Campo di pressione medio e perturbato nello strato limite tridimensionale

Perturbative and mean pressure field in the three-dimensional boundary layer

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1) **Ludwig Prandtl’s definition** on *Third Mathematics Congress*, Heidelberg, 8 August 1904:

«The boundary layer is a thin region near the surface, where effect of friction causes the fluid to stick to the surface (no-slip condition). Outside the boundary layer the flow is essentially inviscid, while very large velocity and pressure gradients are present within it.»

2) **Simplification** for the boundary layer of:

- Euler equations
- Navier–Stokes equations

**Obtaining:** **BOUNDARY LAYER EQUATIONS** *(parabolic behavior)*

**Coupled non-linear partial differential equations**

**Viscosity effects** *(elliptic behavior)*
Three-dimensional boundary layer

- **Generation and development:**
  - Strong pressure gradient at tip
  - "Crossflow" Phenomenon:
    - Motion of fluid particles toward the receding tip
    - Premature Separation

- **Velocity profiles:**
  - "Crossflow" velocity profile \( w \) presents:
    - Inflection point leads to Instability

Swept-back wings
Crossflow instability

**PRIMARY receptivity process**

For very strong inflectional velocity profiles:

**SECONDARY**

1) CROSSFLOW VORTICES

2) SECONDARY VORTICES

**FINAL BREAKDOWN** (TERTIARY VORTICES)

Theory of stability

- Method of small disturbances:
  \[ u = U + \tilde{u} \quad v = V + \tilde{v} \quad w = W + \tilde{w} \quad p = P + \tilde{p} \]

- Orr-Sommerfeld equation:
  \[
  (U - c)(\phi'' - \alpha^2 \phi) - U'' \phi = -\frac{i}{\alpha R} (\phi'''' - 2\alpha^2 \phi'' + \alpha^4 \phi)
  \]

EIGENVALUE PROBLEM

\[
  c = c_r + ic_i
\]

\[
  \begin{cases}
  c_i > 0 \\
  c_i < 0
  \end{cases}
\]
**Mathematical model**

- **BASE FLOW**

  Crossflow component generates $V_\infty$:

  \[
  \begin{align*}
  U_s &= U(x, Re) \\
  W_s &= U(x, Re)
  \end{align*}
  \]

- **Linearized PERTURBED SYSTEM**

  \[
  \tilde{u}, \tilde{v}, \tilde{w} \ll U_\infty, U_\infty \]

  \[
  k = \sqrt{\alpha_r^2 + \gamma^2} \quad \rightarrow \quad \lambda = \frac{2\pi}{k}
  \]

  Obliquity angle: $\phi = \tan^{-1}\left(\frac{\gamma}{\alpha_r}\right)$

  Made DIMENSIONLESS with:

  \[
  \begin{align*}
  \delta^* &= \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy \\
  Re_{\delta^*} &= \frac{U_\infty \delta^*}{v}
  \end{align*}
  \]

  NAVIER-STOKES equations (Fourier space)

  Initial and boundary conditions (y-coordinate) for $\tilde{v}, \tilde{w}$
Pressure field: NUMERICAL SIMULATION

Two methods implemented by MATLAB® software:

- Mutual comparison and validity control
- Temporal and efficiency evaluations: METHOD 2

Spatial LAPLACE-FOURIER decomposition

\[
\begin{align*}
\frac{\partial \varphi}{\partial x} & \rightarrow i\alpha \cdot \varphi \\
\frac{\partial \varphi}{\partial z} & \rightarrow i\gamma \cdot \varphi \\
\nabla^2 \varphi & \rightarrow \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \cdot \varphi
\end{align*}
\]

\[
\begin{align*}
\hat{p} = -\frac{\partial \hat{u}}{\partial t} - i\alpha \hat{u}U_s - \hat{v} \frac{\partial U_s}{\partial y} - i\gamma \hat{w}W_s + \frac{1}{Re_{\delta^*}} \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \hat{u} \\
\frac{\partial \hat{p}}{\partial y} = -\frac{\partial \hat{v}}{\partial t} - i\alpha \hat{v}U_s - i\gamma \hat{w}W_s + \frac{1}{Re_{\delta^*}} \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \hat{v} \\
\frac{\partial \hat{p}}{\partial t} = -\frac{\partial \hat{w}}{\partial t} - i\alpha \hat{w}U_s - \hat{v} \frac{\partial W_s}{\partial y} - i\gamma \hat{w}W_s + \frac{1}{Re_{\delta^*}} \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \hat{w}
\end{align*}
\]

NUMERICAL INTEGRATION by MATLAB®:

\[\hat{p} = \hat{p}(y, t, \alpha, \gamma) \xrightarrow{F^{-1}} \hat{p} = \hat{p}(x, y, z, t)\]
Simulation results: PARAMETRIC ANALYSIS

\[ p = P(x) + \tilde{p}(x, y, z, t) \]

PERTURBATIVE PRESSURE FIELD

- Effects of crossflow CHARACTERISTIC PARAMETERS:

