Activity on Momentum Transport

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Small scale localization in the Large Eddy Simulation of a compressible turbulent jet at $M = 5$. 

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Motivation: LES for high-Re turbulent flows

Compressible turbulent flows may have very high Reynolds numbers. For high-Re LES, a tool to detect such regions is needed to insert LES models where turbulence is fully developed do not fill the entire domain. LES and shock capturing are not compatible (Ducros, 1999). Explicit LES modeling is needed. Hopefully, the large scales only may be simulated.

Regions where turbulence is fully developed do not fill the entire domain. Explicit LES modeling is needed. LES subgrid terms is opportune to suppress LES subgrid terms, where it is necessary to locate the shock regions, where explicit numerical dissipation is expected sensors are necessary to locate the shock regions, where it is opportune to suppress LES subgrid terms.
The only model that attempts to locate regions under-resolved turbulent regions is the Selective Structure Function model by Lesieur et al. (1996-1999). It is based on:

$$f(\langle \omega \rangle) = \langle \omega \rangle \cdot \langle \langle \omega \rangle \rangle^2 \delta |\langle \omega \rangle| |\langle \langle \omega \rangle \rangle|$$

when $f$ is close to 1 $\iff$ no subgrid terms are inserted into filtered equations

when $f$ is far to 1 $\iff$ subgrid terms are inserted into filtered equations

$$E \in [I-1, I] \quad \frac{2\langle \langle m \rangle \rangle}{2\langle \langle m \rangle \rangle \cdot \langle m \rangle} = (\langle m \rangle)f$$

The only model that attempts to locate regions under-resolved turbulent regions with small (under-resolved) scale
Small scale localization criterium

When the flow is turbulent and not fully resolved, the smallest resolved scales in the simulation:

- are highly three-dimensional
- they are within the inertial range, and then:
  - have significant level of energy
  - non-linear terms are important

So we consider the following functional:

\[
0 < \frac{0 \varepsilon + \frac{1}{2} \left| \langle \mathbf{m} \rangle \right|}{\left| \langle \mathbf{n} \rangle_{\Delta} \cdot \langle \mathbf{m} \rangle \right|} = \left( \langle \mathbf{m} \rangle, \langle \mathbf{n} \rangle \right) f
\]

Main features:

- information from only one filtering level – with filter scale $\Delta$ – is used.
- it is always positive, ranging from 0 when there is no flow to high values when there is high stretching in relation to enstrophy.
- small constant $\varepsilon > 0$ is used only to assure that $f \rightarrow 0$ when $\omega \rightarrow 0$.

The smallest resolved scales, the smallest scales resolved in the simulation:

When the flow is turbulent and not fully resolved, the smallest resolved scales...
Prior test: homogeneous isotropic turbulence

The distribution of functional $f$ has been tested on a turbulent field coming from a DNS at $Re \approx 280$, where $f$ is filtered at different resolutions: the probability that $f$ is larger than a given threshold $t_\omega$ is shown.

$0.0 0.5 1.0 1.5$

$0.0$ $0.2$ $0.4$ $0.6$ $0.8$ $1.0$

$P(f > t_\omega)$

$64$ $128$ $256$ $512$ $1024$

• Larger values of $f$ have higher probability in less resolved fields.

• It is possible to define a threshold $t_\omega$ such that turbulence can be considered fully resolved when $f < t_\omega$ and unresolved when $f > t_\omega$.

• $t_\omega \approx 0.3$. 

When $f < t_\omega$ the $\omega$ and unresolved.

When $f > t_\omega$ the $\omega$ and unresolved.
Application to compressible jets simulations

Functional $f$ has been applied to results from numerical simulations of the temporal evolution of a 3D cylindrical jet:

- Euler equations with PPM finite-volume numerical method, two resolutions: $128^3$ and $256^3$.
- Initial conditions: uniform jet with unstable perturbations.
- Initial density ratio 0.1, initial jet Mach number $M = 5$.

Initial conditions: uniform jet with unstable perturbations.

