Some Aeroelastic Aspects of Slender Wings Design

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U.S.A. Project for High Altitude Platform

www.dfrc.nasa.gov/Projects/Erast (1994)
HAVE-UAV (High Altitude Very-long Endurance/Unmanned Air Vehicles) capable of remaining aloft for very long period of time (9-12 months) by a solar-powered and fuel cells system.
HELIPLAT® (*HELlos PLATform*)

- WT = 8000N; Wpl = 1000N; Preq = 6kW; Ppl = 0.8kW; Vc = 71 km/h (TAS); n max = 3.1
- Sw = 176.5m²; b = 73m; ARw = 30.2
- Croot = 2.97m; taper r = 0.32
- S ht = 28m²; b = 17.5m; AR ht = 11; C = 1.6m.

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**CASA:**
1) Single CFRP elements: wing tubular spars and ribs, horizontal and vertical tail tubular spars and ribs, booms. 2) Metal fittings.

**POLITO-DIASP:**
1) Assembling of different parts of the aircraft (wing, horizontal and vertical tails, booms). 2) Assembling the whole aircraft. 3) Perform static tests up to the design loads 4) Find the correlation with the numerical analysis.
TYPICAL AEROELASTIC PROBLEMS:

- Flutter
- Load Distribution
- Buffeting
- Dynamic Loads
- Divergence
- Control Effectiveness and Reversal
**LOAD DISTRIBUTION** – During the flight the deflection of structure tends to redistribute the air-loads in a significantly different way. It is of course necessary a specific verification of the structural design. The case of a swept wing is presented in fig.1-5. The elastic deflection increases the lift within the fuselage, moving the center of pressure inboard rather than outboard as for straight wings.

(figure 1.5 - after Bisplinghoff)  
(figure 5.17 - after Chiocchia)
Qualitative comparison of critical speeds

(figure 1.8 - after Bisplinghoff)
Aeroelastic behavior: General trends

(figures after Dowell)
SLENDER WING : Structural considerations

The concept “slender” has to be defined in a coherent way avoiding not justified simplification in the model. The isotropic beam structural problem with a solid or closed form section is analyzed according to Cicala’s point of view as an example (finite modulus and maximum stress applied to the structure is reduced to the order \( \varepsilon \)). Let us introduce:

\[
\frac{h}{L} = \varepsilon^x \quad \rho \approx \varepsilon^y \quad k = \varepsilon^z
\]

If a bending strain of order 1 is expected, then \( x+z=1 \) : that is \( X=1, Z=0 \) if dimensional ratio is of order \( \varepsilon \). This configuration is defined as ROD. It assumes a finite curvature variation and displacements of the same order as the beam length. The Euler critical stress in this case is:

\[
\sigma_c = \frac{\pi^2 EI}{Al^2} \approx \varepsilon^2
\]

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If again a bending strain of order 1 is expected, then \( x+z=1 \), but with \( X=0.5, Z=0.5 \) then the finite curvature variation and displacements remain of the same order as the section dimensions. The Euler critical stress in this case is:

\[
\sigma_C = \pi^2 EI / Al^2 \approx \varepsilon
\]

This configuration is defined as BEAM (classical CLT beam).

An interesting observation regards the applied loads. Considering a ROD cantilever with a transversal load \( F \), the maximum bending moment is \( M=FL=2\sigma I/h \). If the maximum tension is of \( \varepsilon^1 \), \( M \) is \( \varepsilon^4 \) as \( F \). Adding a longitudinal load of the same order 4, the longitudinal traction stress is of order 2 lower than the maximum bending stress and for this reason negligible. This is a not-extensional configuration.
INFLUENCE OF AEROELASTIC PHENOMENA ON DESIGN

- Modern aircraft design and advanced aircraft design have to cope with aeroelastic effects.

- The aeroelasticity is defined (Cox and Pugsley) as:
  Substantial interaction among the aerodynamic, inertial, structural forces that act upon and within the flight vehicle.

