

Dispersive to nondispersing transition in the plane wake and channel flows

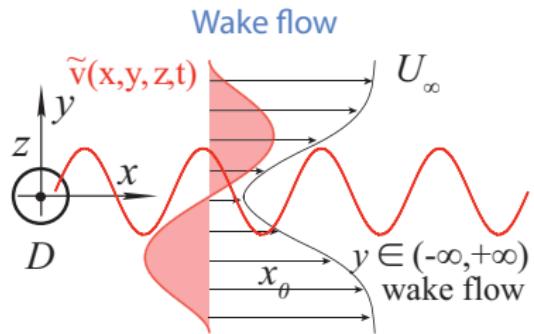
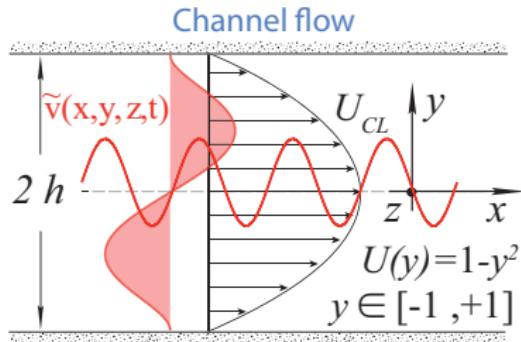
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Problem formulation



Fourier/Laplace decomposition

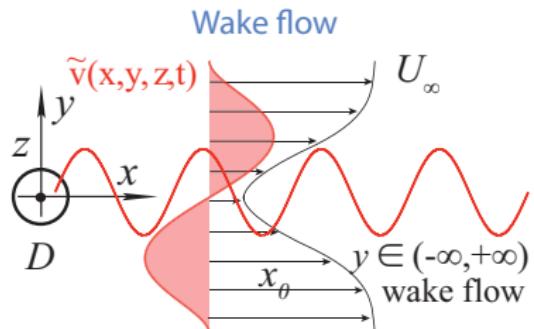
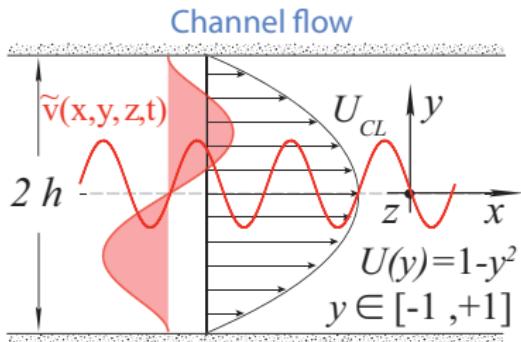
$$\hat{g}(y, t; \alpha, \gamma) = \int_{-\infty}^{+\infty} \int_{0, -\infty}^{+\infty} \tilde{g}(x, y, z, t) e^{-i\alpha x - i\gamma z} dx dz$$

Wave frequency and phase velocity

$$\theta_w(y, t; \alpha, \gamma) = \arg(\hat{v}(y, t; \alpha, \gamma))$$
$$\omega(t; y_0, \alpha, \gamma) = |d\theta(t; y_0, \alpha, \gamma)|/dt$$

$$\mathbf{c} = (\omega/k)\hat{\mathbf{k}} \quad (1)$$

Problem formulation



Orr-Sommerfeld/Squire non modal equations (wavenumbers space)

$$[(\partial_t + i\alpha U)(\partial_y^2 - k^2) - i\alpha U'' - \frac{1}{Re}(\partial_y^2 - k^2)^2]\hat{v} = 0$$

$$[(\partial_t + i\alpha U) - \frac{1}{Re}(\partial_y^2 - k^2)]\hat{\eta} = -i\gamma U'\hat{v}$$

Boundary conditions

$$\hat{v}(\pm 1, t) = \partial_y \hat{v}(\pm 1, t) = \hat{\eta}(\pm 1, t) = 0 \quad \text{channel}$$

$$\partial_y^2 \hat{v} = k^2 \hat{v}, \quad \hat{\eta} = 0 \quad y \rightarrow \pm\infty, \forall t \quad \text{wake}$$