Temporal evolution of a 3D cylindrical jet:

Application to compressible jets simulations.
Distribution of functional $f$:

Areas where small, under-resolved scales are present according to the criterion:

$$\forall 0 < \langle \mathbf{m}, \mathbf{n} \rangle_f$$

Simulation 128:

Simulation 256:

Simulation 256:

Simulation 128:

: $f$
Fraction of space where sub-filter scales are present according to a selected threshold $t_\omega$ on the functional $f$ (simulation 2563, $t = 22.7$):

$1 = 22.7$

$0 = 0.4$

$2 = 0.2$

Fraction of space where sub-filter scales are present according to a selected threshold $t_\omega$ on the functional $f$ (simulation 2563, $t = 22.7$):
Explicit numerical dissipation for shock capturing has been used:

\[ \frac{1 - I \langle d \rangle + I \langle d \rangle \neq + I + I \langle d \rangle}{1 - I \langle d \rangle + I \langle d \rangle \neq - I + I \langle d \rangle} \quad \forall = \mathcal{G} \quad \mathcal{Z} \langle m \rangle + \mathcal{Z} \langle \mathbf{n} \rangle \cdot \Delta = \alpha \quad \mathcal{G}/\alpha = \mathcal{S} \]

Following shock sensor proposed by Ducros et al. (1999) for the insertion of LES and numerical dissipation are not compatible, so that numerical dissipation should be explicit.
Strong compressions are present when the functional is of order 1. This can be accomplished by structuring the Euler pseudo-direct-simulation equations with terms of the kind:

\[ \Delta (\mathcal{M} \mathbf{f} - f) H(s - 1) + a \mathbf{D} s \]

where \( H \) is the Heaviside step function.

The identification of the regions where the shocks are present will then coincide with setting of the numerical artificial dissipation and setting aside the subgrid scale model.
We estimate the effect of the selective insertion of SGS terms on the jet spreading rate.

A self-similar stage of evolution is reached after 10 time scales, so that:

$$\frac{l_x}{\delta_s} \frac{\tau}{\tau_s} = \frac{\tau}{\tau_s} = \frac{\bar{u}\bar{u}}{\bar{u}\bar{u}} = \frac{\bar{u}\bar{u}}{\bar{x}}$$

The ensemble average equation (Favre averages) for a temporal jet in the $x$ direction is

$$\frac{d}{dt} \left( \frac{\bar{u}}{\bar{u}} \right) = \frac{\bar{u}\bar{u}}{\bar{x}} \frac{\tau}{\tau_s} = \frac{\bar{u}\bar{u}}{\bar{x}} \frac{\tau}{\tau_s} \frac{\bar{u}}{\bar{u}}$$

Subgrid terms correction to jet pseudo-DNS simulations.
By inserting these similarity transformations into the longitudinal ensemble averaged balance, we obtain

\[
\frac{\partial}{\partial t} \left( \frac{f(\eta)}{\delta U_0} \right) = \frac{\partial}{\partial \eta} \left( \frac{f(p)}{\partial p} \right)
\]

and

\[
\frac{\partial}{\partial t} \left( \frac{\tau(\eta)}{\delta U_0} \right) \propto \frac{\partial}{\partial \eta} \left( \frac{g'(\eta)}{\partial \eta} + \frac{g(\eta)}{\partial \eta} \right)
\]

so that the temporal spreading rate is proportional to total stress, sum of the resolved Reynolds stress and of the sub-grid contribution:

\[
\frac{\partial}{\partial t} \left( \frac{1}{\delta U_0} \tau(\eta) \right) \propto \frac{\partial}{\partial \eta} \left( \frac{1}{\partial \eta} \right)
\]

Consequently we can relate the spreading rate with and without the correction of SGS terms:

\[
\frac{\partial}{\partial t} \left( \frac{1}{\delta U_0} \tau(\eta) \right) = \left( \frac{\partial}{\partial t} \left( \frac{1}{\delta U_0} \tau(\eta) \right) \right)_{nc} + \left( \frac{\partial}{\partial t} \left( \frac{1}{\delta U_0} \tau(\eta) \right) \right)_{sg}
\]

where the index \text{nc} (non corrected) indicates the spreading obtained from the under-resolved direct numerical simulation.
Comparison with experimental results

With a Taylor hypothesis the temporal spreading rate (Brown & Roshko, 1974) is transformed into a spatial spreading rate:

\[ \frac{\partial t}{\partial t} = \frac{\partial t}{\partial x} \frac{\xi}{\xi} \]

Experimental comparison has been deduced from the scaling for compressible shear-layer growth by Slessor et al. (2001) in terms of the parameter:

\[ \Pi = \max_{i=1,2} \frac{1 - \gamma_i a_i}{\gamma_i - 1} \]

where:

- \( \Delta U \) is the difference of the mean velocity in the jet and out of the jet,
- \( \gamma_i \) are the isentropic coefficients,
- \( a_i \) are sound velocities, and then corrected with the density ratio (Brown & Roshko, 1974).
A criterium for the localization of under-resolved regions in simulations of turbulent flows is proposed and is consistent with data from incompressible homogeneous turbulence. It is local. It can be coupled with shock capturing numerical schemes with explicit artificial dissipation. A priori tests on a global parameter (the jet growth rate) show that the selective introduction of subgrid terms brings a spreading rate nearer to the experimental reference value.