Turbulent anisotropic transport in a model cloud interface

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UFS Schneefernerhaus - Zugspitze
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Temporal evolpution of a cloud-clear air interface

Initial value problem
## Context

### Turbulent mixings

- A kinetic energy gradient creates an intermittent region (shearless mixing layer)
- It creates an additional compression of fluid elements in the direction of $\nabla E$ and a stretching in the other directions

### Cloud droplet collisions

- Warm cloud have more turbulent kinetic energy than the surrounding clear air ($\Rightarrow \nabla E$ at the interface)
- Above 30-40 $\mu$m droplet growth is mainly determined by collisions
- Droplets accumulate in regions with high strain
- Can a shearless mixing layer change the collision rate of droplets?
Working hypothesis

- Top/bottom cloud-clear air interfaces can be seen as turbulent shearless mixing regions
- The compression of fluid elements at small scale, typically met across a shearless mixing layer, may increase the collision rate and particle numerical density
- Considering the bottom interface, gravity favours droplet exit from the cloud. Large droplets may become rain.
The role of the integral scale inhomogeneity

Uniform kinetic energy, inhomogeneous scale

*Physica D*, 2012.
Passive scalar transport

- Physical problem
  - Turbulent shearless mixings
  - The effect of the stratification
- Appendix
- Proposed model
- Preliminary results

Diagram:
- High energy turbulence
- Low energy turbulence
- Energy flow
- Mixing layer
- Intermittent velocity fluctuations
- Intermittent scalar fluctuations
- Scalar interface
- Intermittent scalar fluctuations
Velocity derivative skewness

General behaviour

\[ \xi = \frac{\partial u_i}{\partial x_i}, \ i = x, y_1 \text{ and } y_2 \]

\[ (Re_\lambda = 150, \ t/\tau = 3.5) \]

Increase of fluid filaments compression in the energy gradient direction, reduction of fluid filaments compression in the other directions.
– we introduce collisions in the dynamics of the droplets, even a simple rough model of inelastic collisions

– a feedback on the flow is proposed
Droplet dynamics model - small scale DNS

**Droplet motion: Stokes drag & gravity**

\[
\frac{dx_k}{dt} = v_k, \quad \frac{dv_k}{dt} = \frac{u(x_k, t) - v_k}{\tau_p} + g
\]

**Evaporation-Condensation**

Each droplet can change its mass by condensation and evaporation:

\[
\frac{dR_k}{dt} = C \frac{\varphi(x_k, t) - 1}{R_k}
\]

where \( \varphi \) is the relative humidity, \( \varphi = \rho_v/\rho_{sat} \) (Mason, 1971)

**Collisions**

Droplets are assume to coalesce when \( |x_i - x_j| \leq R_i + R_j \):

\[
m_i + m_j = m^*, \quad m_i v_i + m_j v_j = m^* v^*
\]
Physical problem
Turbulent shearless mixings
The effect of the stratification
Appendix

The cloud-clear air interface
Proposed model
Preliminary results

\[ C = f(r, \kappa, D, \rho \ell) \]

- \( r \) = latent heat of evaporation condensation
- \( \kappa \) = thermal diffusivity in the air \((Pr \approx 0.7)\)
- \( D \) = diffusivity of vapour in air \((Sc \approx 0.5)\)
- \( \rho \) = density
- \( St = \tau_p / \tau_\eta = 2 \) corresponds to about 30\( \mu \)m
Flow model

Navier-Stokes, Boussinesq approximation, plus vapour transport

\[
\nabla \cdot \mathbf{u}' = 0
\]
\[
\frac{D\mathbf{u}'}{Dt} = -\nabla \tilde{p} \rho + \nu \nabla^2 \mathbf{u}' + \alpha g\theta' + \mathbf{f}
\]
\[
\frac{D\theta'}{Dt} = \kappa \nabla^2 \theta'
\]
\[
\frac{D\varphi}{Dt} = \kappa_v \nabla^2 \varphi + S\varphi
\]