<table>
<thead>
<tr>
<th>REYNOLDS NUMBER (Re_\delta)</th>
<th>(Re=100)</th>
<th>(Re=5000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE GRADIENT (\beta)</td>
<td>1, -0.1988</td>
<td></td>
</tr>
<tr>
<td>CROSSFLOW ANGLE (\theta)</td>
<td>(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})</td>
<td></td>
</tr>
<tr>
<td>OBLIQUITY ANGLE (\phi)</td>
<td>0, (\frac{\pi}{4}, \frac{\pi}{2})</td>
<td></td>
</tr>
<tr>
<td>WAVELENGTH NUMBER (k)</td>
<td>0.02, 0.06, 0.1, 0.6, 1, 1.2, 1.6, 2</td>
<td></td>
</tr>
</tbody>
</table>

- Comparison with AMPLIFICATION FACTOR:

\[ G(t; \alpha, \gamma) = \frac{e(t; \alpha, \gamma)}{e(t = 0; \alpha, \gamma)} \]
1. Flow configuration: $\theta = 30^\circ$, $\phi = 45^\circ$, $\beta = 1$ for $Re = 100$ ➞ STABLE

2. Flow configuration: $\theta = 60^\circ$, $\phi = 90^\circ$, $\beta = 1$ for $Re = 5000$ ➞ UNSTABLE

Perturbative and mean pressure field in the three-dimensional boundary layer

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• Flow configuration: \( \theta = 60^\circ, k = 0.4, \beta = 1 \) for \( Re = 5000 \)

Obliquity angle \( \phi \)

OBLIQUE WAVE for \( \phi = 45^\circ \)

ORTHOGONAL WAVE for \( \phi = 90^\circ \)
Crossflow angle $\theta$

- High values increase the **TRANSIENT GROWTH** in amplitude and time
  
  Flow configuration: $\phi = 45^\circ$, $k = 0, 4$, $\beta = 1$ for $Re = 5000$

- Unstable configurations:
  1. $\theta = 30^\circ, \phi = 0^\circ$ for $Re=5000$
  2. $\theta = 60^\circ, \phi = 90^\circ$ for $Re=5000$

Pressure gradient $\beta$

- $\beta = 1$ (negative gradient) $\Rightarrow \frac{\Delta p}{\Delta x} < 0$ (flow acceleration) **STABILIZING**
- $\beta = -0.1988$ (positive gradient) $\Rightarrow \frac{\Delta p}{\Delta x} > 0$ (flow deceleration) **DESTABILIZING**

Flow configuration: $\phi = 45^\circ$, $k = 0, 4$, $\beta = 1$ for $Re = 5000$
Simulation results: PRESSURE TOTAL FIELD

Effects of perturbative pressure field on mean pressure field:

\[ p = P(x) + \tilde{p}(x, y, z, t) \]

**Simulation settings:**

- Mean pressure \( P(x) \) computation:
  \[
  \begin{align*}
  U(x) &= U_\infty x^m \\
  P(x) &= -\rho U_\infty x^{2m} + c
  \end{align*}
  \]

- Quantities made **dimensional** with:
  \[ \rho U_\infty^2, \delta, \delta^*, t_{car} \]

**Simulation domain**

\[ \Delta P / \Delta x > 0 \]

\[ p = p_\infty^{1.0} \]

\[ (\beta = -0.1988) \]

**Parameters**:
- \( c = 1 \, m \)
- \( P_\infty = 1 \, atm = 101350 \, Pa \)
- \( \rho_\infty = 1.225 \, kg/m^3 \)
- \( \nu = 1.5 \times 10^{-5} \, m^2/s \)
- \( U_\infty = 210.4626 \, m/s \)
- \( T = T_{amb} \)
- \( Re_{\delta^*} = 5000 \)
- \( \delta(0.5) = 0.00086 \, m \)
- \( \delta^*(0.5) = 0.0002975 \, m \)
1. Flow configuration: \( \theta = 30^\circ, \ \phi = 0^\circ \)

Mean field

\[ P [\text{Pa}] \]

\[ x [\text{m}] \]

\[
\begin{array}{cccc}
    k=0.06 & k=0.4 & k=1 & k=1.6 \\
\end{array}
\]

For \( k=0.06 \):
\[ \Delta p = 1000 \text{ Pa} \]
\[ \delta = 18 \text{ mm} \]

2. Flow configuration: \( \theta = 60^\circ, \ \phi = 90^\circ \)

Total field

\[ P [\text{Pa}] \]

\[ x [\text{m}] \]

\[
\begin{array}{cccc}
    k=0.06 & k=0.1 & k=0.4 & k=1.6 \\
\end{array}
\]

For \( k=0.06 \):
\[ \Delta p = 1800 \text{ Pa} \]
\[ \delta = 30 \text{ mm} \]
1. Flow configuration: $\theta = 30^\circ$, $\phi = 0^\circ$

2. Flow configuration: $\theta = 60^\circ$, $\phi = 90^\circ$
Conclusions

1. PARAMETRIC ANALYSIS

• Correlation between $G(t)$ and $\tilde{p}(t)$

• Combination of destabilizing parameters

2. DIMENSIONAL ANALYSIS

• Strong temporal pressure growth

• Elevated spatial $(x,z)$ and temporal oscillations

3. FURTHER DEVELOPMENT

• Increased computational domains $(x,y,z,t)$

• Different fly conditions included
Grazie per la cortese attenzione