Small scale localization in the Large Eddy Simulation of compressible dishomogeneous turbulent flows

D. Tordella[1], M.Iovieno[1], S.Massaglia[2]

[1] Dipartimento di Ingegneria Aeronautica e Spaziale, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino, Italy

[2] Università degli Studi di Torino, Dipartimento di Fisica Generale
Via P. Giuria 1, 10125 Torino, Italy

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Motivation: LES for high Re turbulent flows

- Compressible turbulent flows may have very high Reynolds numbers.
  e.g.: astrophysical jets have $Re > 10^{10} \div 10^{13}$ and M up to $50 \div 100$
- hopefully, the large scales only may be simulated
  $\Rightarrow$ explicit LES modeling is needed.
- regions where turbulence is fully developed do not fill the entire domain
  $\Rightarrow$ a tool to detect such regions is needed to insert LES models
- LES and shock capturing are not compatible (Ducros, 1999)
  $\Rightarrow$ explicit numerical dissipation
  $\Rightarrow$ detection sensors are necessary to locate the shock regions, where it is opportune to suppress LES subgrid terms

Detection of turbulent regions with small (under-resolved) scales

The only model that attempts to locate regions under-resolved turbulent regions is the *Selective Structure Function* model by Lesieur *et al.* (1996-1999). It is based on:

$$f(\langle \boldsymbol{\omega} \rangle) = \frac{\langle \boldsymbol{\omega} \rangle \cdot \langle \langle \boldsymbol{\omega} \rangle \rangle_{2\delta}}{| \langle \boldsymbol{\omega} \rangle | \, | \langle \langle \boldsymbol{\omega} \rangle \rangle_{2\delta} |} \in [-1, 1]$$

when f is close to 1 $\Rightarrow$ no subgrid terms
when f is far to 1 $\Rightarrow$ subgrid terms are inserted into filtered equations

Problems:

- only the disalignement of vorticity vector on scale δ is used.
- it is not easy to define a threshold, which in turn seems to depend on the resolution.

Small scale localization criterium

When the flow is turbulent and not fully resolved, the smallest resolved scales in the simulation:

- are highly three-dimensional
- they are within the inertial range, and then:
  - ◇ have significant level of energy
  - ◇ non linear terms are important

So we consider the following functional

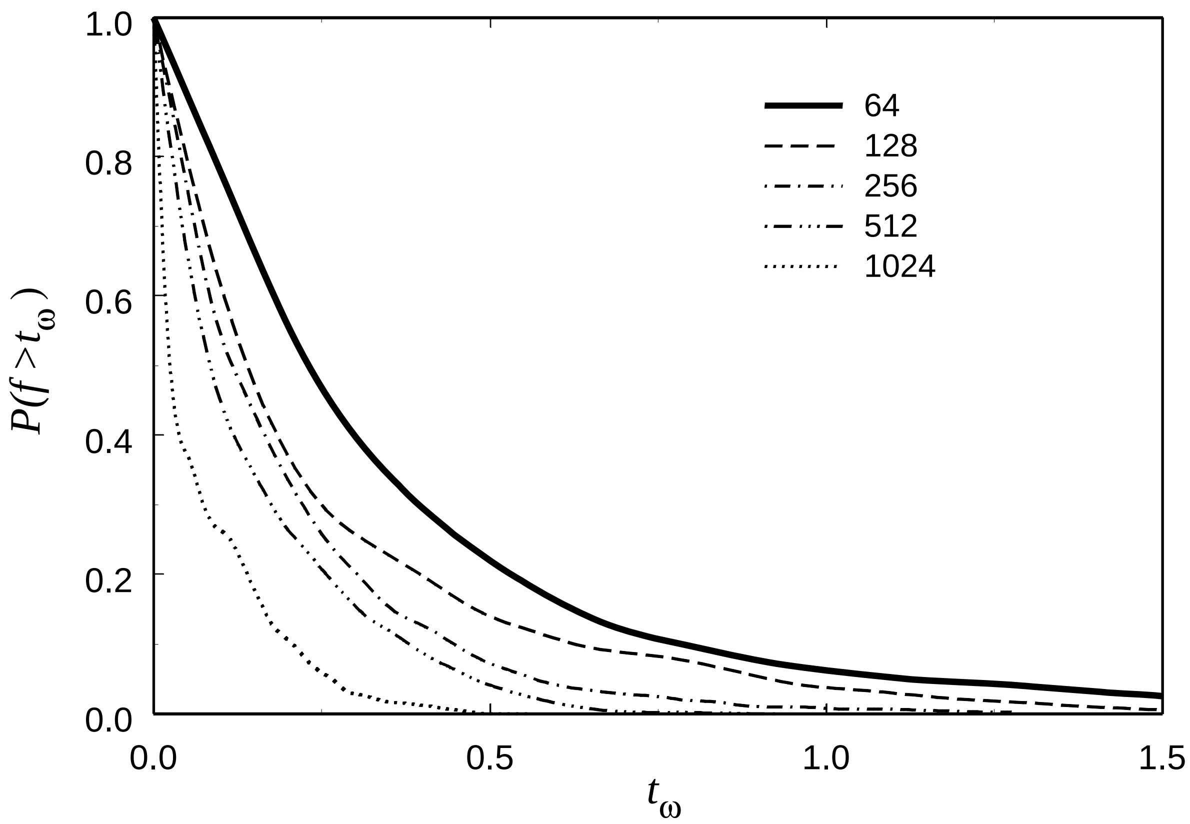
$$f(\langle \mathbf{u} \rangle, \langle \boldsymbol{\omega} \rangle) = \frac{|\ \langle \boldsymbol{\omega} \rangle \cdot \nabla \langle \mathbf{u} \rangle\ |}{|\ \langle \boldsymbol{\omega} \rangle\ |^2 + \varepsilon_0} \geq 0$$