Coupling (source) terms

\[
f = -\frac{1}{V_{I(x,\delta)}} \sum_{\mathbf{x}_k \in I(x,\delta)} m_k \frac{d\mathbf{v}_k}{dt} = -\sum_k m_k \frac{\mathbf{u}(\mathbf{x}_k, t) - \mathbf{v}_k}{\tau_p}
\]
\[
S\varphi = -\frac{1}{V_{I(x,\delta)}} \sum_{\mathbf{x}_k \in I(x,\delta)} \frac{1}{\rho_{sat}} \frac{dm_k}{dt} = -\frac{4\pi \rho L}{\rho_{sat} V_{I(x,\delta)}} \sum_k R_k (\varphi(\mathbf{x}_k, t) - 1)
\]
Preliminary results – Particle movement

Flow

\[ Re_\lambda \approx 50 \]
\[ E_1/E_2 = 6.7 \]

Particles

\[ N_p = 10^6, \ St = 2, \] collisions and coalescence
Preliminary results – Particle movement

Flow

\[ Re_\lambda \approx 50 \]
\[ E_1 / E_2 = 6.7 \]

Particles

\[ N_p = 10^6, \; St = 2, \]
collisions and coalescence
Physical problem
Turbulent shearless mixings
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Appendix

The cloud-clear air interface
Proposed model
Preliminary results

Particle density

\[ \langle n \rangle = \frac{t}{\tau} \]

- Shearless mixing, \( E_1/E_2 = 6.7 \)
- DNS \( Re_\lambda \approx 50 \)
- \( N_p = 10^6 \) particles
- collisions and coalescence
- \( St = 2 \)
Physical problem
Turbulent shearless mixings
The effect of the stratification
Appendix

Particle velocity

low energy
\( \frac{x_3}{L} = 1/4 \)

mixing
\( \frac{x_3}{L} = 1/2 \)

high energy
\( \frac{x_3}{L} = 3/4 \)
$P_{\geq k}(t) =$ fraction of particles which underwent at least $k$ collisions

$-dN_p/dt =$ collision rate (exponent $m \approx 1.56$)
The underlying shearless flow

General flow configuration:

periodic boundary condition $\Rightarrow$ 2 mixing layers
The underlying shearless flow

Shearless mixing layers show the following properties:

- no gradient of mean velocity, thus no kinetic energy production
- the mixing is generated by the inhomogeneity in the turbulent kinetic energy and integral scale
- the mixing layer becomes very intermittent at both large and small scales [Tordella-Iovieno *Phys.Rev.Lett.* 2011]
- the presence of a gradient of energy is a sufficient condition for the onset of intermittency [Tordella and Iovieno *JFM* 2006; Tordella et al. *Phys. Rev.* 2008]
- 2D and 3D mixings: different asymptotic layer thickness growth exponent
3D mixing: Self-similarity

\[ \frac{E_1}{E_2} = 6.7, \ell_1 = \ell_2 \]

\[ \Delta(t) \text{ is the conventional mixing layer thickness, } \Delta(t) \sim t^{0.46} \]
Large scale intermittency

\[ S = \frac{\overline{u^3}}{\overline{u^2}^{3/2}} \]

\[ K = \frac{\overline{u^4}}{\overline{u^2}^2} \]

\[ u = \text{velocity component in the mixing direction} \]

\[ S_{\text{max}}, K_{\text{max}} = \text{maximum of Skewness and Kurtosis in the mixing layer} \]

\[ \eta_{\text{max}} = \text{normalized position of the maximum in the mixing layer} \]

(Figures: sample data from simulations with \( E_1/E_2 = 6.7, \ell_1 = \ell_1, Re_\lambda = 45 \))
We define the penetration as the position of the maximum of the skewness normalized over the mixing layer thickness: \( \eta = \frac{x_s(t)}{\Delta(t)} \)
Velocity derivative

\[ Re_\lambda = 45 \]

\[ Re_\lambda = 150 \]
Velocity derivative skewness

General behaviour

\[ \xi = \frac{\partial u_i}{\partial x_i}, \; i = x, y_1 \text{ and } y_2 \]
\[ (Re_\lambda = 150, \; t/\tau = 3.5) \]

Increase of fluid filaments compression in the energy gradient direction, reduction of fluid filaments compression in the other directions
Small scale anisotropy

Shear flows: large transversal skewness
Shearless mixings: strong differentiation of the longitudinal skewness

(1) Sreenivasan-Antonia

(2,3) Warhaft-Shen
• PRE 2008, PRL 2011, JoT 2014
The role of the integral scale inhomogeneity

Uniform kinetic energy, inhomogeneous scale

*Physica D*, 2012.
Energy gradient generation

Different decay exponents of the homogenous regions
⇒ generation of an energy gradient
Skewness vs. Kurtosis during the decay

