Does the Kolmogorov scaling bridge hydrodynamic linear stability and turbulence?

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Energy spectrum in fully developed turbulence

- Phenomenology of turbulence **Kolmogorov 1941**: 
  \(-5/3\) power-law for the energy spectrum over the inertial range;
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Energy spectrum in fully developed turbulence

- Phenomenology of turbulence **Kolmogorov 1941**: $-5/3$ power-law for the energy spectrum over the inertial range;
- Common criterium for the production of a fully developed turbulent field to verify such a scaling (e.g. Frisch, 1995; Sreenivasan & Antonia, ARFM, 1997; Kraichnan, Phys. Fluids, 1967).

Energy spectrum and linear stability analysis

- We study the state that precedes the onset of instability and transition to turbulence:
Energy spectrum and linear stability analysis

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  - To understand how spectral representation can effectively highlight the nonlinear interaction among different scales;
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• The set of small 3D perturbations:
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  - Includes all the processes of the perturbative Navier-Stokes equations;
  - Leaves aside the nonlinear interaction among the different scales;
- The perturbative evolution is ruled by the initial-value problem associated to the Navier-Stokes linearized formulation.
Spectral analysis through initial-value problem

- The linear transient dynamics offers a great variety of very different behaviors (*Scarsoglio et al.*, 2009, 2010, 2011):
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- Understand how the energy spectrum behaves;

- Is the linearized perturbative system able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument?
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⇒ Understand how the energy spectrum behaves;

• Is the linearized perturbative system able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument?

• We determine the energy decay exponent of arbitrary perturbations in their asymptotic states and we compare it with the -5/3 Kolmogorov decay.
Perturbation scheme

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Perturbation scheme

- Laplace-Fourier (wake) and Fourier-Fourier (channel) transform in the $x$ and $z$ directions.
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- Laplace-Fourier (wake) and Fourier-Fourier (channel) transform in the $x$ and $z$ directions.
Perturbative equations

- Perturbative linearized system:

\[
\frac{\partial^2 \hat{v}}{\partial y^2} - k^2 \hat{v} = \hat{\Gamma} \\
\frac{\partial \hat{\Gamma}}{\partial t} = i\alpha \left( \frac{d^2 U}{dy^2} \hat{v} - U \hat{\Gamma} \right) + \frac{1}{Re} \left( \frac{\partial^2 \hat{\Gamma}}{\partial y^2} - k^2 \hat{\Gamma} \right) \\
\frac{\partial \hat{\omega}_y}{\partial t} = -i\alpha U \hat{\omega}_y - i\gamma \frac{dU}{dy} \hat{v} + \frac{1}{Re} \left( \frac{\partial^2 \hat{\omega}_y}{\partial y^2} - k^2 \hat{\omega}_y \right)
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The transversal velocity and vorticity components are \( \hat{v} \) and \( \hat{\omega}_y \) respectively, \( \hat{\Gamma} \) is defined as \( \hat{\Gamma} = \partial_x \hat{\omega}_z - \partial_z \hat{\omega}_x \).
Perturbative equations

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The transversal velocity and vorticity components are $\hat{v}$ and $\hat{\omega}_y$ respectively, $\hat{\Gamma}$ is defined as $\hat{\Gamma} = \partial_x \tilde{\omega}_z - \partial_z \tilde{\omega}_x$.

• Initial conditions: symmetric and asymmetric inputs;
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- Initial conditions: symmetric and asymmetric inputs;
- Boundary conditions: \( (\hat{u}, \hat{v}, \hat{w}) \to 0 \) as \( y \to \pm\infty \) and at walls.
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Perturbation energy

- **Kinetic energy density** $e$:

$$e(t; \alpha, \gamma) = \frac{1}{2} \int_{-y_d}^{+y_d} (|\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2) dy$$
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- Amplification factor $G$:
  $$G(t; \alpha, \gamma) = \frac{e(t; \alpha, \gamma)}{e(t = 0; \alpha, \gamma)}$$
Perturbation energy

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  $$r(t; \alpha, \gamma) = \frac{\log[e(t; \alpha, \gamma)]}{2t}$$
Perturbation energy