Main features:

- information from only one filtering level – with filter scale Δ – is used.
- it is always positive, ranging from 0 when there is no flow to high values when there is high stretching in relation to enstrophy
- small constant $\varepsilon_0 > 0$ is used only to assure that $f \longrightarrow 0$ when $\boldsymbol{\omega} \longrightarrow 0$.

A priori test: homogeneous isotropic turbulence

The distribution of functional f has been tested on a turbulent field coming from a DNS at $Re = 280$, filtered at different resolutions: the probability that f is larger than a given threshold t_ω is shown.

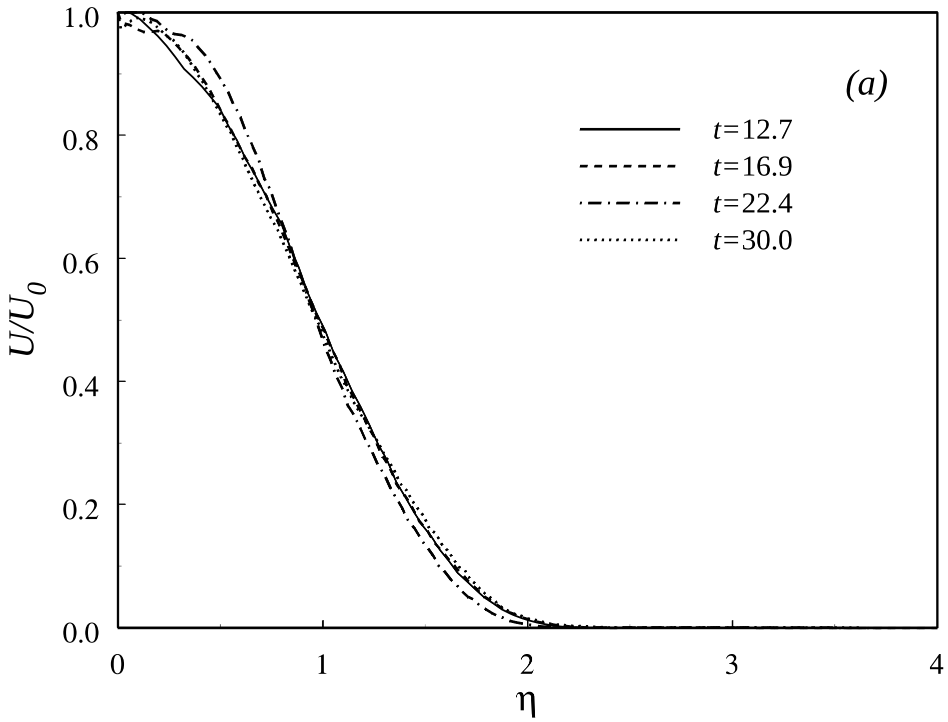


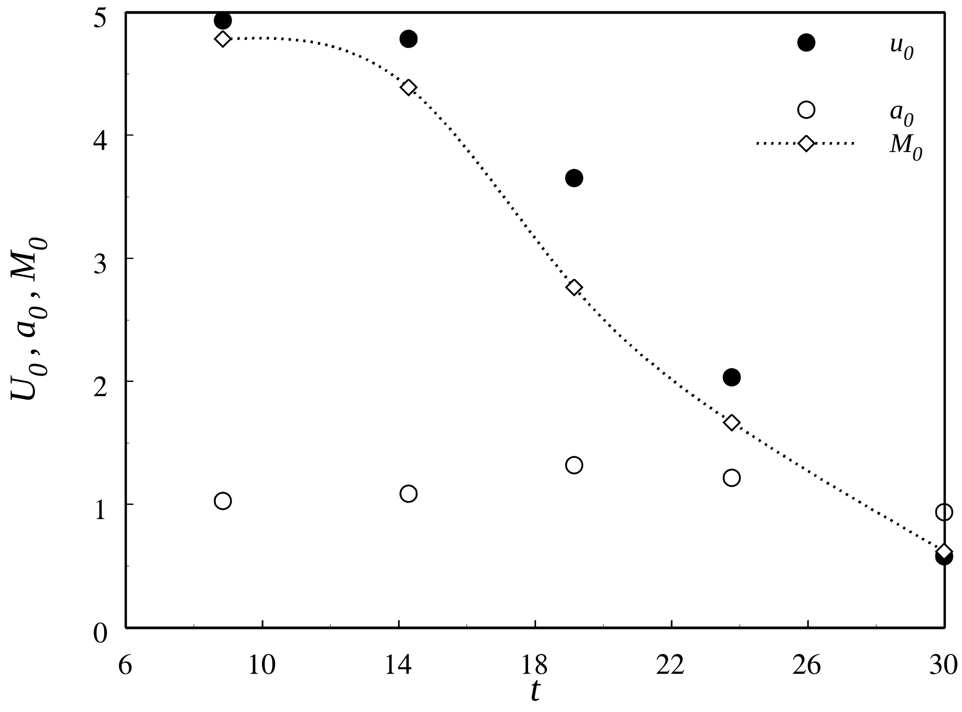
- Larger values of f have higher probability in less resolved fields.
- It is possible to define a threshold t_ω such that turbulence can be considered fully resolved when $f < t_\omega$ and unresolved when when $f > t_\omega$
- $t_\omega \approx 0.3$

Application to compressible jets simulations

Functional f has been applied to results from numerical simulations of the temporal evolution of a 3D cylindrical jet:

- Euler equations with PPM finite-volume numerical method, two resolutions: 128^3 and 256^3 .
- Initial conditions: uniform jet with unstable perturbations.
- Initial density ratio 0.1, initial jet Mach number $M = 5$.