(figure from Kolonay 2001)
\[
\sigma_{00} - \nu/(1 + \nu)[\sigma_{00} + \sigma_{11} + \sigma_{22}] = 2G(e_0 + u_{0,0} + \Delta \omega_1 \xi_2 - \Delta \omega_2 \xi_1)
\]
\[
\sigma_{11} - \nu/(1 + \nu)[\sigma_{00} + \sigma_{11} + \sigma_{22}] = 2G u_{1,1}
\]
\[
\sigma_{22} - \nu/(1 + \nu)[\sigma_{00} + \sigma_{11} + \sigma_{22}] = 2G u_{2,2}
\]
\[
\sigma_{01} = G(u_{0,1} + u_{1,0} - \Delta \omega_0 \xi_2)
\]
\[
\sigma_{02} = G(u_{0,2} + u_{2,0} + \Delta \omega_0 \xi_1)
\]
\[
\sigma_{12} = G(u_{1,2} + u_{2,1})
\]
\[
\sigma_{11,22} + \sigma_{22,11} - 2\sigma_{12,12} = \nu(\ast_{11} + \ast_{22})(\sigma_{11} + \sigma_{22} + 2G u_{0,0})
\]
\[
\sigma_{00,0} + \sigma_{01,1} + \sigma_{02,2} = \sigma_{01,0} + \sigma_{11,1} + \sigma_{12,2} = \sigma_{02,0} + \sigma_{12,1} + \sigma_{22,2} = 0
\]
\[ \frac{h}{L} = \epsilon \quad M_{1,2} = N_{1,2} = \epsilon^5 \quad \Delta \omega_{1,2} = \epsilon \quad N_0 = M_0 = \epsilon^5 \quad \Delta \omega_0 = \epsilon \]  

(33)

Tabella di classificazione per la trave

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(after Cicala 1985, Frulla 1999, Frulla 2001)
Simple model:

(figure from Isaaks)
Table TA1 (P is the ratio of the initial pitch freq. with flap freq., Q the ratio of the lag freq. with flap freq., alfa is the flap freq. assumed unitary)

<table>
<thead>
<tr>
<th>Static position(rad)</th>
<th>P</th>
<th>Q</th>
<th>Alfa</th>
</tr>
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<tr>
<td>0</td>
<td>6.007</td>
<td>0.5</td>
<td>0.998</td>
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<tr>
<td>0.05</td>
<td>6.005</td>
<td>0.497</td>
<td>1.0023</td>
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<tr>
<td>0.1</td>
<td>5.99</td>
<td>0.489</td>
<td>1.009</td>
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<tr>
<td>0.15</td>
<td>5.98</td>
<td>0.475</td>
<td>1.022</td>
</tr>
<tr>
<td>0.2</td>
<td>5.97</td>
<td>0.455</td>
<td>1.04</td>
</tr>
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<td>0.25</td>
<td>5.95</td>
<td>0.43</td>
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<td>5.93</td>
<td>0.399</td>
<td>1.09</td>
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</tbody>
</table>

Table TA2 (P is the ratio of the initial pitch freq. with flap freq., Q the ratio of the lag freq. with flap freq., alfa is the flap freq. assumed unitary)

<table>
<thead>
<tr>
<th>Static position(rad)</th>
<th>P</th>
<th>Q</th>
<th>Alfa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.004</td>
<td>1.98</td>
<td>0.999</td>
</tr>
<tr>
<td>0.05</td>
<td>2.202</td>
<td>1.821</td>
<td>1.0015</td>
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<td>2.453</td>
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<td>2.756</td>
<td>1.563</td>
<td>1.02</td>
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<tr>
<td>0.2</td>
<td>3.114</td>
<td>1.462</td>
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<td>0.25</td>
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<td>0.3</td>
<td>4.005</td>
<td>1.285</td>
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DYNAMIC BEHAVIOR OF A CANTILEVER BEAM: TIP MASS EFFECT (by Da Silva and Hodges). A comparison between different approximation was done in order to verify if the usual procedure is correct.

(figure 1 from Da Silva – Hodges)

The effect of tip displacement in function of non-dimensional tip mass is reported in table for zero pitch (see specific paper for other beam data):

<table>
<thead>
<tr>
<th>v/l</th>
<th>Freq.a</th>
<th>Freq.b</th>
<th>P</th>
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<tbody>
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<td>0.011</td>
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<td>0.0218</td>
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<td>0.0325</td>
<td>1.38</td>
<td>0.341</td>
<td>1.419</td>
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</table>
(figure 8, figure 9 after Da Silva, Hodges)
SLENDER WINGS : Aerodynamic considerations

Specific theoretical-experimental study about the effect of finite span on the aerodynamic forces of an oscillating airfoil was developed in 1938 (Cicala) and these concepts were improved by other researchers as Jones, Biot, Kussner, Reissner, Wasserman. Because of the complexity of calculations and since finite span effects for wings with moderate aspect ratios are in general small, most of flutter analyses employ two-dimensional aerodynamic coefficients.

The first solution to the unsteady theoretical aerodynamic problem were obtained in two dimensions by Theodorsen (1935) and by Kussner and Schwarz (1940).

An early method of flutter analysis (by Smilg and Wasserman -1942) divided the wing into a number of strips. The aerodynamic loads on each strip were calculated on the basis of the two-dimensional coefficients evaluated at the centerline of the strip.

