A measure of turbulent diffusion in two and three dimensions

F. De Santi¹, L. Ducasse¹, J. von Hardenberg², M. Iovieno¹, D. Tordella¹

¹Politecnico di Torino, Torino, Italy
²Istituto di Scienze dell’Atmosfera e del Clima, CNR, Torino, Italy

September 14, 2010

European Fluid Mechanics Conference - 8
Presentation of the problem

2 turbulent flows put aside with different kinetic energies:
- a high energy field on the left of energy $E_1$
- a low energy field on the right of energy $E_2$

Mixing layer thickness: $\Delta(t)$

$\Delta(0) \approx l$ (integral scale)

$l \approx D/80$

Periodic boundary conditions: 2 mixing layers in the simulation
Presentation of the problem

Main goals:

▶ Study the turbulent diffusion through the evolution in time of the mixing layer
▶ Compare 2D and 3D cases
Presentation of the problem

Main goals:
- Study the turbulent diffusion through the evolution in time of the mixing layer
- Compare 2D and 3D cases

Shearless mixing layers show the following properties:
- No gradient of mean velocity → no kinetic energy production
- Mixing generated by the inhomogeneity in the turbulent kinetic energy
- Intermittent behavior at both large and small scales (EC-512, 2009)
- Gradient of energy: sufficient condition for the onset of intermittency (Phys.Rev.E, 2008)
- 2D and 3D mixings → show a very different behaviour
A visualisation

Kinetic energy: evolution in time

Initial energy ratio: $E_1/E_2 = 6.6$
Important remarks

Main parameter : Initial energy ratio $E_1/E_2$

The system has been studied using the values : $E_1/E_2 = 6.6, 40, 300, 10^4, 10^6$

In the Navier Stokes equation :

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{\rho} \nabla p + (-1)^{p+1} \nu_n \Delta^{2n} \mathbf{u}$$

**2D** : An hyperviscous coefficient ($n = 2$) has been used

**3D** : The total energy decays faster than in **2D**
Evolution of the mixing layer

Time evolution of the mixing layer thickness $\Delta(t)$:

$2D$ mixes faster!
Velocity statistics

Skewness (computed along the homogeneous $y$ direction)

$E_1/E_2 = 10^4$
Velocity statistics

Kurtosis (computed along the homogeneous y direction)

\[ \frac{E_1}{E_2} = 10^4 \]
Velocity statistics

Position of the maximum of skewness $X_S$

2D

$$X_S(t) \propto t$$ evolves faster than $\Delta(t) \propto t^{0.7}$

3D

$$X_S(t) \propto \Delta(t) \propto t^{0.33}$$
Time evolution

Time evolution of the energy profile:

- Mixing layer
- Position of the maximum of skewness

Total time in both cases: $\sim 22 \tau$
Velocity statistics

Evolution of the penetration $\eta = \frac{X_S}{\Delta}$

$2D \Rightarrow \eta(t)$ diverges

$3D \Rightarrow \eta(t)$ reaches a constant value : $\eta_{\text{max}}$

![Graph showing the evolution of $\eta$ over time for 2D and 3D conditions.](image)

- **2D**
  - $E_1/E_2 = 6.6$
  - $E_1/E_2 = 40$
  - $E_1/E_2 = 300$
  - $E_1/E_2 = 10^4$
  - $E_1/E_2 = 10^6$

- **3D**

(c) $\eta_{\text{max}} = \frac{x_{\text{max}}}{\Delta}$

- DNS $Re_\lambda = 45, 4\pi$ domain
- DNS $Re_\lambda = 45, 8\pi$ domain
- V&W, Ref.[2]
- Briggs et al., Ref.[3]
Proposal of a memory measure as a global quantity referred to its own time derivative, for example

\[ \text{MEM} = \frac{\Delta}{\Delta'} \]

2D : \( \frac{d\Delta(t)}{dt} \sim t^{-0.3} \),  3D : \( \frac{d\Delta(t)}{dt} \sim t^{-0.67} \)

2D : \( \text{MEM} = \frac{\Delta(t)}{\Delta(t)_t} \sim 1.4t \),  3D : \( \text{MEM} = \frac{\Delta(t)}{\Delta(t)_t} \sim 3t \)

Different dimensionality, same trend (qualitative universality?), with a different coefficient

3D has a slightly longer memory than 2D
Conclusions

Comparison between the 2D and 3D situation:

**Similarities:**
- $\Delta(t)$ evolves asymptotically in time as a power law
- A strong intermittency $\rightarrow$ visible on the high order moments

**Differences:**
- Mixing is faster in 2D
- No autosimilarity in time in the 2D case
Conclusions

Comparison between the 2D and 3D situation:

Similarities:
- $\Delta(t)$ evolves asymptotically in time as a power law
- A strong intermittency $\rightarrow$ visible on the high order moments

Differences:
- Mixing is faster in 2D
- No autosimilarity in time in the 2D case

Possible explanation:

The evolution of $\Delta(t)$ is essentially led by the large scales
2D $\rightarrow$ energy tends to concentrate to the large scales (inverse cascade)