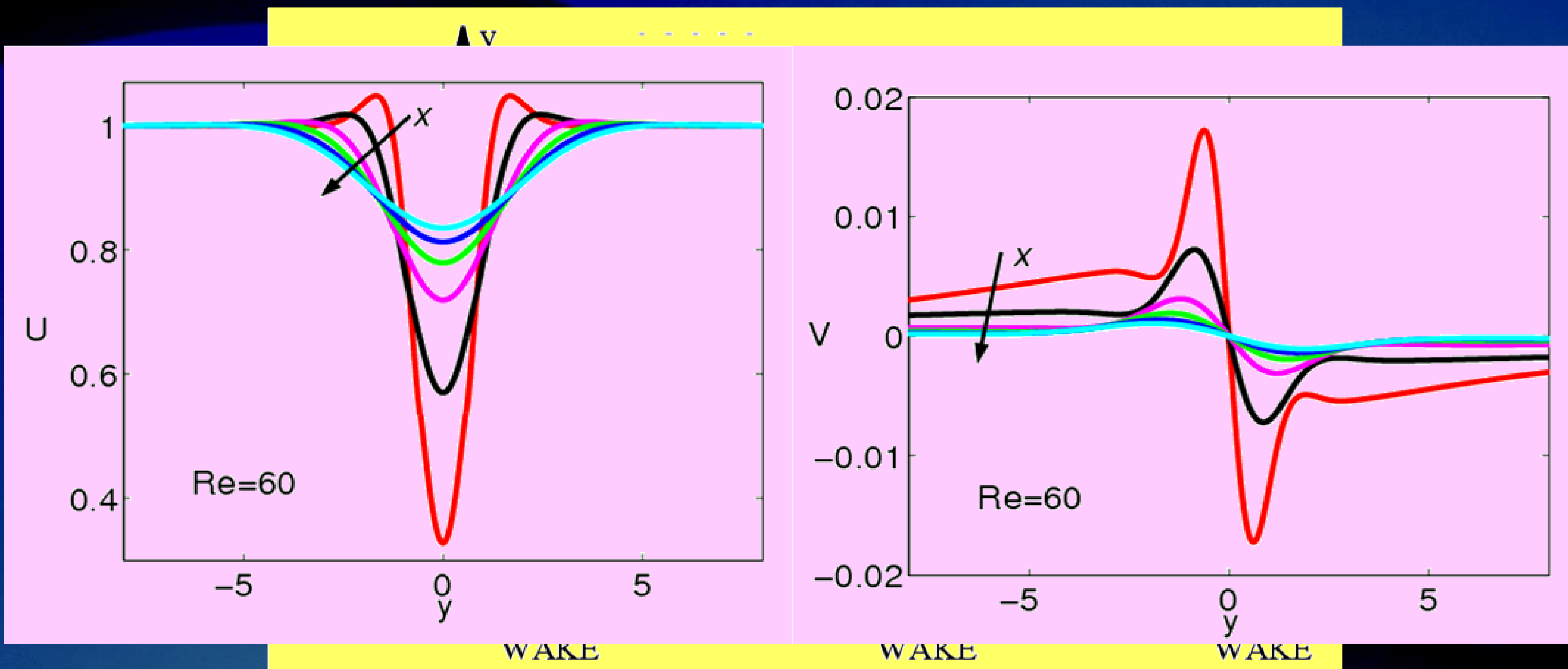
Marburg, Germany
September 7-10, 2009

Introduction

- Appearance of different time scales during the transient growth of a small two-dimensional or three-dimensional perturbation applied to the cylinder wake:
  - Temporal scales different each other and different from the values obtained in the asymptotic limit;
  - Variety of time scales associated to a given specific value of the instability wavelength;
  - Universal behaviour revealed by asymmetric oblique or longitudinal disturbances;
- These phenomena are developing in the context of the linear dynamics;
- The frequency determination is validated through the asymptotic comparison with normal mode theory and experimental results.