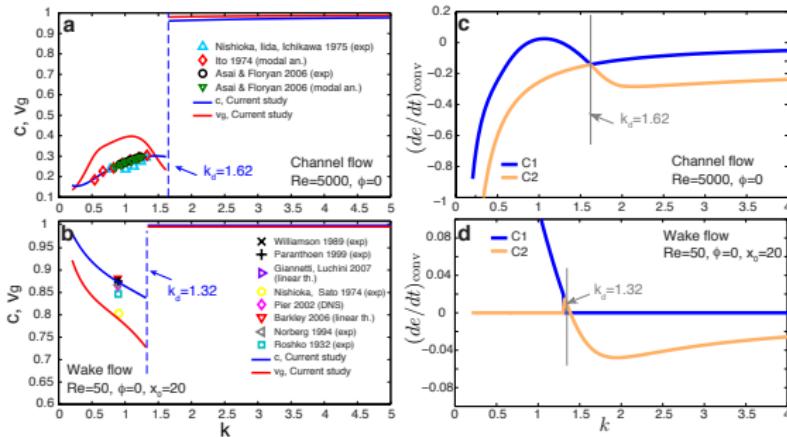
Basic flows

$$U(y) = 1 - y^2 \quad \text{channel}$$

$$U(y) = 1 - \frac{1}{\sqrt{x_0}}(1.22 + 6.7 \cdot 10^{-5} Re^2)e^{-Re y^2/(4x_0)} \quad \text{wake}$$



Dispersion relations of the least-stable mode



Kinetic energy equation (wavenumbers space)

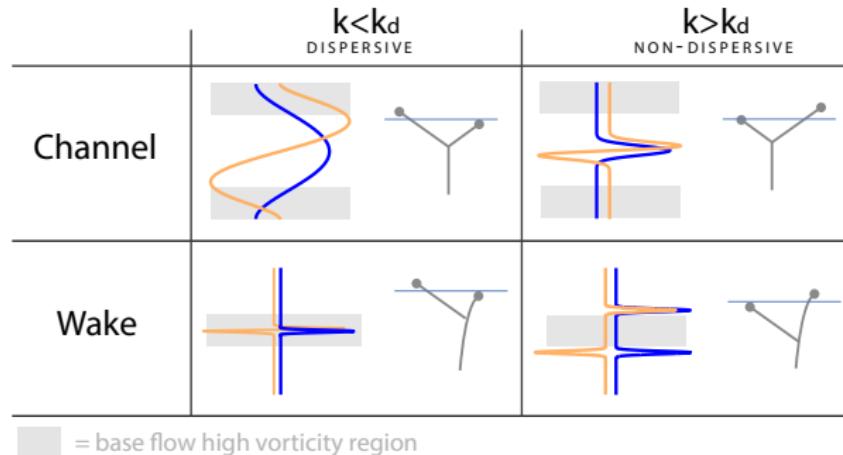
$$e(t; \alpha, \beta) = \frac{1}{2k^2} \int_{-y_f}^{+y_f} (|\partial_y \hat{v}|^2 + |k \hat{v}|^2 + |\hat{\eta}|^2) dy$$

$$\frac{de}{dt} = \underbrace{\frac{1}{k^2} Imag \int U' \left(\alpha \bar{v} \partial_y \hat{v} - \beta \bar{v} \hat{\eta} \right) dy}_{Convection} - \underbrace{\frac{1}{Re k^2} \int \left(|\partial_y^2 \hat{v}|^2 + 2k^2 |\partial_y \hat{v}|^2 + k^4 |\hat{v}|^2 + |\partial_y \hat{\eta}| + k^2 |\hat{\eta}|^2 \right) dy}_{Dissipation}$$

Dispersive/Nondispersive behavior

Existence of wavenumber threshold between dispersive/ nondispersive behavior, named k_d . Asymptotic perturbation:

- ▶ $k < k_d \rightarrow$ “shear modes” (“wall modes” for channel, “in-wake modes” for the wake): the perturbation highest vorticity is located where base flow has high vorticity. (shear region)
- ▶ $k > k_d \rightarrow$ “external modes” (“central modes” for channel, “out-of-wake modes” for the wake): the perturbation highest vorticity is located where base flow has low vorticity (out of shear region)
- ▶ Convective energy term is relevant $\rightarrow k_d$ is located at the crossing of least stable right and left eigenmodes curves



Trends for the threshold wavenumber k_d

Channel flow

- ▶ k_d increases with the wave angle ϕ
- ▶ k_d decreases with the Reynolds number Re

Re	$\phi = 0$	$\phi = \pi/6$	$\phi = \pi/4$	$\phi = \pi/3$
1000	2.071	2.111	2.168	2.256
2000	1.883	1.922	1.979	2.073
3000	1.764	1.803	1.866	1.960
4000	1.686	1.725	1.784	1.878
5000	1.623	1.662	1.721	1.815
6000	1.576	1.615	1.670	1.765
7000	1.536	1.568	1.627	1.720
8000	1.497	1.536	1.589	1.682

Wake flow, $x_0 = 20$

- ▶ k_d decreases with the wave angle ϕ
- ▶ k_d increases with the Reynolds number Re
- ▶ k_d decreases with the streamwise position x_0

Re	$\phi = 0$	$\phi = \pi/6$	$\phi = \pi/4$	$\phi = \pi/3$
20	0.756	0.732	0.691	0.616
30	0.968	0.943	0.896	0.815
40	1.153	1.118	1.086	0.987
50	1.325	1.294	1.250	1.159
60	1.471	1.441	1.400	1.318
70	1.616	1.587	1.550	1.463
80	1.748	1.719	1.686	1.596
90	1.881	1.851	1.809	1.728
100	1.992	1.983	1.945	1.860