![Graph showing skewness vs. kurtosis during decay](image-url)
Velocity derivative
Longitudinal derivative Skewness and Kurtosis

\[ \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y} \]

\[ S_{\partial u/\partial x}, S_{\partial u/\partial y} \]

\[ \frac{\partial u/\partial x}{\partial v/\partial y} \]

\[ K_{\partial u/\partial x}, K_{\partial u/\partial y} \]

\[ \ell_1/\ell_2 = 1.5 \]
\[ \ell_1/\ell_2 = 2.1 \]
\[ \ell_1/\ell_2 = 2.8 \]
Velocity derivative

Longitudinal skewness vs. longitudinal kurtosis

Filled symbols $\partial u/\partial x$, empty symbols $\partial v/\partial y$
Conclusions - scale inhomogeneity

- different scales generate different decays and then an energy gradient concurrent to the scale gradient
- the transient lifetime of the kinetic energy gradient is almost proportional to the initial scale ratio
- intermittency in the interaction layer grows as the flow decays
- anisotropy and intermittency are, with a certain lag, spread also to small scales
- small scale anisotropy: strong differentiation of the longitudinal skewness but no transversal skewness
Passive scalar transport

Physical problem
- Turbulent shearless mixings
- The effect of the stratification

Appendix
- Velocity statistics
  - The role of the integral scale
  - Passive scalar transport

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Passive scalar concentration

- $t/\tau = 1$
- $t/\tau = 5$
- $t/\tau = 10$

2D flow

Re$_{\lambda}$ = 150

Re$_{\lambda}$ = 250

J. Turb. 2014

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Mean Scalar Diffusion

2D Mixing

3D Mixing

Energy ratio $E_1 / E_2 = 6.7$, Schmidt number = 1.
Scalar mixing layer thickness

2D Mixing

Scalar layer thickness: $\Delta \vartheta = x_{\vartheta=0.75} - x_{\vartheta=0.25}$

3D mixing: $\Delta \vartheta \sim t^{0.46}$, 2D mixing: $\Delta \vartheta \sim t^{0.7}$
Scalar variance and scalar flux

\[ \frac{(x - x_c)}{\Delta \theta} \]

\[ \frac{t}{\tau} \]

Scalar flow direction

Energy flow direction

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Scalar intermitteny
Small scale intermittency

\[ \frac{t}{\tau} \]

\[ \frac{S_\theta / \partial x}{\left( x - x_c / \Delta \theta \right)} \]

\[ \frac{K_\theta / \partial x}{\left( x - x_c / \Delta \theta \right)} \]

\[ Re_1 = 150 \ 250 \]

\[ t/\tau = 1 \ 5 \ 10 \ 12.5 \]

\[ \text{energy flow} \]

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Scalar spectra

2D flow

3D flow ($Re_\lambda = 250$)
Scalar transport - Conclusions

2D/3D Passive scalar diffusion across an energy step

- all moments profiles are skewed towards the higher kinetic energy region
- self-similar profiles of first and second order moments
- large intermittency and non-gaussian behaviour on both sides of the mixing, even where the scalar flux is small.
- larger asymmetry in moment distributions in 2D mixing
- 2D: no stretching, inverse cascade, long-range interaction which penetrate more against the energy gradient
Mixing in presence of stratification

Temporal decay of a vapor-clear air interface

Computational domain: $L_z \approx 12 \text{ m}$
Initial vapor/clean air interface: $\Delta \theta \approx 0.3 \text{ m}$
Computational grid: $1024^2 \times 2048$

standard temperature lapse rate
$G_0 \approx 0.0065 \text{ K/m}$

Unstable Stable

$z \approx 1000 \text{ m}$

E₂ - Lower kinetic energy
E₁ - Higher kinetic energy

(unsaturated) water vapor concentration transported as passive scalar
**Turbulence data (reference altitude 1000 m s.l.)**

High energy region $E_1$: $u_{\text{rms}} = 0.2 \text{ m/s}$, $\ell = 0.3 \text{ m}$, $Re_\lambda \approx 250$