Base flow: 2D cylinder wake

- Flow behind a circular cylinder ➡ steady, incompressible and viscous;
- Approximation of 2D asymptotic Navier-Stokes expansions (Belan & Tordella, 2003) ➡ $U(y; x_0, Re)$



The initial-value problem formulation

- Linear, three-dimensional perturbative equations in terms of velocity and vorticity (Criminale & Drazin, 1990);
- Laplace-Fourier transform of perturbation quantities in x and z directions, α complex, γ real (Scarsoglio et al., 2009);

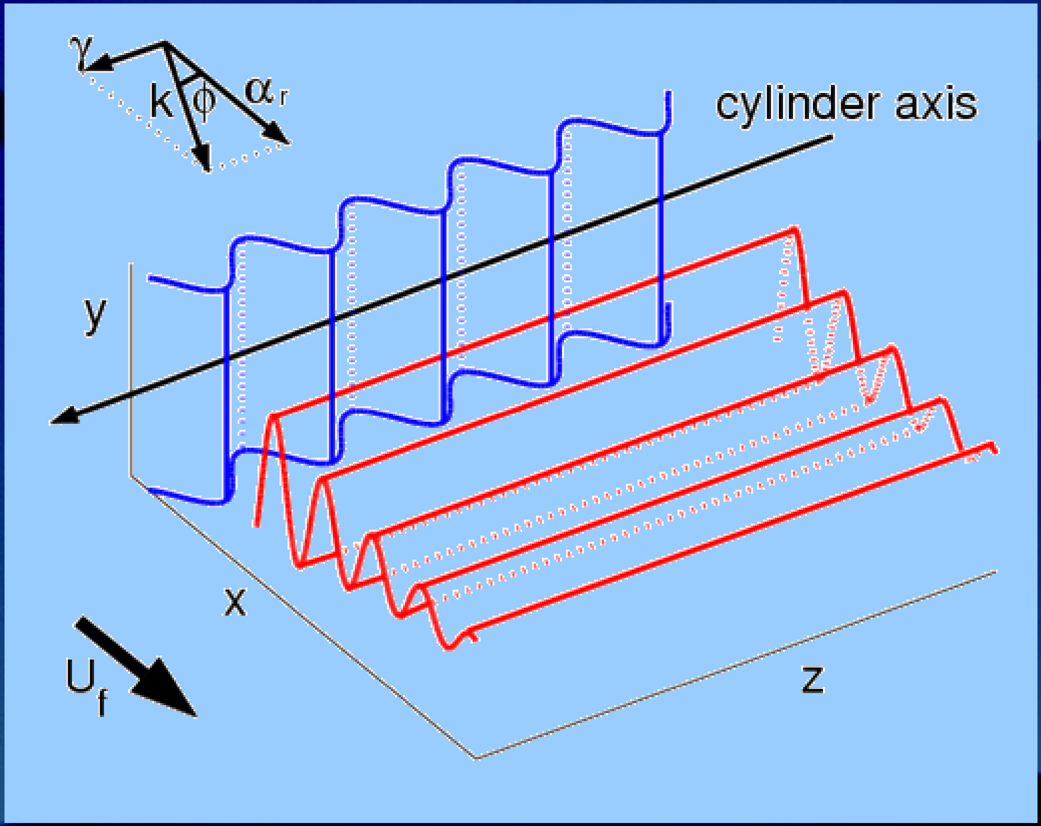
α_r = longitudinal wavenumber

γ = transversal wavenumber

Φ = angle of obliquity

k = polar wavenumber

α_i = spatial damping rate



- The governing partial differential equations are:

$$\begin{cases} \dfrac{\partial^2 \hat{v}}{\partial y^2} & - \quad (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{v} = \hat{\Gamma} \\ \dfrac{\partial \hat{\Gamma}}{\partial t} & = \quad (i\alpha_r - \alpha_i)(\dfrac{d^2U}{dy^2}\hat{v} - U\hat{\Gamma}) + \dfrac{1}{Re}[\dfrac{\partial^2\hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\Gamma}] \\ \dfrac{\partial \hat{\omega}_y}{\partial t} & = \quad -(i\alpha_r - \alpha_i)U\hat{\omega}_y - i\gamma\dfrac{dU}{dy}\hat{v} + \dfrac{1}{Re}[\dfrac{\partial^2\hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\omega}_y] \end{cases}$$

where the transversal velocity and vorticity components are indicated as $\hat{v}$ and $\hat{\omega}_y$ respectively, while $\hat{\Gamma}$ is defined through the kinematic relation $\tilde{\Gamma} = \partial_x\tilde{\omega}_z - \partial_z\tilde{\omega}_x$.