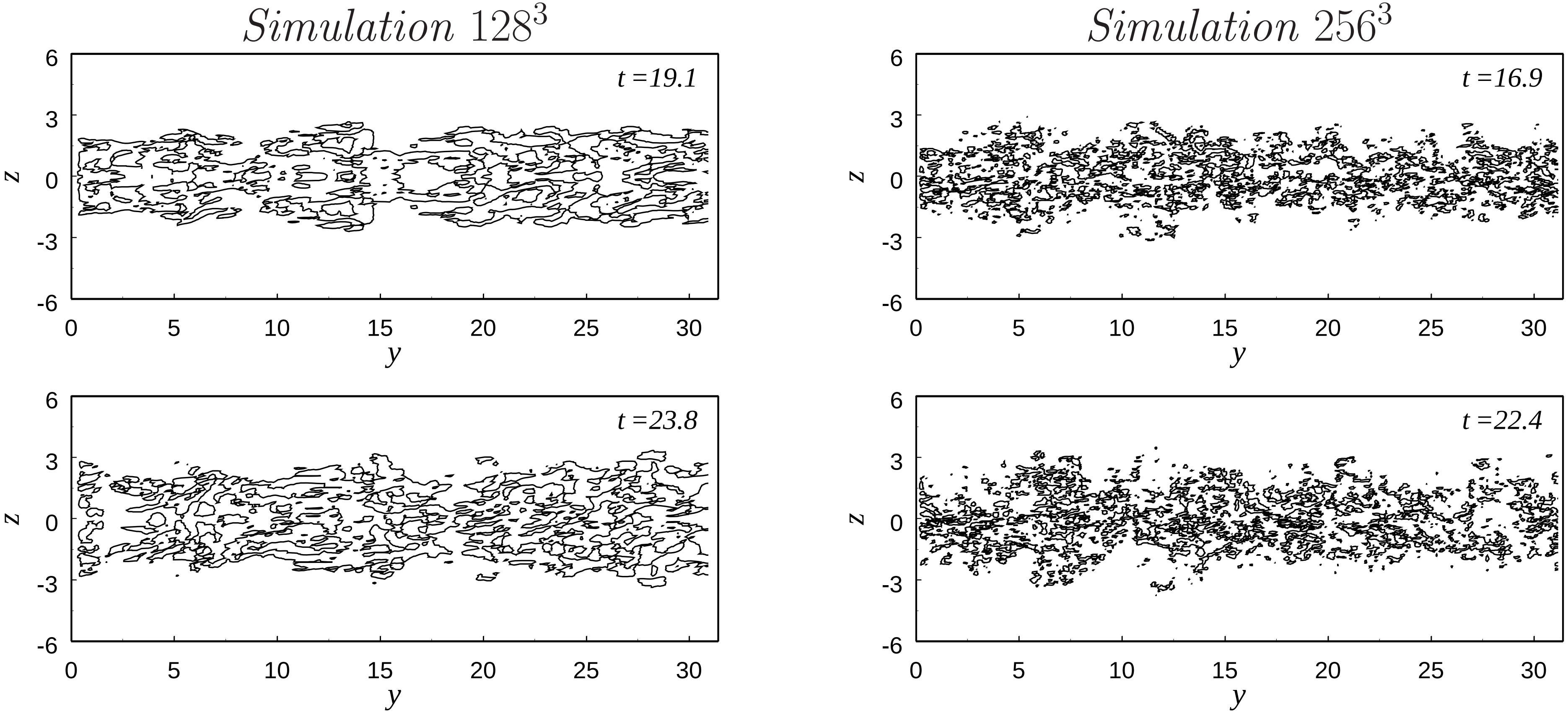




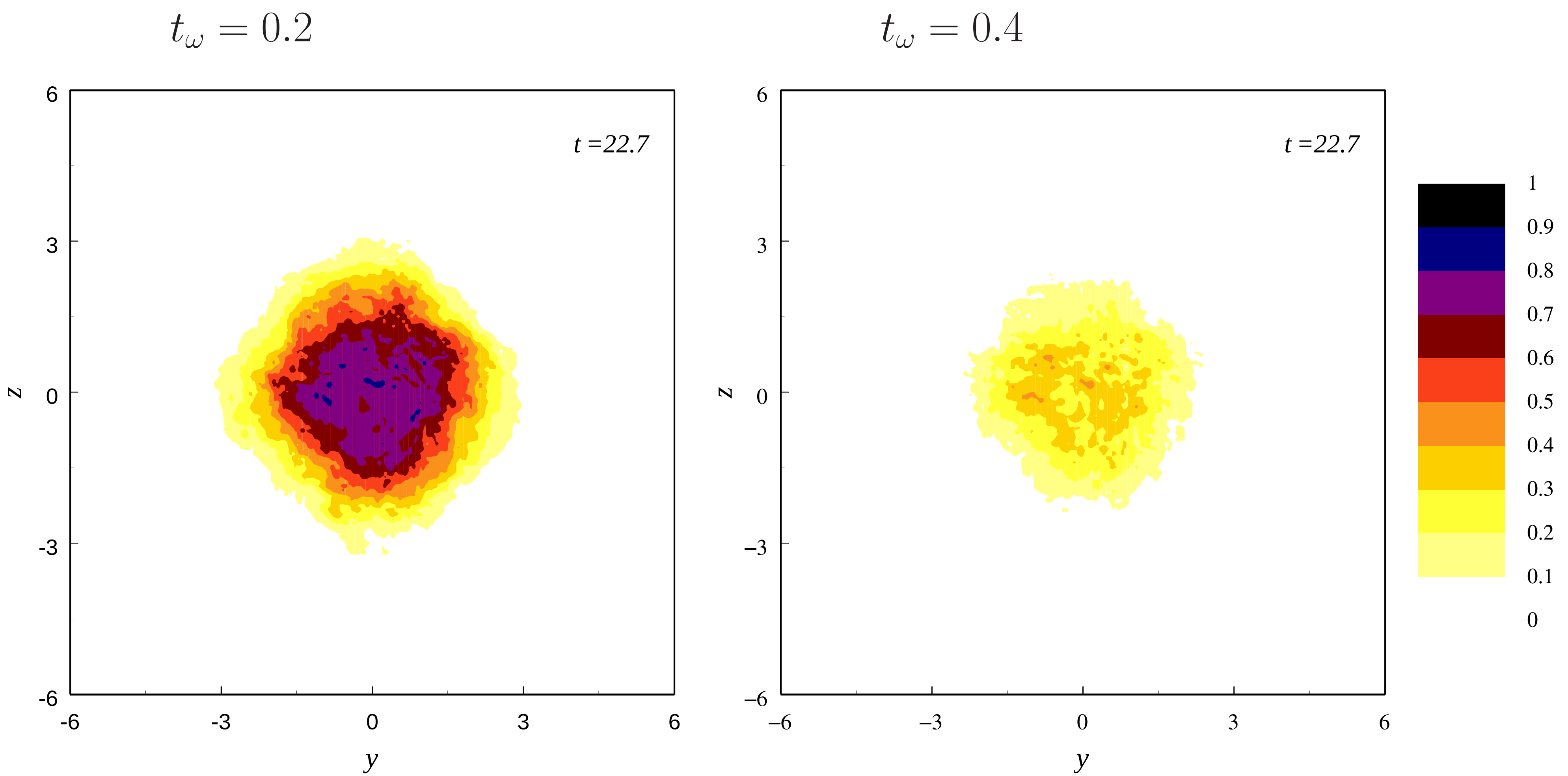
Distribution of functional f:

Areas where small, under-resolved, scales are present according to the criterium

$$f(\langle \mathbf{u} \rangle, \langle \boldsymbol{\omega} \rangle) > t_\omega, \quad t_\omega = 0.4$$



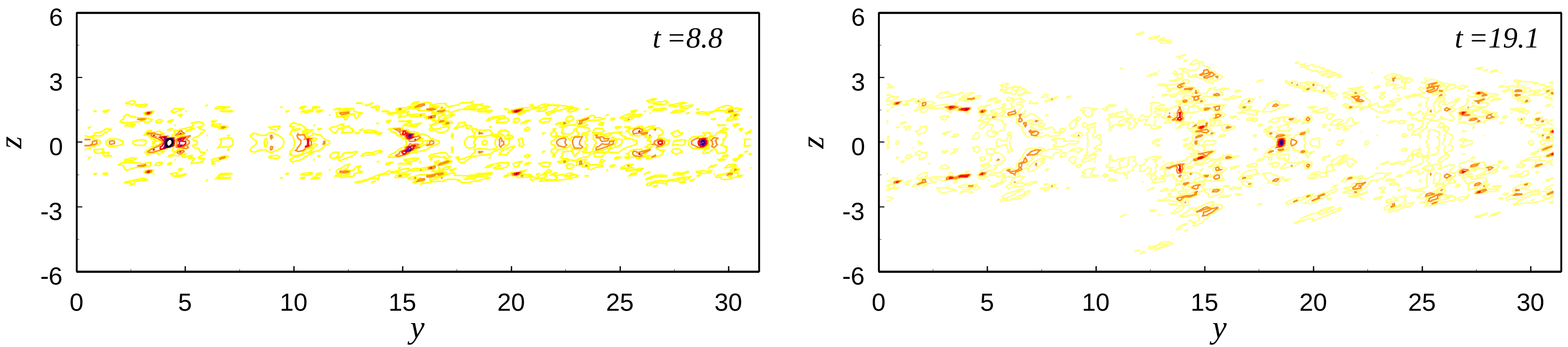
Fraction of space where sub-filter scales are present according to a selected threshold t_ω on the functional f (simulation 256^3, $t = 22.7$):