$E_1/E_2 \approx 6.7$, $Pr = 0.72$, $Sc = 0.61$

---

**Parametric study on Initial Stratification**

<table>
<thead>
<tr>
<th>$\nabla \theta$ [K/m]</th>
<th>$\Delta \theta$ [K]</th>
<th>$N_{ic}$ [s$^{-1}$]</th>
<th>$Fr_T^2$</th>
<th>$Re_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.004</td>
<td>0.021</td>
<td>970</td>
<td>7</td>
</tr>
<tr>
<td>0.20</td>
<td>0.06</td>
<td>0.052</td>
<td>160</td>
<td>112</td>
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<tr>
<td>0.65</td>
<td>0.2</td>
<td>0.150</td>
<td>19</td>
<td>273</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>0.335</td>
<td>3</td>
<td>833</td>
</tr>
<tr>
<td>30.0</td>
<td>10.0</td>
<td>1.060</td>
<td>0.4</td>
<td>2635</td>
</tr>
<tr>
<td>-6.5</td>
<td>-0.2</td>
<td>/</td>
<td>-19</td>
<td>-273</td>
</tr>
<tr>
<td>-3.0</td>
<td>-1.0</td>
<td>/</td>
<td>-3</td>
<td>-833</td>
</tr>
</tbody>
</table>

$N_{ic} = \sqrt{\alpha g \frac{d\theta}{dx_3}}$ is the Brunt-Väisälä frequency

$Fr_T^2 = \frac{u'_{\text{rms}}^2}{N_{ic}^2 \ell^2}$ is the ratio between kinematic and buoyancy forces

$Re_b = \frac{\varepsilon N_{ic}^2}{\nu}$ is the ratio between diffusivity and buoyancy
Velocity and temperature variance $t/\tau = 6$

Stable cases
- Formation of a pit of kinetic energy (strong strat)
- Reduction of scalar fluctuation

Unstable cases
- Enhance of kinetic energy
- Mild reduction of scalar fluctuation

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Flow structure

Vertical Velocity ($Fr = 1.8$, $t/\tau = 8$)

Run flow visualization
High order moments $t/\tau = 6$

**Stable cases**
- General reduction of intermittency
- Strong stratification produces two velocity intermittent sublayers

**Unstable cases**
- Increase of velocity intermittency
- Negligible effects on scalar
- No changes in behaviour
Creation of a pit of energy in the centre of the mixing:

- Presence of two opposite mean turbulent kinetic energy gradients
- Very low energy inside the pit (reduced transport)
- The pit onset and intensity depend on the stratification level
Pit of kinetic energy

Creation of a pit of energy in the centre of the mixing:

- Presence of two opposite mean turbulent kinetic energy gradients
- Very low energy inside the pit (reduced transport)
- The pit onset and intensity depend on the stratification level

\[
Fr = 1.8
\]

\[
\frac{t/\tau}{\approx 1.0}
\]

\[
\frac{t/\tau}{\approx 2.0}
\]

\[
\frac{t/\tau}{\approx 4.0}
\]

\[
\frac{t/\tau}{\approx 6.0}
\]

\[
\frac{t/\tau}{\approx 8.0}
\]

\[
\frac{x_3}{L_3}
\]

\[
\frac{E_p}{E_m}
\]

\[
1 + 0.008(t/\tau)^{1.74}
\]

\[
1 + 0.013(t/\tau - 1.5)^{1.42}
\]
Instability growth factor

\[ Fr^2 = -19 \]

Growth factor

- Growth given by the ratio respect to the unstratified case
- \( \zeta = \frac{E_{Fr^2 = -3}}{E_{Fr = 31}} - 1 \)
- Instability effects becomes relevant after \( t = 2\tau \)
- The instability exponent is equal to 1.532

Computed values
Exponential fit \( \zeta = 0.7(t/\tau)^{1.532} \)
Dissipation

\[ Fr = 1.8 \]

Dissipation rate

- \( \varepsilon \) turbulence dissipation rate
  \[ \varepsilon = \frac{1}{2} \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

- In the energy pit the dissipation is higher than its isotropic value \( E^{3/2} / \ell \) (about 30%)

- Self-similarity in PDFs regardless of vertical position
Dissipation

\[ F r^2 = -19 \]

Dissipation rate

- \( \varepsilon \) turbulence dissipation rate
  \[ \varepsilon = \frac{1}{2} \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]
- Dissipation remains almost constant inside and outside the mixing region
- Self-similarity in PDFs regardless of vertical position
Entrainment

**Entrainment of dry air over time**

\[ w_e = \frac{d\zeta(\theta=0.25)}{dt} \]