Initial value problem: initial conditions

I.C.	Wake flow	Channel flow
SI/SC	$\hat{v}_0 = e^{-y^2} \cos(y)$	$\hat{v}_0 = e^{-\frac{y^2}{0.01}} \cos(3y)$
AI/AC	$\hat{v}_0 = e^{-y^2} \sin(y)$	$\hat{v}_0 = e^{-\frac{y^2}{0.01}} \sin(3y)$
SO/SW	$\hat{v}_0 = e^{-(y-10)^2} + e^{-(y+10)^2}$	$\hat{v}_0 = (1-y^2)^2$
AO/AW	$\hat{v}_0 = e^{-(y-10)^2} - e^{-(y+10)^2}$	$\hat{v}_0 = y(1-y^2)^2$

S=symmetric

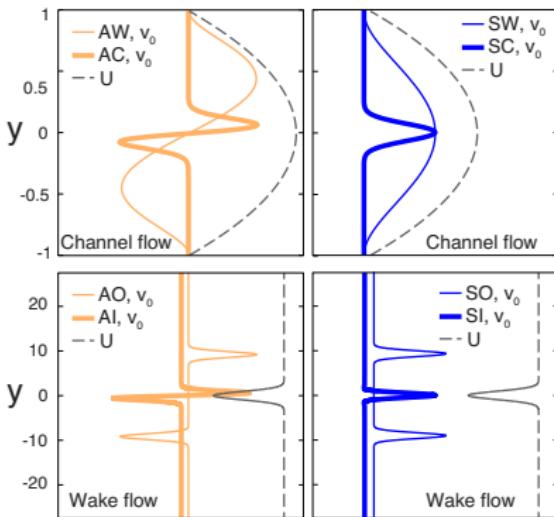
A=antisymmetric

W=wall

C=central

I=inside-wake

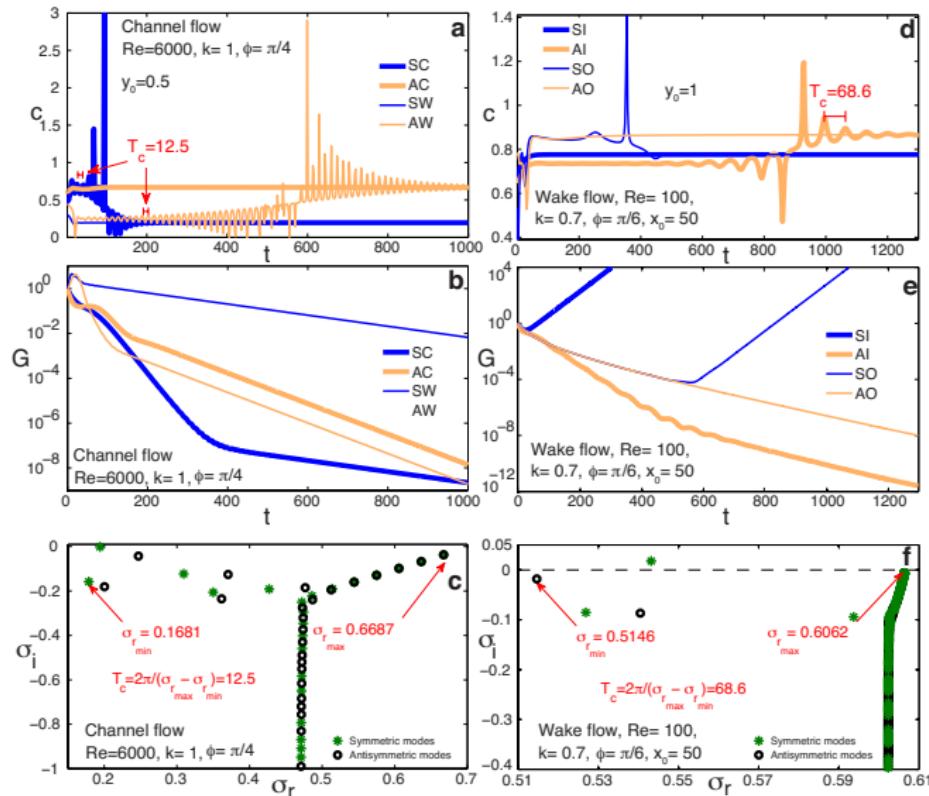
O=out-of-wake



Relevant features:

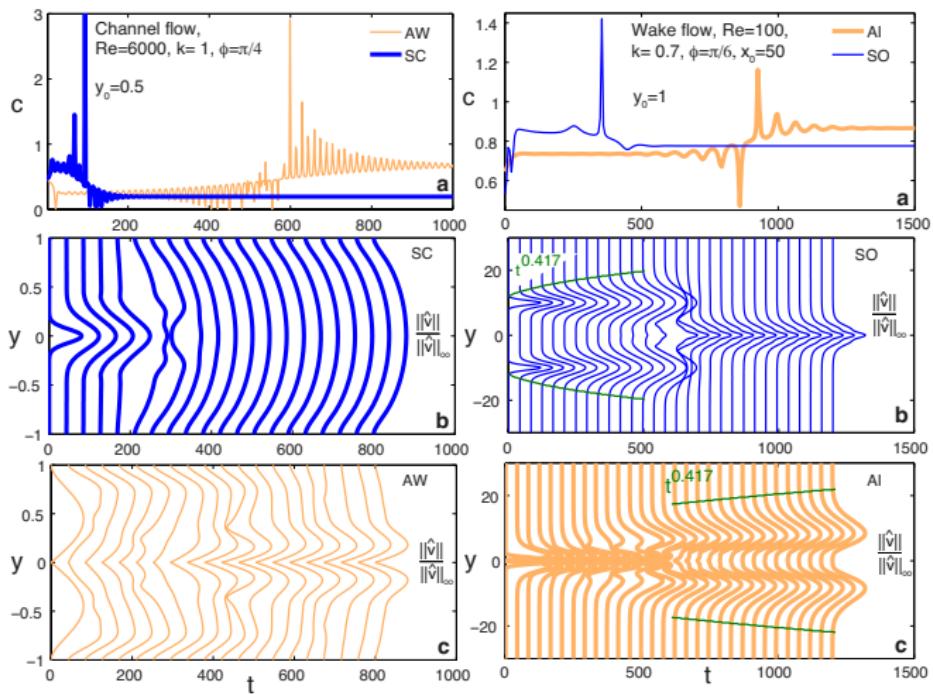
- ▶ Location of the perturbation hi-vorticity region with respect to the base flow one
- ▶ Parity of the initial condition

Initial value problem: phase velocity



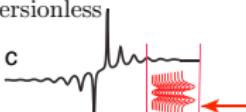
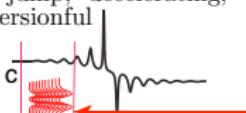
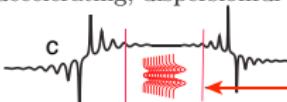
k_d and the spectrum width are global properties affecting the transient lives of the perturbations

Initial value problem: similarity of solution



Frequency jumps are symptoms of a change of identity of the solution. In the wake case, when the perturbation reaches the base flow shear region it is trapped in it (b) or, viceversa, an initial perturbation inside the wake can escape from the shear region, as in (c).

Initial value problem: classification of transients

Phase velocity qualitative transient	Wake flow	Channel flow
No jumps Short transient	<ul style="list-style-type: none">Initial conditions SI or AO and $k < k_d$Initial conditions SO or AO and $k > k_d$	<ul style="list-style-type: none">Initial conditions SW or AC, and $k < k_d$Initial conditions SC or AC, and $k > k_d$
One jump, accelerating, dispersionless 	<ul style="list-style-type: none">Initial conditions AI and $k < k_d$Initial conditions SI or AI, and $k > k_d$	<ul style="list-style-type: none">Initial conditions AW, and $k < k_d$Initial conditions SW or AW, and $k > k_d$
One jump, decelerating, dispersionful 	Initial conditions SO , and $k < k_d$	Initial conditions SC , and $k < k_d$
Two jumps, accelerating-decelerating, dispersionful 	Mixed initial conditions with dominant antisym. component, and $k < k_d$	Mixed initial conditions with dominant antisym. component, and $k < k_d$

Summary

- ▶ For $k < k_d$ the behavior (asymptotic, $t \rightarrow \infty$) is dispersive ($v_g \neq c$). For $k > k_d$ the behavior is not dispersive ($v_g = c$).
- ▶ The asymptotic velocities for $k < k_d$ are “shear-modes” (solutions with high vorticity in correspondence to the base flow high shear region), the opposite for $k > k_d$ (“central modes” for channel, “out-of-wake modes” for the wake).
- ▶ The threshold value k_d is the wavenumber at which the energy convective curves related to the least-stable modes of the left and right branch, respectively, encounter. As k increases, $C \rightarrow 0$.
- ▶ Different type of transients have been identified, depending on two features of the initial condition: the location of the perturbation and its parity.
- ▶ Frequency jumps and oscillations characterize the transient evolution
- ▶ Similarity properties show up in the intermediate term. The scaling exponent tends to the diffusive value of 0.5 as $\phi \rightarrow \pi/2$, as $Re \rightarrow 0$ and as $k \rightarrow \infty$



Scaling exponent for the wake transients