- Periodic initial conditions:

$$\hat{\omega}_y(0,y) = 0 \quad \begin{cases} \hat{\Gamma}(0,y) = e^{-y^2}\sin(y) & \text{asymmetric} \\ \text{or} & \\ \hat{\Gamma}(0,y) = e^{-y^2}\cos(y) & \text{symmetric} \end{cases}$$

- The perturbation velocity field has to vanish in the free stream.

Early transient and asymptotic behaviour

- Kinetic energy density *e* of the perturbation:

$$e(t;\alpha,\gamma,Re) = \frac{1}{2}\frac{1}{2y_d}\int_{-y_d}^{+y_d}(|\widehat{u}|^2+|\widehat{v}|^2+|\widehat{w}|^2)dy$$
$$= \frac{1}{2}\frac{1}{2y_d}\frac{1}{|\alpha^2+\gamma^2|}\int_{-y_d}^{+y_d}(|\frac{\partial\widehat{v}}{\partial y}|^2+|\alpha^2+\gamma^2||\widehat{v}|^2+|\widehat{\omega}_y|^2)dy$$

- The amplification factor *G*:

$$G(t;\alpha,\gamma) = \frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

measures the growth of the perturbation at time *t*.

- The temporal growth rate *r* (Lasseigne et al., 1999) is

$$r(t;\alpha,\gamma) = \frac{log|e(t;\alpha,\gamma)|}{2t}, \quad t > 0$$

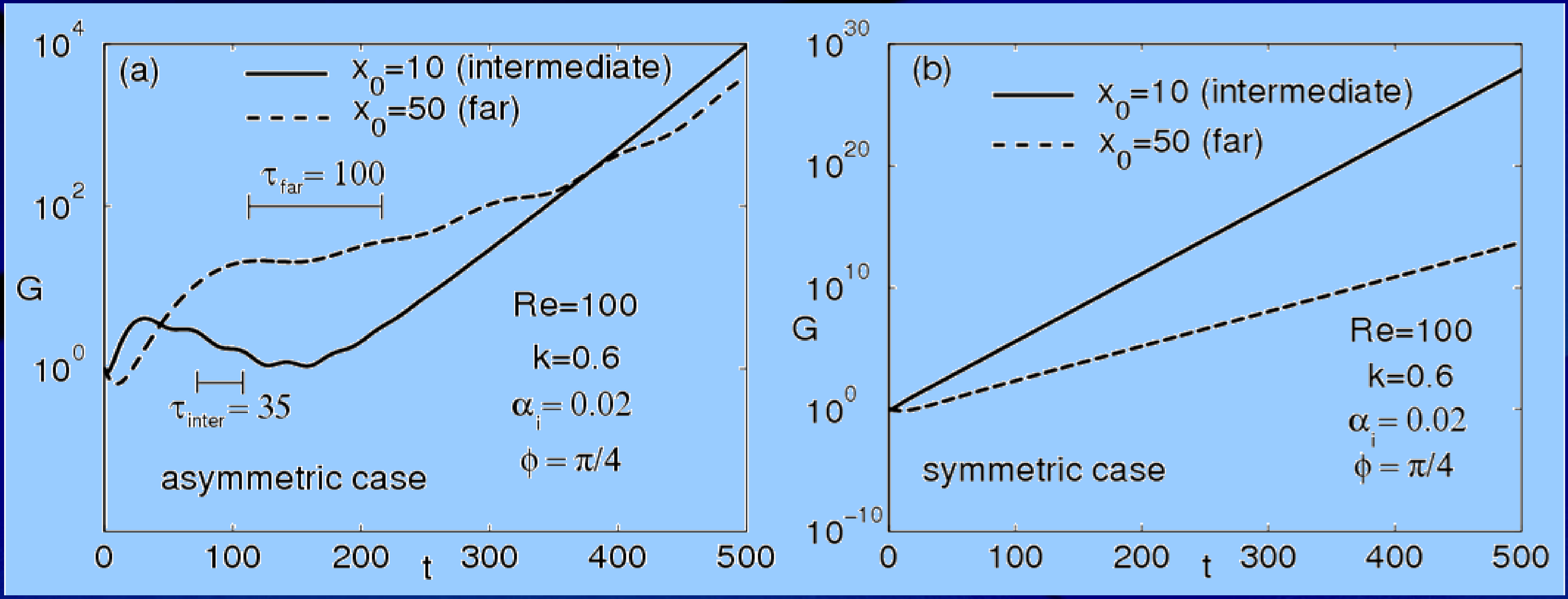
- The angular frequency (pulsation) ω (Whitham, 1974) is