- \( w_e \) is the vertical velocity of the water vapor front \( x_{3,1} \) (where mean vapor concentration is 25% of the maximum)
- Exponential decay for weak stratification
- Faster damping for strong stratification
- Slower damping for unstable stratification

**Entrainment velocity**

- \( Fr = 12.7 \)
- \( Fr = 4.4 \)
- \( Fr = 1.8 \)
- \( Fr = 0.6 \)
- \( Fr^2 = -19 \)
- \( Fr^2 = -3 \)
Mixing layer thickness

\[ \Delta \chi = x_{3,1} - x_{3,2} \]

- \( x_{3,1} \) is the water vapor front (where \( \langle \chi \rangle = 0.25 \))
- \( x_{3,2} \) is the clear air front (where \( \langle \chi \rangle = 0.75 \))
- Thickening stops on pit onset with mild transient for stable cases
- Overgrowth for unstable cases
Mixing layer thickness

Thickness of the vapor layer

- $\Delta \chi = x_{3,1} - x_{3,2}$
- $x_{3,1}$ is the water vapor front (where $\langle \chi \rangle = 0.25$)
- $x_{3,2}$ is the clear air front (where $\langle \chi \rangle = 0.75$)
- Thickening stops on pit onset with mild transient for stable cases
- Overgrowth for unstable cases
Conclusions - stable stratification

- Horizontally layered structure characterized by a low kinetic energy sublayer in case of local, stable, intense stratification (pit of energy)
- The pit of energy acts as a barrier and blocks entrainment
- Two highly intermittent regions with opposite local kinetic energy gradient
- This situation strongly reduces the entrainment of dry air

Conclusion - unstable stratification

- Exponential growth of the energy in the mixing region respect to the external region.
- Greater intermittency in the mixing layer
- Enhancing of the entrainment after an initial transient (when buoyancy forces overcome inertial forces)
- Faster thickening of the mixing layer
- No relevant differences in dissipation respect to unstratified cases
Dissipation

\[
\frac{\epsilon}{\langle \epsilon \rangle} \cdot \text{PDF}(\epsilon)
\]

\( Fr = 1.8 \)

- Pit
- High energy
- Low energy
- Main gradient
- Secondary gradient
• Thickening stops at the pit onset
• Milder transient for advected scalars
• Reduction for energy due to the enlargement of the pit width
  (vertical kinetic energy flux makes the gradient increasingly steeper)
A posteriori estimate of the velocity divergence following Durran (JFM 2007): \(-\partial_z \bar{\rho} u_z / \bar{\rho} = \nabla \cdot \mathbf{u}\)
Possible models

Mellado-Stevens *J. Atm. Sci.* 2014

\[
\nabla \cdot \mathbf{u} = 0
\]

\[
\frac{D\mathbf{u}}{Dt} = -\nabla \frac{p}{\rho_a} + \nu \nabla^2 \mathbf{u} - b(g/g)
\]

\[
\frac{D\chi}{Dt} = \kappa \nabla^2 \chi, \quad b = b(\chi)
\]

\(\chi = \text{mixture fraction, } b = \text{buoyancy}\)

Umid air:

\[
\nabla \cdot \mathbf{u} = 0 \tag{1}
\]

\[
\frac{Du}{Dt} = -\nabla \frac{p}{\rho_a} + \nu \nabla^2 \mathbf{u} + f \tag{2}
\]

\[
\frac{D\chi}{Dt} = \kappa \nabla^2 \chi - \dot{C}_{\text{cond}}, \quad \chi = \frac{\rho_v}{\rho_a} \tag{3}
\]

Water droplets:

\[
\frac{D^2 X_\beta}{Dt^2} = \frac{1}{\tau_\beta} \left[ \mathbf{u}(X_\beta, t) - \dot{X}_\beta \right] + g \quad \forall \beta = 1 \ldots N_p \tag{4}
\]

\[
r_\beta \frac{Dr_\beta}{Dt} = K (\varphi - 1), \quad \dot{C}_{\text{cond}} \propto \sum_{\beta \in \Delta} (\varphi(X_\beta, t) - 1) r_\beta \tag{5}
\]

- no energy equation (temperature) and no stratification;
- necessity to follow a huge number of particles;
- droplet collisions?
Lalas-Einaudi *J.Appl.Meteor.* 1973