This “Strip Theory” was surprisingly accurate in many cases, although its successes were primarily in applications to unswept wings with high aspect ratios (corrections were introduced for finite span effect and mach effect). A control surface is also introduced and both the airfoil and the control surface are assumed rigid in the chord-wise direction.
SLENDER WINGS : Aerodynamic considerations

\[
\begin{align*}
\{\alpha_g\} &= [A]\{lC_L\} \\
\begin{bmatrix}
    b_{11}/4b + 1/l_1 C'_L \inf & -b_{12} & \cdots & -b_{1m} \\
    -b_{21} & b_{22}/4b + 1/l_2 C'_L \inf & \cdots & -b_{2m} \\
    \cdots & \cdots & \cdots & \cdots \\
    -b_{m1} & -b_{m2} & \cdots & b_{mm}/4b + 1/l_m C'_L \inf
\end{bmatrix}
\end{align*}
\]

\[
\{\alpha_g\} &= [A_{ST}]\{lC_L\} \\
\begin{bmatrix}
    1/l_1 C'_L & 0 & \cdots & 0 \\
    0 & 1/l_2 C'_L & \cdots & 0 \\
    \cdots & \cdots & \cdots & \cdots \\
    0 & 0 & \cdots & 1/l_m C'_L
\end{bmatrix}
\]
SLENDER WINGS: Aero-structural considerations

\[ M_b = 0.25 \frac{\rho_a}{\rho} E \frac{V^2}{E/\rho} \frac{c}{L} C_L^1 \alpha L^3 \]

\[ \frac{c}{L} = \varepsilon^{0.5}; \frac{\rho_a}{\rho} = \varepsilon^{1.5}; \frac{V^2}{E/\rho} = \varepsilon^2; \alpha = \varepsilon \]

dimensional ratio with a value typical of an aspect ratio of more than 25.

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Classical flutter behavior: preliminary calculations

(after Frulla 2002)
PRELIMINARY NON-LINEAR ANALYSIS OF “SCALED” SIMILAR WING:

A cantilever slender wing has been studied (Frulla 2002) in order to spot some characteristics of this type of structure. A simple assumption has been made: constant stiffness and mass properties. Stiffness ratio : EI_{lag}/EI_{flap}=0.2 ; GJ/EI_{flap}=0.1 ; mass: m=4.22 kg/m

A uniform distributed load has been applied in order to have a reasonable static deflection. The tip deflection to length ratio is : w/L=3.24/12.

Modal analysis of the two configuration:

<table>
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<th>Linear modes (Hz)[type]</th>
<th>Non-linear modes(Hz)[type]</th>
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<td>1.47[in-plane]</td>
<td>1.45[in-plane +T]</td>
</tr>
<tr>
<td>1.55[flap]</td>
<td>1.61[flap]</td>
</tr>
<tr>
<td>4.54[in-plane]</td>
<td>3.01[in-plane + flap + T]</td>
</tr>
<tr>
<td>7.05[flap + T]</td>
<td>7.47[flap + T + inp]</td>
</tr>
<tr>
<td>10.2[flap + T]</td>
<td>10.2[flap + T + inp]</td>
</tr>
</tbody>
</table>
PRELIMINARY ANALYSIS OF “SCALED” SIMILAR WING:

First linear

First non-linear

Second linear

Second non-linear

(after Frulla 2002)
PRELIMINARY ANALYSIS OF “SCALED” SIMILAR WING:
Modal shape comparison: third mode linear versus non-linear

(after Frulla 2002)
PRELIMINARY ANALYSIS OF “SCALED” SIMILAR WING:
Modal shape comparison: fourth mode linear versus non-linear

Fourth linear

Fourth non-linear

(after Frulla 2002)

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PRELIMINARY ANALYSIS OF “SCALED” SIMILAR WING:
FLUTTER ANALYSIS: V-g plots  linear
PRELIMINARY ANALYSIS OF “SCALED” SIMILAR WING:

FLUTTER ANALYSIS: V-g plots non-linear

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(after Frulla 2002)
FLUTTER CALCULATION FOR A SLENDER WING AIRCRAFT (Patil, Hodges, Cesnik 2001). The geometrical data are: chord/L=1/16, m=0.75kg/m. I=0.1kgm, GJ/EIf=0.5; EIlag/EIf=100. First torsion/first flap freq.=13.86; firstlag/first flap freq.=14.13.

