Does the Kolmogorov scaling bridge hydrodynamic linear stability and turbulence?

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Energy spectrum in fully developed turbulence

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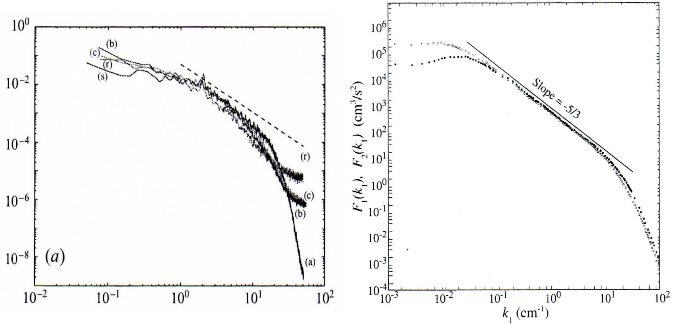
  $-5/3$ power-law for the energy spectrum over the inertial range;
- Common criterium for the production of a fully developed turbulent field to verify such a scaling (e.g. *Frisch, 1995*; *Sreenivasan & Antonia, ARFM, 1997*; *Kraichnan, Phys. Fluids, 1967*).



(left) *Evangelinos & Karniadakis, JFM 1999*. (right) *Champagne, JFM 1978*.

Energy spectrum and linear stability analysis

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- The perturbative evolution is ruled by the **initial-value problem** associated to the Navier-Stokes linearized formulation.

Spectral analysis through initial-value problem

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- **Is the linearized perturbative system able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument?**
- We determine the **energy decay exponent of arbitrary perturbations in their asymptotic states** and we compare it with the -5/3 Kolmogorov decay.

Perturbation scheme

- Linear 3D perturbative equations in terms of velocity and vorticity (*Criminale & Drazin, Stud. Appl. Math., 1990*);

Perturbation scheme

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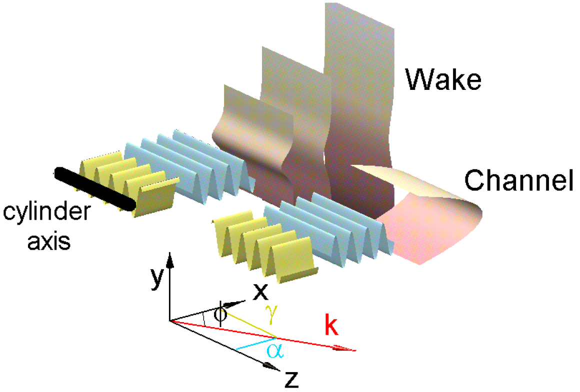

Perturbative equations

- Perturbative linearized system:

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\frac{\partial^2 \hat{v}}{\partial y^2} &- k^2 \hat{v} = \hat{\Gamma} \\
\frac{\partial \hat{\Gamma}}{\partial t} &= i\alpha \left(\frac{d^2 U}{dy^2} \hat{v} - U \hat{\Gamma} \right) + \frac{1}{Re} \left(\frac{\partial^2 \hat{\Gamma}}{\partial y^2} - k^2 \hat{\Gamma} \right) \\
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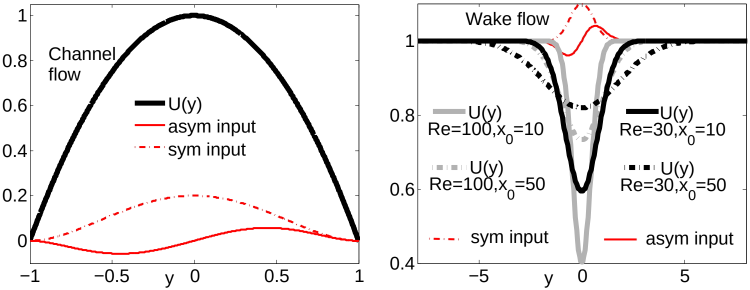

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Perturbation energy

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- Kinetic energy density e:

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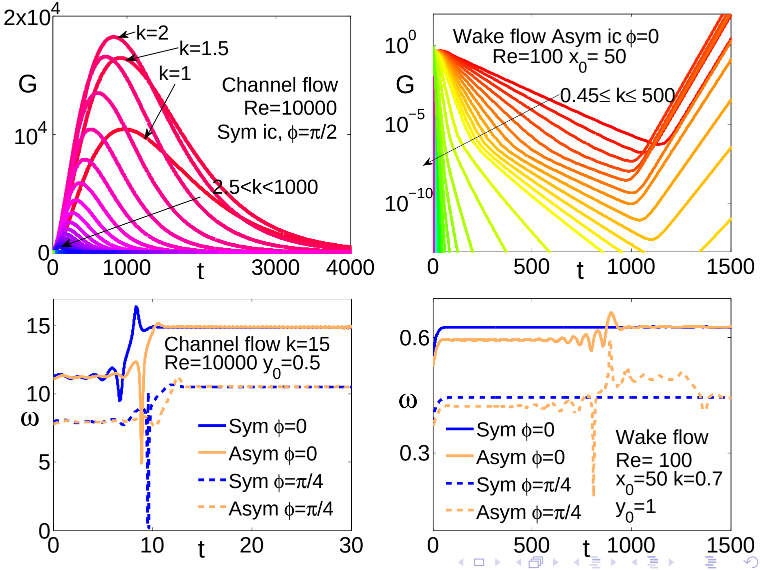
- Angular frequency (pulsation) ω:

$$\omega(t;y=y_0,\alpha,\gamma) = \frac{d\varphi(t;y=y_0,\alpha,\gamma)}{dt}, \qquad \varphi \text{ time phase}$$

