Statistics of the interaction of two isotropic turbulent fields

7th European Fluid Mechanics Conference
Manchester, 14-18 September 2008

Daniela Tordella, Michele Iovieno

Dipartimento di Ingegneria Aeronautica e Spaziale
Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129 Torino, Italy
Turbulent shearless mixing


1-High energy turbulence
2-Low energy turbulence

Mixing layer
State of the art

- Grid turbulence experiments:
  - Gilbert *JFM* 1980
  - Veeravalli-Warhaft *JFM* 1989
State of the art

- Grid turbulence experiments:
  - Gilbert *JFM* 1980
  - Veeravalli-Warhaft *JFM* 1989

- Numerical experiments:
  - Briggs *et al.* *JFM* 1996
  - Knaepen *et al.* *JFM* 2004
  - Tordella-Iovieno *JFM* 2006
  - Iovieno-Tordella-Bailey *PRE* 2008)

- Periodic b.c.
- Temporal decay

$E(x, t)$

$\Delta(t)$
Main features

- High intermittency, function of:
  - gradient of turbulent kinetic energy
  - gradient of integral scale
- A gradient of kinetic energy is a sufficient condition for the onset of intermittency (*PRE* 2008)
- Intermittency is (*JFM* 2006)
  - *ENHANCED* if the energy gradient is concurrent with the integral scale gradient
  - *REDUCED* if the energy gradient is opposite to the integral scale gradient
Aim and Method

- **Aim**: to study the intermittency features of the large as well of the small scales
- Three different Reynolds numbers: $Re_\lambda = 45, 71$ and $150$.
- Energy ratio $\mathcal{E} = E_1/E_2$ from $6$ to $10^4$ with uniform integral scale.
- Velocity and velocity derivative statistics
- **Method**: DNS
  - parallelepiped domain, $2\pi \times 2\pi \times 4\pi$
  - Fourier-Galerkin pseudospectral space discretization
  - RK-4 time integration
Anisotropy of velocity statistics

$Re_\lambda = 45$

**Left:** second order moment anisotropy
Anisotropy of velocity statistics

\( Re_\lambda = 45 \)

Left: second order moment anisotropy
Right: triple moment anisotropy
Large scale intermittency

\[ Re_\lambda = 45, \quad \mathcal{E} = 6.7 \]

\[ S = \frac{u^3}{u^2}\frac{3}{2} \quad K = \frac{u^4}{u^2} \]

\[ S_{\text{max}}, \quad K_{\text{max}} = \text{maximum of Skewness and Kurtosis in the mixing layer} \]

\[ \eta_{\text{max}} = \text{position of the maximum in the mixing layer} \]
Large scale intermittency

\[ Re_\lambda = 150, \mathcal{E} = 6.7 \]

\[ S = \frac{u^3}{u'^2}^{3/2} \]

\[ K = \frac{u^4}{u'^2}^2 \]

\( S_{\text{max}}, \quad K_{\text{max}} = \text{maximum of Skewness and Kurtosis in the mixing layer} \)

\( \eta_{\text{max}} = \text{position of the maximum in the mixing layer} \)
Small scale intermittency

Velocity component in the mixing direction, longitudinal moments: \( E_1/E_2 = 6.7, \ell_1/\ell_2 = 1 \)
\( Re_\lambda = 45 \)

\( \eta = (x-x_c)/\Delta \)

\( \Delta \) is the mixing half-width

\( \eta \) is the dimensionless coordinate along the mixing
Small scale intermittency

Velocity component in the mixing direction, longitudinal moments: \( E_1 / E_2 = 6.7, \ell_1 / \ell_2 = 1 \)

\( Re_\lambda = 150 \)

\( \eta = x / \Delta \) is the dimensionless coordinate along the mixing

\( \Delta \) is the mixing half-width
Small scale intermittency

Velocity component normal to the mixing direction, longitudinal moments: \( E_1/E_2 = 6.7, \ell_1/\ell_2 = 1 \)
\( Re_\lambda = 45 \)

\( \eta \) is the dimensionless coordinate along the mixing
\( \Delta \) is the mixing half-width
Small scale intermittency

Velocity component normal to the mixing direction, longitudinal moments: $E_1/E_2 = 6.7$, $\ell_1/\ell_2 = 1$

$Re_\lambda = 150$

$\eta = x/\Delta$

$\Delta$ is the mixing half-width

$\eta$ is the dimensionless coordinate along the mixing
Asymptote for $E_1/E_2 \rightarrow +\infty$

Skewness:

$S_u$, $x$ in the mixing direction
Asymptote for $E_1/E_2 \rightarrow +\infty$

Kurtosis:

$u, x$ in the mixing direction
Longitudinal skewness

Scheme of the general behaviour

$S_{\partial v/\partial y}$

$S_{\partial u/\partial x}$

Velocity component normal to the mixing

Homogeneous turbulence

Component in the mixing direction
Longitudinal skewness

Comparison between the variation of the longitudinal derivative skewness of the component along the mixing and normal to the mixing

\[ \Delta S = \left| S_{mixing} - S_{HIT} \right| \]
Comparison with homogeneous turbulence

Comparison of longitudinal moments inside the mixing with longitudinal moments in homogeneous and isotropic turbulence

- **HIT, present simulations**
- **Shearless mixings, present simulations**
Comparison with homogeneous turbulence (II)

Comparison with the upper bound

\[-S_{\partial u/\partial x} \leq 2 \left( \frac{K_{\partial u/\partial x}}{21} \right)^{\frac{1}{2}}\]

of the longitudinal skewness in homogeneous turbulence
Conclusions

Over a range of energy ratios, for \( \text{Re}=45, 7 \leq \varepsilon \leq 10^4 \), and for \( \text{Re}_\lambda = 71 \) and 150, \( \varepsilon = 7 \), we observed:

- an intermittency increase with the energy ratio:
  - velocity \textbf{Skewness} and \textbf{Kurtosis} as large as 2.3 and 11, respectively
  - longitudinal derivative \textbf{Skewness} and \textbf{Kurtosis} as large as -5 and 50

- anisotropy quantitative data:
  - velocity: negligible for the second moments, significant for triple moments, for higher moments
  - longitudinal velocity derivatives (3\textsuperscript{rd} and 4\textsuperscript{th} moments): significant, it increases with the Reynolds number.
# Appendix: Shearless mixing statistics

<table>
<thead>
<tr>
<th>$\frac{u_1u_3u_5}{u_3^{3/2}}$</th>
<th>Velocity Kurtosis</th>
<th>Long. derivative skewness</th>
<th>Long. derivative kurtosis</th>
<th>Trans. Moments</th>
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<tbody>
<tr>
<td>$i=1$</td>
<td>$K_{u_1}$</td>
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<td>$i=3$</td>
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<td>$S_{\partial_3u_3}$</td>
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Mixings with $\ell_1/\ell_2 = 1$, $R_\lambda = 45$

<table>
<thead>
<tr>
<th>$E_1/E_2$</th>
<th>$\frac{u_1u_3u_5}{u_3^{3/2}}$</th>
<th>$K_{u_1}$</th>
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<tbody>
<tr>
<td>6.6</td>
<td>0.34 0.36 0.82</td>
<td>3.6 3.4 4.07</td>
<td>-0.11 -0.10 -1.04</td>
<td>5.0 4.85 6.95</td>
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<tr>
<td>40</td>
<td>0.54 0.59 1.34</td>
<td>5.6 6.0 5.66</td>
<td>0.52 0.70 -2.08</td>
<td>7.1 6.77 12.0</td>
<td>0.50 8.50</td>
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<tr>
<td>80</td>
<td>0.63 0.69 1.57</td>
<td>6.4 6.7 6.97</td>
<td>0.95 1.1 -2.60</td>
<td>8.5 8.6 17.1</td>
<td>0.60 13.0</td>
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<tr>
<td>300</td>
<td>0.78 0.87 1.91</td>
<td>7.5 8.2 8.93</td>
<td>1.5 2.0 -3.37</td>
<td>16 14 24.5</td>
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<tr>
<td>$10^4$</td>
<td>0.92 0.95 2.20</td>
<td>7.8 8.2 11.6</td>
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<td>20 26 37.3</td>
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Mixings with $\ell_1/\ell_2 = 1$, $R_\lambda = 71$

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<tr>
<th>$E_1/E_2$</th>
<th>$\frac{u_1u_3u_5}{u_3^{3/2}}$</th>
<th>$K_{u_1}$</th>
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<td>0.42 0.37 0.81</td>
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Mixings with $\ell_1/\ell_2 = 1$, $R_\lambda = 150$

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<td>6.7</td>
<td>-- -- 0.96</td>
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<td>-0.30 -0.28 -1.16</td>
<td>5.7 5.8 7.20</td>
<td>0.12 7.30</td>
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Veeravalli and Warhaft (1989), $E_1/E_2 \approx 7$, $\ell_1/\ell_2 \approx 1.5 \div 1.7$

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Legend:

$3 = \text{inhomogeneous direction}, 1,2 = \text{homogeneous directions}$

$S_{u_1}, K_{u_1} = \text{skewness and kurtosis of } u_i$

$S_{\partial_j u_1}, K_{\partial_j u_1} = \text{skewness and kurtosis of } \partial u_i/\partial x_j$