Effects of arrhythmias on arterial fluid dynamics

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Atrial fibrillation: the most common form of arrhythmia

- HEART CONTRACTION COMPROMISED
- IRREGULAR / HIGH FREQUENCY HEARTBEATS

- 7 million of people affected in the USA and Europe
  - higher incidence with age
  - risks: heart failure and stroke
    (responsible for 15-20% of total ischemic strokes)

Several key points on AF consequences are not still clear...
Aim: arterial pressure and flow rate responses in AF

What’s the problem under the fluid mechanics point of view?

- unsteady viscous motion
- not perfect incompressible fluid: blood
- variable pulsating pressures as forcing
- deformable vessels: arteries
  - anisotropic non-linear viscoelastic behaviour
  - complicated geometry

How to solve such a complex problem?

- a multi-scale mathematical model coupling sub-models of different dimensions (0D e 1D)
Domain

PRESENT ELEMENTS
- left heart
- 48 arteries
- 18 distal groups
- 24 arterial junctions
- both arms
- one leg

NEGLECTED ELEMENTS
- right heart
- venous return
- coronary circuit
- cerebral circuit
Resolution large-to-medium arteries

- Axisymmetric vessel geometry and flow field
- Laminar flow (mean Re=1000-100 along the arterial tree)
- Longitudinally tethered arterial walls
- Homogeneous and Newtonian blood

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
\]

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( 2\pi \int_0^R r^2 \, rd\tau \right) = -\frac{A}{\rho} \frac{\partial P}{\partial x} + 2\pi \nu \left[ \frac{\partial u}{\partial r} \right]_{r=R}
\]

\[
u(r,x,t) = \begin{cases} 
  u(x,t), & 0 < r < R(x,t) - \delta(x,t) \\
 \frac{R^2(x,t) - r^2(x,t)}{2R(x,t)\delta(x,t) - \delta^2(x,t)} u(x,t), & r \geq R(x,t) - \delta(x,t)
\end{cases}
\]

\[
P(x,t) = P_e(x,t) + P_v(x,t) = f(A(x,t))
\]

\[
[u \cdot \hat{r}]_{r=R} = 0, \quad [u \times \hat{r}]_{r=R} = 0
\]
Resolution left ventricle
- a time-varying elastance model

Resolution mitral and aortic valves
- an ideal diode for mitral valve
- a pressure-flow relation for aortic valve

Resolution microcirculation districts

Resolution arterial junctions
- conservation of the total pressure and mass
Numerical resolution

- Discontinuous-Galerkin approach to discretize 1D space
- Second order Runge-Kutta explicit scheme to march in time
  - Time step: 1E-4s
  - Mean element length: 2.5cm
- Initial conditions: P=100mmHg, NO FLOW
- Convergence after around 7 heart cycles
- Pressure and flow rate time series everywhere, as output

Calibration of model parameters

Expected hemodynamic results, for a healthy young man, without AF

<table>
<thead>
<tr>
<th></th>
<th>EDV [ml]</th>
<th>ESV [ml]</th>
<th>SV [ml]</th>
<th>SW [J]</th>
<th>CO [L/min]</th>
<th>Psys [mmHg]</th>
<th>Pdia [mmHg]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>120.2</td>
<td>52.9</td>
<td>67.4</td>
<td>0.92</td>
<td>5.0</td>
<td>120.5</td>
<td>71.0</td>
</tr>
</tbody>
</table>
Fibrillated sequence of heartbeat periods RR

NEGLECTED EFFECT
- variation in mean heartbeat period/frequency in AF

EFFECTS CONCERNED
With respect to the Normal Sinus Rhythm (NSR):
- reduced temporal correlation
- increased temporal variability (higher $\sigma$)
Results from the AF simulation & data elaborations

2000
PRESSURE & FLOW RATE signals, at each arterial site

Example at the beginning of aorta:

Interesting results are drawn from the Probability Density Functions (PDFs) for
- systolic pressures
- diastolic pressures
- mean flow rates

Each PDF can be characterized through
- its mean value, $\mu$
- its standard deviation, $\sigma$
- its coefficient of variation, $cv = \sigma/\mu$
- its skewness, $S$, and kurtosis, $K$, values
Results I

Example at the beginning of aorta:

- SLIGHTLY HIGHER
  - mean systolic pressure
  - mean diastolic pressure
  - mean values of mean flow rate than without AF, for equal mean heartbeat period (0.8s)

Short and long beats of the simulated RR sequence in AF produce mean changes in all quantities of interest as if we had a mean heartbeat period shorter than 0.8s!
Results II

FLUCTUATIONS IN

- systolic pressure: 7-8%
- diastolic pressure: 13-16%
- mean flow rate: 12-33%

Arterial system tends to amplify and damp diastolic and systolic pressure oscillations, respectively, going towards distal regions!
Results III

S & K for systolic/diastolic pressure and mean flow rate, significantly grow with the distance from the heart.

Disorders introduced by AF are particularly amplified at specific arterial zones!
Why?

Pressure and flow signals are nothing but waves
- travel at a finite speed (waves speed or phase velocity within the high frequency range)
- are reflected (especially at the arterial bifurcations)

TOTAL PRESSURE AND FLOW SIGNALS at a generic site b depend on:

1) pressure and flow signals (at point a)
2) the local phase velocity (at point b)
3) how waves are reflected (at point c)
4) distance to the nearest site of reflection (bc)

- oscillations in local phase velocities: 6.7-8.7%
- fluctuations in local magnitudes of reflections: 5-35%

These variations are magnified with the number of bifurcations per unit of length...
To Conclude...

Results in Atrial fibrillation:
- small growth in mean values of systolic/diastolic pressures and mean flow rates
- significant oscillations in all these quantities
- normal wave propagation and reflection along aorta altered

Limitations:
- neglected elements in the chosen domain (e.g. coronary and cerebral circulations)
  - hypothesis on which some sub-models are based
  - only one mean heartbeat frequency for the RR sequence in AF

Future applications:
- introducing the missing parts of the actual domain
- inquiring into the role played by the mean heartbeat frequency on arterial fluid dynamics
- studying effects of pathologies such as hypertension on circulatory system
- entering the world of space medicine
Thanks you for your attention!
The boundary layer thickness is calculated through a not-dimensional parameter: the Womersley number

- Womersley number \( \alpha = R \sqrt{\frac{\omega}{\nu}} \)
- Cardiac pulsation \( \omega = \frac{2\pi}{T} \)

\[ \alpha = 1 \Leftrightarrow \delta \approx 0.7 \text{mm} \quad \text{if} \quad T = 0.8 \text{s} \]
APPENDIX 2)

Sound velocity in blood: \( a = 1570 \text{m/s} \)

Mach numbers:

- For mean blood velocity
  \[ M = \text{from } 2 \times 10^{-6} \text{ to } 6 \times 10^{-7} \]

- For maximal blood velocity
  \[ M = \text{from } 2 \times 10^{-5} \text{ to } 4 \times 10^{-6} \]

- For phase velocity
  \[ M = \text{from } 3 \times 10^{-3} \text{ to } 4 \times 10^{-3} \]
Lacal wave speed at b

Conditions of reflection at c

\[ RI = \frac{|P_{\text{backward}}|}{(|P_{\text{forward}}| + |P_{\text{backward}}|)} \]