Shock detection

LES and numerical dissipation are not compatible, so that numerical dissipation should be explicit.
Following shock sensor proposed by Ducros *et al.* (1999) for the insertion of explicit numerical dissipation for shock capturing has been used:

$$s_j = \alpha\beta_j, \quad \alpha = \frac{(\nabla \cdot \langle \mathbf{u} \rangle)^2}{(\nabla \cdot \langle \mathbf{u} \rangle)^2 + \langle \omega \rangle^2}, \quad \beta_j = 4\left|\frac{\langle p \rangle_{j+1} - 2\langle p \rangle_j + \langle p \rangle_{j-1}}{\langle p \rangle_{j+1} + 2\langle p \rangle_j + \langle p \rangle_{j-1}}\right|$$



• Strong compressions are present when the functional s is of order 1
• The identification of the regions where the shocks are present will then coincide with setting of the numerical artifcial dissipation and setting aside the subgrid scale model.

This can be accomplished by structuring the Euler pseudo-direct-simulation equations with terms of the kind

$$sD_a + (1 - s)H(f - t_\omega)\nabla \cdot \tau^{sgs}.$$

where H is the Heaviside step function.

Subgrid terms correction to jet pseudo-DNS simulations

We extimate the effect of the selective insertion of SGS terms on the jet spreading rate.

The ensemble average equation (Favre averages) for a *temporal* jet in the x direction is

$$\frac{\partial\overline{\langle v_x\rangle_F}}{\partial t} = -\frac{\partial\overline{\langle v_x\rangle_F\langle v_r\rangle_F}}{\partial r} - \frac{\overline{\langle v_x\rangle_F\langle v_r\rangle_F}}{r} + \frac{\partial\overline{\tau_{xr}^{sgs}}}{\partial r} + \frac{\overline{\tau_{xr}^{sgs}}}{r}$$

A self-similar stage of evolution is reached after 10 time scales, so that:

$$\overline{\langle v_x\rangle_F} = U_0(t)f(\eta), \quad -\overline{\langle v_x\rangle_F\langle v_r\rangle_F} = \overline{\tau} = \overline{\tau_0(t)}g(\eta), \quad \overline{\tau^{sgs}} = \tau_0^{sgs}(t)g^{sgs}(\eta)$$

where η is the similarity variable $\eta = r/\delta(t)$.

By inserting these similarity transformations in the longitudinal ensemble averaged balance, we obtain

$$f(\eta) - \frac{\delta' U_0}{\delta U_0'}\,\eta f'(\eta) = \frac{\overline{\tau_0(t)}}{\delta U_0'}\left[g'(\eta) + \frac{g(\eta)}{\eta}\right] + \frac{\overline{\tau_0^{sgs}(t)}}{\delta U_0'}\left[g^{sgs\prime}(\eta) + \frac{g^{sgs}(\eta)}{\eta}\right]$$

so that the temporal spreading rate is proportional to total stress, sum of the resolved Reynolds stress and of the sub-grid contribution:

$$\frac{d\delta}{dt} \propto \frac{\tau_0 + \tau_0^{sgs}}{U_0}$$

Consequently we can relate the spreading rate with and without the correction od SGS terms:

$$\frac{d\delta}{dt} = \left(\frac{d\delta}{dt}\right)_{nc} \frac{\tau_0 + \tau_0^{sgs}}{\tau_0}$$

where the index nc (non corrected) indicates the spreading obtained from the under-resolved direct numerical simulation.