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  \[
  r(t; \alpha, \gamma) = \frac{\log[e(t; \alpha, \gamma)]}{2t}
  \]

- Angular frequency (pulsation) \( \omega \):
  \[
  \omega(t; y = y_0, \alpha, \gamma) = \frac{d\varphi(t; y = y_0, \alpha, \gamma)}{dt}, \quad \varphi \text{ time phase}
  \]
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Spectral representation

- The energy spectrum is evaluated as the wavenumber distribution of the amplification factor, $G(k)$.
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- The spectral representation is determined by comparing the energy of the waves when they are exiting their transient state.
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- The energy spectrum is evaluated as the wavenumber distribution of the amplification factor, $G(k)$;
- The spectral representation is determined by comparing the energy of the waves when they are exiting their transient state;
- Every perturbation has a characteristic transient exit time, $T_e$;
Spectral representation

- The energy spectrum is evaluated as the wavenumber distribution of the amplification factor, $G(k)$;
- The spectral representation is determined by comparing the energy of the waves when they are exiting their transient state;
- Every perturbation has a characteristic transient exit time, $T_e$;
- The asymptotic condition is reached when the perturbative wave exceeds the transient exit time, $T_e$, that is when $r \sim \text{const}$ is satisfied for stable and unstable waves.

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Energy $G(k)$ at the asymptotic state ($r \sim \text{const}$)

Present results (Re values: see panels)
- $\triangle$ Asym ic, $\phi=\pi/2$
- $\triangle$ Asym ic, $\phi=\pi/4$
- $\triangle$ Asym, $\phi=0$
- $\bigcirc$ Sym ic, $\phi=\pi/2$
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- $\square$ *(E_x, E_y, E_z)* Moser et al. (MKM), 1999, Re=12390, DNS
- $\blacksquare$ Saddoughi & Veeravalli (SV), 1993, Re*=4994, lab data
- $\star$ *(E_u, E_v)* Ong & Wallace (OW), 1996, Re=3900, lab data
- $\blacktriangle$ Cerutti & Meneveau (CM), 2000, Re=572080, DNS & LES

Channel flow
Re=500

Channel flow
Re=10000
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Concluding remarks

• Spectrum determined by evaluating the energy of the waves when they are exiting their transient state;
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• Regardless the symmetry and obliquity of perturbations, there exists an intermediate range of wavenumbers in the spectrum where the energy decays with the same exponent observed for fully developed turbulent flows ($-5/3$), where the nonlinear interaction is considered dominant;
Concluding remarks

- Spectrum determined by evaluating the energy of the waves when they are exiting their transient state;
- Regardless the symmetry and obliquity of perturbations, there exists an intermediate range of wavenumbers in the spectrum where the energy decays with the same exponent observed for fully developed turbulent flows \((-5/3)\), where the nonlinear interaction is considered dominant;
- Scale-invariance of \(G\) and \(T_e\) at different (stable and unstable) Reynolds numbers and for different shear flows;
Concluding remarks

• Spectrum determined by evaluating the energy of the waves when they are exiting their transient state;
• Regardless the symmetry and obliquity of perturbations, there exists an intermediate range of wavenumbers in the spectrum where the energy decays with the same exponent observed for fully developed turbulent flows \((-\frac{5}{3})\), where the nonlinear interaction is considered dominant;
• Scale-invariance of \(G\) and \(T_e\) at different (stable and unstable) Reynolds numbers and for different shear flows;
• The \(-\frac{5}{3}\) spectral power-law scaling of inertial waves seems to be a general and intrinsic dynamical property of the NS solutions encompassing the nonlinear interaction.
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Coming next...

• Analysis of the perturbation transient dynamics in the 2D and 3D boundary layer (W. O. Criminale, University of Washington);

⇒ Study of the intermediate and long term asymptotics.

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Coming next...

• **Analysis of the perturbation transient dynamics in the 2D and 3D boundary layer** (*W. O. Criminale, University of Washington)*;

• **Analytical integration of the kinetic energy equation based on the perturbed velocity and vorticity field** (*G. Staffilani, MIT*)

⇒ Study of the intermediate and long term asymptotics.