$$\omega(t) = \frac{d\varphi(t)}{dt}$$

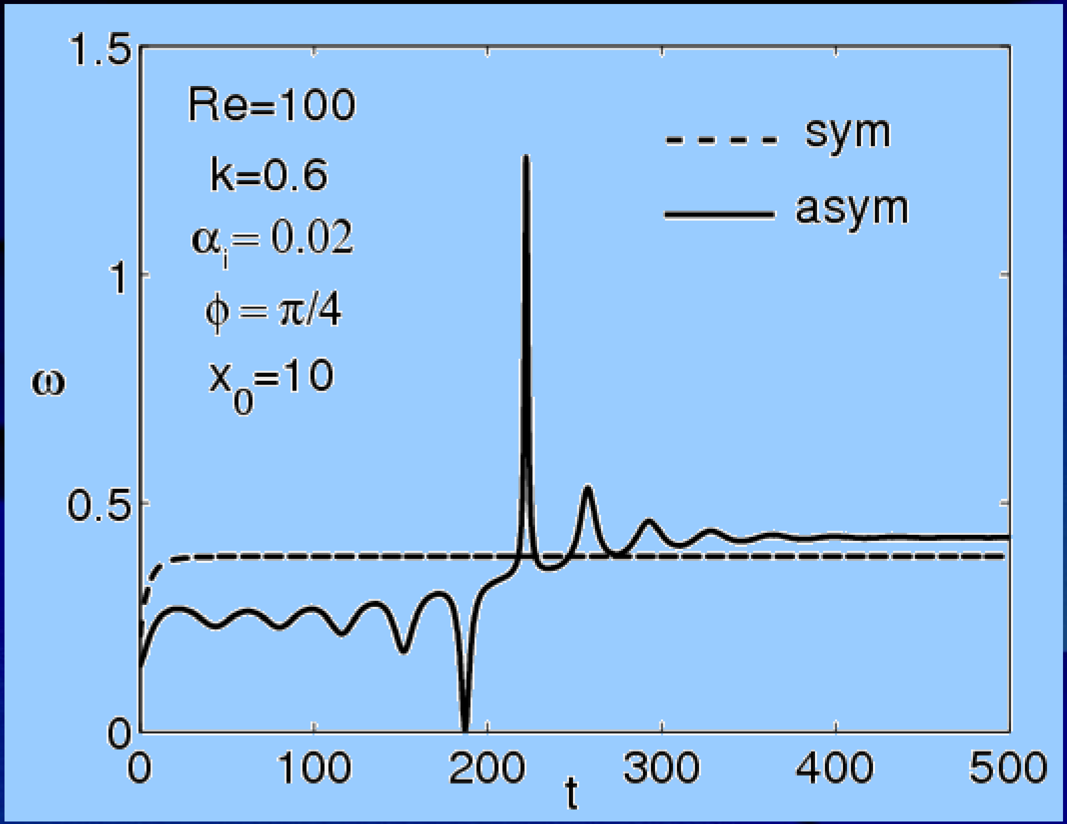
φ time phase

$$\hat{v}(y,t;\alpha,\gamma,Re) = A_t(y;\alpha,\gamma,Re)e^{i\varphi(t)}$$

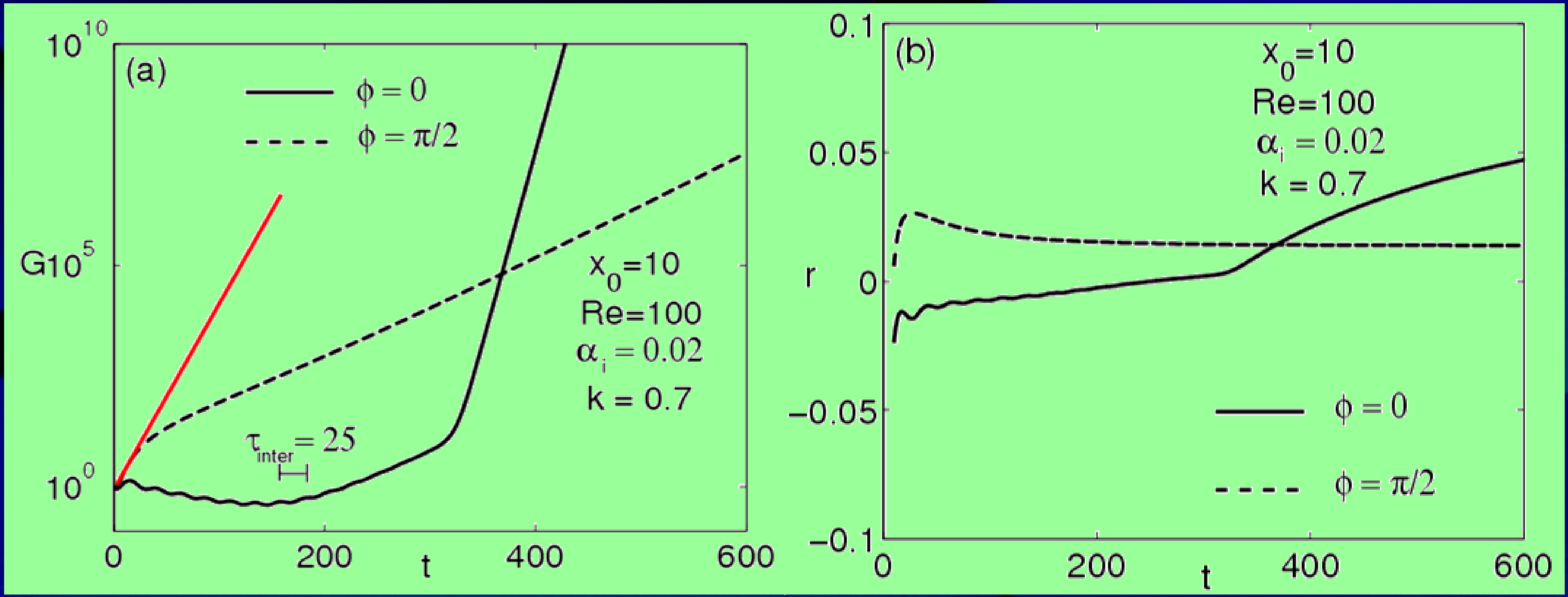
Results



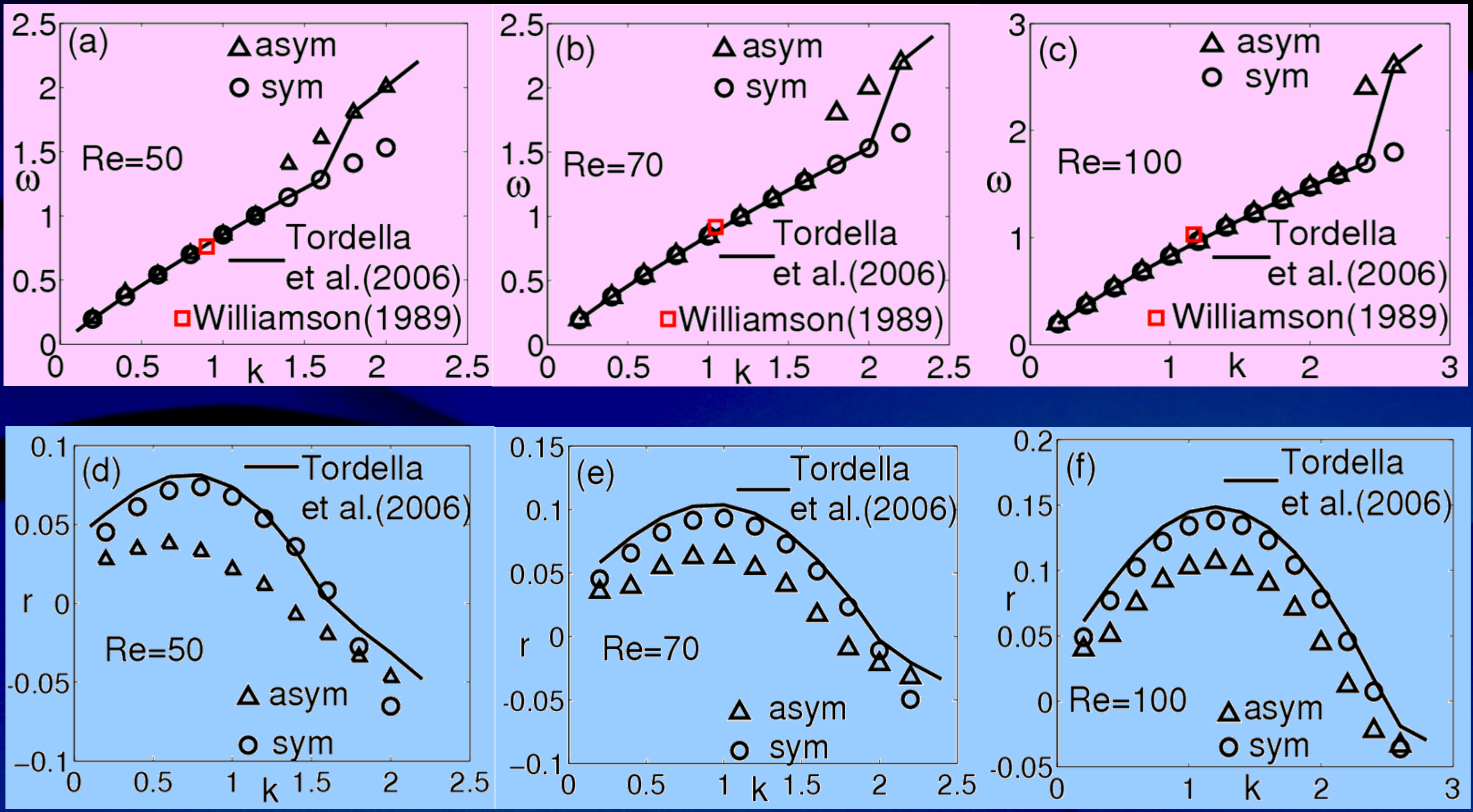
(a)-(b) Effect of the symmetry of the perturbation. The amplification factor G. (a) asymmetric initial condition, (b) symmetric initial condition. Intermediate (x_0=10, solid curves) and far field (x_0=50, dashed curves) wake configurations.



Pulsation ω. Intermediate wake configuration (x_0=10), symmetric and asymmetric cases.



(a)-(b) Effect of the angle of obliquity Φ. (a) The amplification factor G and (b) the temporal growth rate r as functions of time. Asymmetric initial condition, $\Phi = 0$ (solid curves), $\Phi = \pi/2$ (dashed curves).



(a, b, c) pulsation ω and (d, e, f) temporal growth rate r for present results (asymmetric case: triangles, symmetric case: circles), modal analysis (Tordella et al. 2006) (solid curves) and experimental data (Williamson 1989) (squares). α_i=0.05, Φ=0, x_0=10, Re=50, 70, 100.

Conclusions

- Variety of temporal scales shown by the transient in the context of the linear dynamics;

- For asymmetric oblique or longitudinal waves it is possible to count up to five different time scales:
  - (i) The temporal scale D/U~1 related to the base flow;
  - (ii) The length of the transient (200-300 time units);
  - (iii)-(iv) The scales associated to the instability frequency in the early transient (about 25 time units) and in the asymptotic state (about 15 time units);
  - (v) The modulation of the pulsation in the early transient (about 35-40 time units);

- Asymptotically good agreement with normal mode theory and experimental data.