(frequencies behavior versus tip displacement - after Patil Hodges Cesznik 2001)
A significant reduction (up to 20%) in flutter speed is obtained with a tip deflection of about 0.1-0.15 of the half-span. Aeroelastic tailoring and a certain degree of opposite curvature in the wing can reduce this deleterious effect (down pre-curvature is limited by the ground clearance).

(flutter speed versus tip displacement - after Patil Hodges Cesznik 2001)
**Whirl flutter** (propeller-nacelle):
It is important to note that if the nacelle is rigid, and the aerodynamic is not present, the system (4-blade rotor) is always stable. Since the whirl mode produce some angle of attack changes, aerodynamic forces provide the mechanism of instability.

(figure 1, figure 2, after Reed 1985)
Dynamic coupling of propeller-nacelle system with the wing motions can alter the whirl flutter boundaries predicted by rigid wing assumption. The general trend is indicated with aerodynamic loads acting on propeller and wing.

(figure 4 - after Reed 1985)
The Dynamic stability of a suspended rigid rotor (without aerodynamic) was studied by the author in a specific application (advanced joined rotor) in order to point out some characteristic of the inertia inequalities and elastic component of the suspension.

\[
[M] \begin{pmatrix} \dot{\xi} \\ \dot{\gamma} \end{pmatrix} + 2\Omega [J] \begin{pmatrix} \dot{\xi} \\ \dot{\gamma} \end{pmatrix} + 2\Omega [M_0] \begin{pmatrix} \dot{\xi} \\ \dot{\gamma} \end{pmatrix} + \omega_0^2 [K] \begin{pmatrix} \zeta \end{pmatrix} = 0
\]

\[
\omega_0^2 = \frac{(K_\zeta + K_\gamma)}{(I_0 + I_1)}; \quad \epsilon_i = \frac{(I_0 - I_1)}{(I_0 + I_1)} \quad A = \frac{I_2}{(I_0 + I_1)} \quad \epsilon_s = \frac{(K_\zeta - K_\gamma)}{(K_\zeta + K_\gamma)}
\]

(after Frulla 2000)

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Referring to unsymmetrical rigid rotor as in this case of two blades, $L$ is the length of a blade, $c, b$ the transversal dimensions and $m$ the blade mass, the inertial parameters are:

$$A = \frac{1 + c^2 / 4L^2}{1 + (2b^2 + c^2) / 4L^2 + 6(h / L)^2} \quad \varepsilon_i = \frac{1 - c^2 / 4L^2}{1 + (2b^2 + c^2) / 4L^2 + 6(h / L)^2}$$

If the blades are slender the transversal dimension can be neglected obtaining:

$$\varepsilon_i = A$$

(Values different from unity refer to rotor with a distance between the rotor and the joint)

(after Frulla 2000)
An elastically symmetric rotor ($\varepsilon_s=0$) is considered. Value of $A=0.5$ divides the region in two fields: for $A$ less than 0.5 there are two critical velocities that include an unstable field. For $A$ equal or higher than 0.5 there is only one critical velocity and the upper part of the plane is unstable.
For rotors with elastic symmetry, the critical velocities are well defined, while for different elastic parameter other unstable fields appear. Two particular cases are reported: 1) $A=0.2715$, $\varepsilon_i=0.234$; 2) $A=0.9$, $\varepsilon_i=0.9$.

(after Frulla 2000)
New tendency in AEROSPACE: AAW Concept

- A new Air Force research project at NASA Dryden has been developing with the aim of taking an element of the Wright brothers into the new designed aircraft.

- This “back to the future” involves the Active Aeroelastic Wing concept (AAW) that is trying to use the elastic twist and warping of a wing to control the aircraft.

- Modern engineers have done everything possible to counteract such kind of deformations from physically stiffening the wings.

- Wings that typically twist in flight are thin and light with high aspect ratio providing good lift and drag characteristics. But they are overstressed. For this reason in the past the response to such kind of phenomena was to remove it.

- Researchers believe AAW concepts can be tailored to control wing twist at high speed and improve roll maneuvering perhaps to the point of eliminating the tail completely. The success of this new wing is connected to the overstress controlling.
New tendency in AEROSPACE: AAW Concept (CONTINUATION)

• The AAW concept can provide a more high-aspect-ratio weight competitive wing and a more efficient one reducing fuel consumption.

• Some applications can involve future airplanes (HALE UAV – supersonic cruiser). The energy of the air is used as a motor to obtain a desired twist: the control surface will be only as tabs in order to initiate the expected effect. The aim of the design will be to use this characteristic instead of trying to overcome control surface losses due to elastic wing twist.

• Some expected figures says that an AAW wing can give a weight saving of about 25% if applied correctly.
SHORT REFERENCE SUMMARY