Comparison with experimental results

With a Taylor hypothesis the *temporal* spreading rate is transformed into a *spatial* spreading rate:

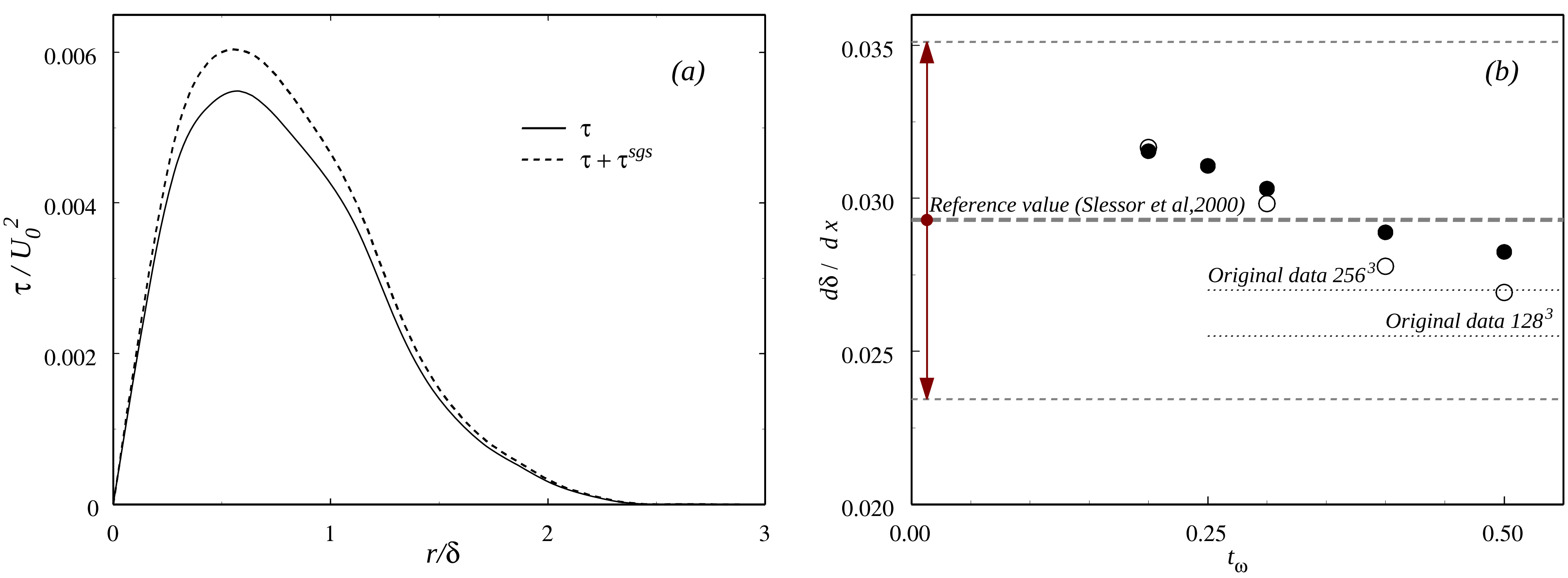
$$\frac{d\delta}{dx} = \frac{d\delta}{dt}\frac{dt}{dx} = \frac{1}{U_0}\frac{d\delta}{dt}$$

Experimental comparison has been deduced from the scaling for compressible shear-layer growth by Slessor *et al.* (2001) in terms of parameter:

$$\Pi_c = max_{(i=1,2)}\left[\frac{1-\gamma_i}{a_i}\right]\Delta U$$

where:
– ΔU is the difference of the mean velocity in the jet and out of the jet,
– index $i = 1, 2$ represents the gases inside and outside the jet,
– γ_i are the isoentropic coefficients
– a_i are sound velocities

and then corrected with the density ratio (Brown & Roshko, 1974)



(a)
τ
τ + τ^sgs
0.006
0.004
0.002
0
0
1
2
3
τ / U_0^2
r/δ
(b)
0.035
0.030
0.025
0.020
0.00
0.25
0.50
Reference value (Slessor et al,2000)
Original data 256^3
Original data 128^3
dδ / d x
t_ω

Final remarks

- A criterium for the localization of under-resolved regions in simulations of turbulent flows is proposed and is consistent with data from incompressible homogeneous turbulence.
- It is local
- It can be coupled with shock capturing numerical schemes with explicit artificial dissipation.
- *A priori* tests on a global parameter (the jet growth rate) show that the selective introduction of subgrid terms brings to a spreading rate nearer to the experimental reference value