Relevant transient behaviors



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- The energy spectrum is evaluated as the wavenumber distribution of the amplification factor, G(k);
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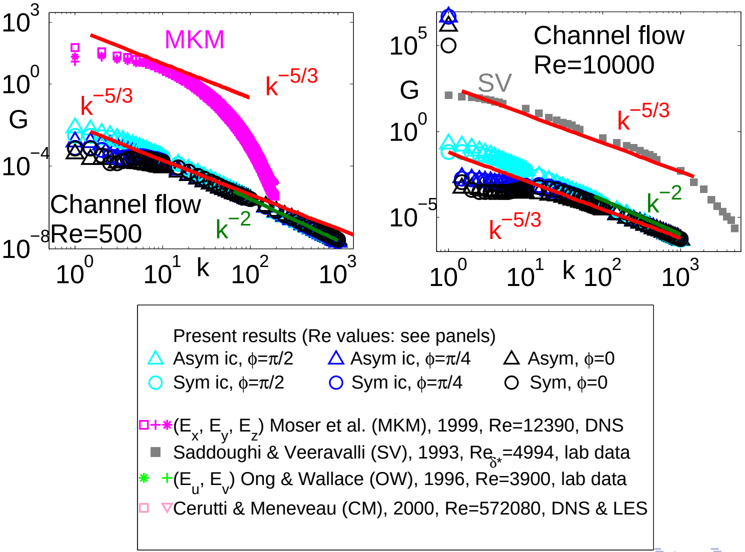
- Every perturbation has a characteristic transient exit time, T_e;
- The asymptotic condition is reached when the perturbative wave exceeds the transient exit time, T_e, that is when $r \sim const$ is satisfied for stable and unstable waves.

Scarsoglio, De Santi & Tordella, submitted to Phys. Rev. Lett., 2011.

Energy $G(k)$ at the asymptotic state ($r \sim$const)



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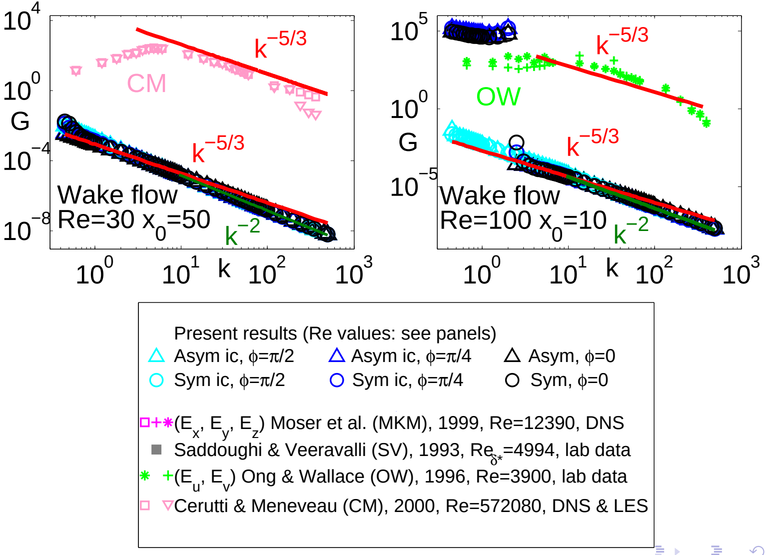
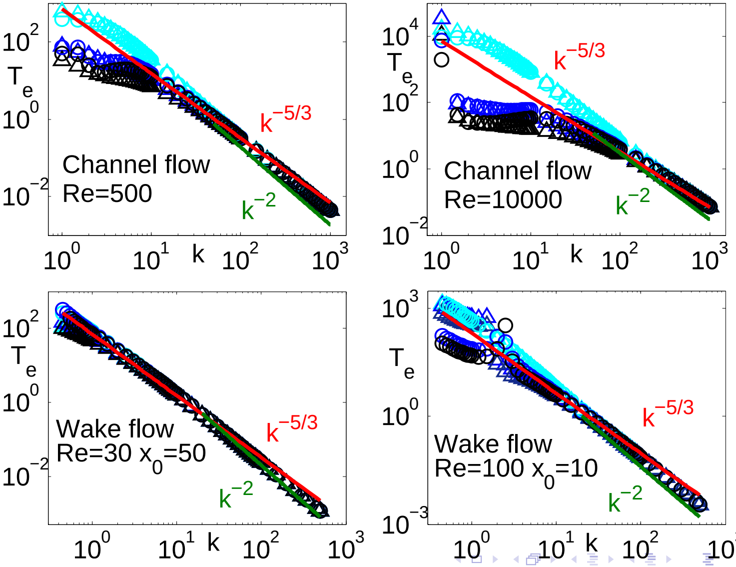


Transient exit time $T_e(k)$



Concluding remarks

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- Scale-invariance of G and T_e at different (stable and unstable) Reynolds numbers and for different shear flows;
- **The $-5/3$ spectral power-law scaling of inertial waves seems to be a general and intrinsic dynamical property of the NS solutions encompassing the nonlinear interaction.**

Coming next...

- **Analysis of the perturbation transient dynamics in the 2D and 3D boundary layer** (*W. O. Criminale, University of Washington);*

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- **Analytical integration of the kinetic energy equation based on the perturbed velocity and vorticity field** (*G. Staffilani, MIT)*
  $\Rightarrow$ **Study of the intermediate and long term asymptotics.**