Three phases system: $\rho_d$, $\rho_v$, $\rho_w$ “density” of dry air, vapour and water droplet phases

$$\partial_t \rho_k + \partial_j (u_j^{(k)} \rho_k) = \Gamma_k \quad \forall k$$

(6)

$\Gamma_k$ = phase change rates, $(\Gamma_w = \Gamma, \Gamma_v = -\Gamma, \Gamma_d = 0)$

$$(\rho_d + \rho_v) D_t u = -\partial_i (p_d + p_v) + (\rho_d + \rho_v) g + (\theta_d + \theta_v) + \Gamma (u - v) / 2$$

(7)

$$\rho_w D_t v = \rho_w g + \theta_w + \Gamma (u - v) / 2$$

(8)

$u = \text{fluid velocity}, \ v = \text{water droplet phase velocity}$

$$(\rho_d c_d + \rho_v c_v) D_t T = -(p_d + p_v) \nabla \cdot u + \Phi_d + \Phi_v + \nabla (\theta_d + \theta_v) \cdot (u - v) + \Gamma (c_v (T_w - T + L_v)$$

(9)

$$\rho_w D_t T_w = \Phi_w + \Phi_v + \Gamma |u - v|^2 / 8 - \Gamma L_v$$

(10)

atmosphere must be saturated at all heights at all times, otherwise $T_w$, $\rho_w$ and $p_w$ are not defined (different set of equations for undersaturated air!)
CSU-GCM model, e.g. Fowler at al. *J.Climate* 1996

$q_v, q_c, q_i = $ water vapour, cloud water and cloud ice content

$$\frac{\partial \pi^* q}{\partial t} + \nabla \cdot (\pi^* u q) + \frac{\partial}{\partial \sigma} (\pi^* u_y q) = \text{source terms} \quad \forall q$$

where $\sigma =$ stretched vertical coordinate (with the pressure), $\pi^*$ pressure scale (used in $\sigma$ definition)

$$\frac{\partial \pi^* \theta}{\partial t} + \nabla \cdot (\pi^* u \theta) + \frac{\partial}{\partial \sigma} (\pi^* u_y \theta) = \text{source terms}$$

$\theta =$ potential temperature

used in general circulation models; many modelled terms which account for many microphysical processes (including phase changes)!
**Physical problem**

Turbulent shearless mixings

The effect of the stratification

**Appendix**

Mixing layer thickness

Other ideas

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### Physical Problem

**Turbulent shearless mixings**

The effect of the stratification

**Appendix**

Mixing layer thickness

Other ideas

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Biona, Druilhet, Benech and Lyra **2001**


Lothon M, Lenschow D H and Mayor S D **2009**

Radkevich, Lovejoy, Strawbridge, Schertzer and Lilley **2008**

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**Graph: Logarithmic Plot**

- **Logarithmic Scale:**
  - $\log_{10}(E [m^3 s^{-2}])$ for $-4 \leq E \leq 2$
  - $\log_{10}(\kappa_z [m^{-1}])$ for $-5 \leq \kappa_z \leq -0.5$

- **Data Points:**
  - **Biona**
    - [elevation 1.3 ÷ 22 m]
  - **Katul**
    - (pineforest) [elevation 30 m]
  - **Katul**
    - (hardwood) [elevation 50 m]
  - **Lothon**
    - (lidar measurement) [altitude s.l. 1÷1.5 km]
  - **Radkevich**
    - (cirrus lidar measurement) [altitude s.l. 8 km]
  - **Radkevich**
    - (aerosol lidar measurement) [altitude s.l. 5 km]
  - **Present work** [altitude 1 km]

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**Legend:**

- **Green Line:** Biona
- **Red Dashed Line:** Katul (pineforest)
- **Red Dotted Line:** Katul (hardwood)
- **Blue Dashed Line:** Lothon (lidar measurement)
- **Pink Dotted Line:** Radkevich (cirrus lidar measurement)
- **Pink Dashed Line:** Radkevich (aerosol lidar measurement)
- **Black Solid Line:** Present work

**Label:** Range simulated around the cloud - clear air interface
Energy spectra exponent

$Fr = 12.7$

$Fr = 1.8$

Creation of a pit of energy in the centre of the mixing:

- Exponent evaluated in inertial range
- Very low energy inside the pit (reduced transport)
- The pit onset and intensity depend on the stratification level